

A NEW
COMPLEAT and UNIVERSAL
SYSTEM or BODY
OF
DECIMAL ARITHMETICK,

CONTAINING,

- I. The Whole *Doctrine of Decimal Numbers*, not only the Plain and Terminate, but also such as *Repeat or Circulate ad Infinitum*; and a Plain but Perfect *Management* of both, laid down and explained in all the *Fundamental Rules of Plain Arithmetick*, and by *Logarithms*.
- II. The Application and Use of *Decimal Arithmetick* in all the *Parts or Branches of Arithmetical Science*; viz. *Vulgar Arithmetick*, *Vulgar Fractions*, *Duodecimal*, and *Sexagesimal Arithmetick*; also in *Algebra* and *Logarithms*. In all which its *Excellency* and absolute *Necessity* is fully evinced.
- III. Its Application and Use in all such *Parts* of the *Mathematics* as absolutely require its *Assistance*; viz. *Plain Trigonometry*, and the Arts depending thereon; as, *Navigation*, *Fortification*, *Altimetry*, and *Longimetry*; Also the *Mensuration* of all Kinds of *Superficies and Solid Bodies*; and the Arts resulting therefrom; as, *Gauging*, *Surveying*, &c.
- IV. A *New and Compleat Set of Decimal Tables* never before published, shewing by Inspection the Value of all Kinds of Decimals (without the tedious Methods of Reductions hitherto used) to four or six Places of Figures; Also all the *Common Tables* very much *enlarged, corrected, and improved*; wherein all the *Circulating Numbers* are marked. With all other *Tables of Interest, Annuities, Exchange, &c.* necessary to render the Work *complete*.
- V. An exact and accurate *Canon of Logarithms* for natural Numbers. And thro' the Whole, several Things new and useful, not here express'd.

By BENJAMIN MARTIN. K

L O N D O N :

Printed for J. Noon, at the *White Hart*, near *Mercers Chappel*,
in *Cheapside*. M.DCC.XXXV.

The PREFACE.

I Presume 'tis entirely needless to write a Panegyric on the superlative Excellency of the noble Science of Decimal Arithmetick, since the World has already been sufficiently apprized thereof in the extream Benefit and Service it has afforded the World of Mathematical Literature, even tho' in its Embryo State; much less doth it need Encomiums to set forth its Nature, Worth, and valuable Properties, which have been discovered and illustrated by late Improvements; of which the ensuing Work is but (as it were) an Instance.

The Book I here present the World withal is a regular System of this valuable Art, according to all the latest Improvements of others, and many (in the several Parts thereof) of my own; the two greatest of which are, The Doctrine of infinite circulating Decimals by the learned Mr. Samuel Cunn; the other, A New Sett of Tables shewing the Value of any Decimal Part of any Integer, whether Money, Weight, Measure, Motion, Time, &c. by Inspection only, to a sufficient Exactness, without the tedious Reductions hitherto necessarily used; which cost me not a little Time and Pains to calculate, but was necessary to render this Art compleat.

The Foundation on which I have built this Superstructure is the abovementioned Gentleman's small, but learned, Treatise of the Doctrine of Decimal circulating Numbers: But that great Master having laid the Foundation deep, and in a great Measure out of the Vulgar Ken; I thought it might be of Service to young Students, a little to disclose and lay it more open to their View, and this was all I at first intended to do; but having done that, Materials came in so fast, that I went on and erected the System of Decimal Science thereupon, as you here see it; of which take the following Account.

The PREFACE.

In the Fundamental Rules of the Art, viz. Addition, Subtraction, Multiplication, and Division, I have been as plain as possibly I could without Prolixity, and shewn the compleat and perfect Management of both Plain and Circulating Decimal Numbers in each of the Rules, in so easy and obvious a Method as the meanest Capacity, with proper Attention, may comprehend; and have taken care, in its proper Place, to give the true Reason, or Rationale, of each particular and different Process, especially of those relating to Circulating or Repeating Numbers of all Kinds, as I went on, omitting nothing that I could communicate towards the perfecting this fundamental and important Part.

In Reduction I have perfected this Art beyond what it hath ever been; as I have not only largely exemplified all the common and vulgar Methods of Reducing to, and from Decimals, in all kinds of Vulgar, Fractional, Duodecimal, and Sexagesimal Numbers, by Arithmetical Operations; but have compleated the Tabular Part, which has been hitherto very deficient, both in the Tables already extant, and in the Want of others. The first of these Defects I have endeavoured to supply by correcting, enlarging, and compleating the common Tables for reducing the various Denominations of all Kinds of Quantities to Decimal Numbers; wherein I have taken care to mark all the Circulating or Repeating Numbers, Single or Compound, so far forth as they came within the Verge or Limits of the said Tables; which hath not till now been done by any. The other Defect, and that which renders this excellent Art most lame and imperfect, viz. The Want of proper Tables to express again the Value of a Decimal in the Vulgar Denominations, or known Parts of its Integer, without tedious and operose Arithmetical Reductions, I have also supplied by the Addition of a compleat Sett of such Tables, and shewn their Use in Examples of all Varieties. The Reader may have a farther Account of these Tables in the Place where they are inserted; of which I shall say no more, but that these are the first Tables of this Kind that were ever published.

*What I have hitherto said, relates purely to the Nature
and*

THE PREFACE.

and Substance of the Art it self; what follows concerns its Application to Arithmetical and Mathematical Sciences.

In Vulgar Arithmetick, I have applied the Doctrine of Decimal Numbers, both Plain and Circulating, in every Part; and shewn its admirable Use, Service, and Expediency thro' the Whole. Particularly I have facilitated the Business of Practice by a new Table and Method of Working by Decimals; whereby the Difficulty and the Intricacy of this Rule by common Arithmetick is avoided.

In Inchange I have been very particular and explicit, having made this Affair (the Basis of Merchandise) more intelligible to meaner Capacities than I have met with it; explained the Nature and Meaning of the Par and Course of Exchange, and have exhibited large and compleat Tables of both; such as are very rare to be found in Books of this Nature, tho' they are an indispensable Part of Decimal Arithmetick. The Rules of Interest, Simple and Compound, being of so great and general Concernment and Importance, and yet so little truly understood, I have taken abundance of Pains and Care to let young Students see the Theory, or true Reason and Nature thereof, by a perspicuous Method of Resolving the most excellent Theorems of the late Mr. Ward, in Decimal Numbers; which in this Part of Arithmetick are absolutely necessary. And to facilitate and expedite Calculations of Interest, I have procured a compleat Set of Decimal Tables of Interest, and shewn their Construction and Use. In the other Parts of common Arithmetick, I have been large and clear, and omitted Nothing that I could contribute to perfect them. In fine, in every Part I have shewn the Necessity and Expediency, and in the Whole, the Preference and superior Utility of Decimal Computations.

In Vulgar Fractions, I have shewn how all Questions are most easily and commodiously resolved by Decimals; and for that End have calculated a Table shewing by Inspection the Decimal equal to any Vulgar Fraction whose Denominator exceeds not Twenty. I have extended the Table no farther, because those small Fractions are most frequent and useful in Business.

THE PREFACE.

In Duodecimal and Sexagesimal Arithmetick, I have shewn the Nature and Rules of the Arts ; and, by many Examples, how Questions are most advantageously wrought by Decimal Numbers, especially in Duodecimals, so much used in Mensuration, I have also inserted compleat Tables for turning Duodecimal and Sexagesimal Numbers into Decimal ones, and such as are not to be found every where ; wherein (as in all my other Tables) I have noted the Circulating Decimals, which no one besides hath done.

In the excellent Art of Logarithms, I have not only explain'd the Principles and Rules of the Art it self, but largely shewn how all Kinds of Decimals are managed and ordered thereby ; others have taught the World the Management of plain or terminate Decimals by Logarithms ; But that of circulating or repeating Decimals of all Kinds, hath fallen to my Province only, since no one before has attempted it. I have here explain'd the Method of finding the Logarithms of any Repetend, whether single or compound, pure or mixed with absolute Numbers, with more Ease and Certainty than can be found for any terminate Decimal exceeding the Numbers in the Canon. I have shewn how to work all Sorts of Decimals in all the Rules of this noble Art ; and to make this Part of the Work compleat, I have inserted a Canon of Logarithms for Natural Numbers ; where by Rules you are taught to find the Logarithms of any Number not exceeding 10000000, and the contrary.

In Algebra, the Use and Necessity of Decimals in raising and resolving Equations, I have demonstrated in a Select Number of the most curious and useful Questions relating to the Theory of Arithmetick, to the Philosophy of Motion, &c. and shewn how lame and imperfect, even this most perfect and perfecting Art would be without the Subservience of Decimal Numbers.

Hitherto of the Use and Application of Decimals in the several Parts of Arithmetical Science ; in the next Place take what concerns its Application to the Principal Parts of Mathematical Knowledge.

*In Plain Trigonometry, I have convinced the Reader how absolutely necessary Decimals are in order to express
the*

The P R E F A C E.

the Quantity of the Sides of all Right-lined Triangles, in the Solution of all the Cases of Right and Oblique-angled Trigonometry. And as this Art is the Foundation and Effence of several others, as Fortification, Navigation, Mensuration of Altitudes and Distances, &c. I have likewise shewn the Nature and Rudiments of those Arts and Sciences, and the Manner of performing Conclusions by them in Decimal Numbers. So that any Person may here both learn the Art of Trigonometrical Calculation, and its Application to the aforesaid Arts, after the best Method, with the same Ease and to as good Purpose, as from many Books wrote purposely on the Subject.

In the Mensuration of Superficies and Solids, no one will pretend to dispute the Superlative Use of Decimal Arithmetick; whereof the small Tract I have here publish'd is a sufficient Instance. I have not only taught how to measure more Superficies and Bodies than any other one Book that I know of, but shew'd how this Art is the very Basis and Substance of Gauging, Surveying, and all other Kinds of Measurements used by Artificers, none of which can be obtain'd to any good Purpose without it; nor any Operations therein so well perform'd as by Decimal Arithmetick; and here I have provided the Gauger with a Table of Multipliers or Divisors for finding the Content of any Superficies, or the Capacity of any Vessel in Wine Gallons, Ale Gallons, Corn Gallons or Corn Bushels, whether the Dimensions be taken in Inches, Feet, or Yards.

Thus I have given a general Account of the Substance of the Book; it would be endless to descend to Particulars: Many Things of Importance in various Parts of the Book will offer themselves to the View of the Reader unexpectedly, and appear in the whole, with the Face of Novelty. I have spared no Pains in Consulting the best modern Authors on each particular Head as I went on, and extracted from them severally whatever I found of value and Worth and would contribute to perfect my Design; so that nothing of Consequence can be found in any other Piece of Decimal Fractions (as this noble Art has been diminutively called)

THE PREFACE.

led) but what may be here met with amidst a great Variety of other novel, but useful and curious Matters.

So that upon the Whole I hope this Book doth truly merit the Title it bears, viz. A New Compleat and Universal System or Body of Decimal Arithmetick.

If then any Person be desirous of a Good and thorough Knowledge of Decimal Numbers of every Kind, and of their Compleat and perfect Management by the Rules of Art, they may be here satisfied. If they would learn its Application, or how to use it to the best Advantage in the various Arts, Trades, and Business of Life, they will here meet with plentiful Instructions, and Examples in every Sort, adapted to particular Cases. Would they learn the true Grounds, or Rationale, of all Arithmetick whether Vulgar, Fractional, Duodecimal, Sexagesimal; and of the Mathematical Arts, Mensuration, Gauging, Surveying, Navigation, Fortification, Altimetry, Longimetry, &c. let them please to spend a few of their spare Hours here. Are they disposed to learn the Use of Logarithms, or the Method of Trigonometrical Calculations, they are here with Ease informed. In short, they may here find in one small Volume, what I have been obliged to turn over many both small and great to procure; and therefore if Variety, Utility, Novelty and Brevity can please, I hope the Publick will candidly accept my Labours. I am not apprised of any Faults therein, and am very sure there are but few material Ones, having taken all the Care and Pains I was able, to prevent them.

But if the well dispos'd and inquisitive Reader, after having perused it thoroughly, shall then judge it deficient; I should be very glad if any thing better of the Kind should offer, that may afford him greater Satisfaction; and till then only, I entreat his kind Acceptation and candid Perusal of This.

VALE.

THE



THE
INTRODUCTION,
CONCERNING

The Nature, Kinds, and Notation, of
DECIMAL NUMBERS.

1. **T**HE excellent Art of *Decimal Arithmetick* derives its Name from a Latin Word (*viz. Decem, Ten*) which denotes the Nature of its Numbers, which represent the Parts of any *Integral Quantity* divided in a *Decuple, Decimal, or tenfold Proportion*.

2. Any Integer, or whole Quantity, being divided into 10, 100, 1000, 10000, &c. Parts, because those Numbers are in a *Decimal* or tenfold Proportion; therefore such Numbers as represent any of those Parts, are call'd *Decimal Numbers, or Decimals*.

3. Thus, suppose I divide any whole Quantity into 10 Parts, and take 7 of them; those are call'd 7 decimal or tenth Parts of that Integer; and are thus vulgarly wrote $\frac{7}{10}$; suppose it divided into 100, 1000, &c. Parts; then 7 of them would be express'd thus $\frac{7}{100}$, $\frac{7}{1000}$, &c. and read as before.

4. Also many Integers and Parts of another, would be express'd thus, $8\frac{7}{10}$, $19\frac{67}{100}$, $475\frac{154}{1000}$, $2\frac{2946}{10000}$, &c. and read thus; 8, and 7 tenths; 19, and 67 Hundreths, or Parts of a 100; 475, and 154 Thousandths, or Parts of a Thousand; 2, and 2946 Parts of ten Thousand. Thus the *Denominator* of Decimal Parts or Numbers, is always an Unite with Cyphers annexed.

B

5. There-

5. Therefore if the Places of Figures in the *Numerator* be equal in Number to the Places of Cyphers in the *Denominator*, (or be made equal thereto, by prefixing Cyphers;) the *Denominator* in such a Case will be known, though it be not written; and therefore in the Notation of Decimal Numbers, is always omitted; and the *Numerator* (ordered as aforesaid) alone is join'd to the *integral Quantity*, with a Comma, or Point, to distinguish it therefrom.

6. Hence $\frac{34}{100}$; $\frac{45}{1000}$; $\frac{57}{10000}$; $\frac{271}{100000}$; $\frac{4}{100000}$; &c. are thus written, .34; .045; .00271; .004. Also $27\frac{3}{10}$; $58\frac{4}{100}$; $129\frac{37}{10000}$; $1\frac{7}{100000}$; are wrote 27.3; 58.04; 129.0132; 1.0017. And on the contrary, by .12; 1.76; .006; 2.003; .001; we understand $\frac{12}{100}$; $1\frac{76}{100}$; $\frac{6}{1000}$; $2\frac{3}{10000}$; $\frac{1}{1000}$.

7. Cyphers prefixed to decimal Numbers, decrease their Value in a decuple or tenfold Proportion; as affixed to Integers, they increase their Value in the same Proportion: thus .5; .05; .005; .0005; &c. are, as they proceed, each one ten times less then the preceding Decimal; as is easy to conceive.

8. When the *Denominator* is an Aliquot Part of the *Numerator* increased by Affixing Cyphers thereto, the Decimal equivalent to such a Fraction, will be compleat and terminate; as, $\frac{5}{10} = .5$; $\frac{5}{20} = .25$; $\frac{5}{30} = .75$; $\frac{5}{40} = .125$; $\frac{5}{50} = .1$; $\frac{5}{60} = .083333$; $\frac{5}{70} = .071428$; $\frac{5}{80} = .0625$; $\frac{5}{90} = .055555$; &c.

9. But if the *Denominator* be no aliquot Part of the *Numerator* thus increased; the Decimal equivalent to such a Fraction will be interminate or endless; that is, it will constantly repeat one Digit only; as $\frac{1}{3} = .333333$, &c. *ad infinitum*; or $\frac{4}{7} = .666666$, &c. or $\frac{7}{12} = .583333$, &c. or $\frac{5}{8} = .625$, &c. or $\frac{3}{8} = .375$, &c.

10. Or else a certain Number of Figures perpetually Circulate, or repeat in the Quotient. Thus $\frac{2}{11} = .181818$, &c. *ad infinitum*; also $\frac{5}{27} = .185185$, &c. And $\frac{2}{9} = .222222$, &c. And $\frac{3}{22} = .136363$, &c. without End. And those Numbers which thus infinitely circulate or repeat, are most fitly termed *Repetends*. Those which circulate a Digit only, are called a *single Repetend*;

Nature, Kinds, and Notation of Decimals. 3

petend; and those in which several Figures circulate, are call'd a *Compound Repetend*, in the following Tract.

11. For the greater Elegance and Perspicuity, in all the Operations of Circulating Numbers, I have dashed the first and last Figure of the *Repetend*; thereby making one Place of the *Repetend* sufficient. Thus the Examples above are thus wrote or expressed; $3; .6; .583; .138; .0322918$. And the *Compound Repetends* thus; $.18; .183; .952380; \text{and } .0138$; herein following the Ingenious Mr. Cunm, the first Improver of this Part of *Decimal Arithmetick*.

12. In a *Compound Repetend*, any one of the circulating Figures may be made the first of the *Repetend*; for Instance, in the *Repetend* $8.6323\ 325\ 325$, &c. it may be made 8.63253 ; or 8.632532 . And by this Means any two or more *Repetends* may be made to begin and end in the same Place; and then they are said to be conterminous.

13. Several other Things relating to the Nature and Properties of circulating Numbers, I have interspersed in the following Treatise in their proper Places, where they may be understood, and which are not to be found in any other Book of *Decimal Arithmetick*.

14. In all *Decimal Numbers*, if the Point of Distinction be removed one Place towards the Right Hand, every Figure, and consequently the Whole Expression, will be increased in a tenfold Proportion; as in those Decimal Expressions 3.756 , 37.56 , 375.6 , 3756 . which are each one 10 times greater than the preceding one. In which Proportion also, 'tis manifest, they decrease in Value, by removing the Decimal Point a Place to the Left Hand.

15. The Nature and Properties of *Decimal Numbers*, are the same with those of *Integers* or *Whole Numbers*, and the Method of Working both the same (excepting *Repetends*). Hence ariseth the Excellency and superior Usefulness of *Decimal Arithmetick*, above all other kinds of Computation.

16. To make the preceding Proposition evident, suppose 'twere required to express the Time since our Saviour's Incarnation to the Year present, in Centuries and Decimal Parts of a Century; it would be thus 17.33 ; where you observe one half of the Number consist of *Integers*, and the other half of *Decimals*. But suppose the Time ex-

4 *The Introduction, concerning the*

Pres'd in Years, the Number consists of the same Figures, 1733; and is whole or integral.

17. Hence 'tis plain the same Number may be either *Integral* or *Decimal*, and that either in Whole or in Part, according to what is made the Integer; for in the foregoing Case, if a Myriad be the Integer, the Time will be expressed by a pure Decimal 0.1733; if a Century be the Integer, by a mix'd Decimal 17.33; if a Year be the Integer, by the integral Number 1733; as before.

18. All the different Species, or Parts of different *Kinds* and *Denominations*, of *Money*, *Weights*, and *Measures*, and all other Quantities, are to be reduced to *Decimals*, or may be expressed in *Decimal Parts* of their respective Integers, by proper Tables calculated for that Purpose; also any *Decimal* may very nearly by Inspection only (without the tedious Reductions hitherto used) be read in the vulgar Parts or Denominations of its respective Integer, by a Set of new Tables, which I have compos'd for the Ease of those who are conversant in this excellent Science.

19. Since then it has been shewn that *Decimals* are the same with *whole Numbers*, as to their Nature and the Manner of Operation; and that all mix'd Numbers, or such as consist of divers and different Denominations, are reducible thereto, and *vice versa*; it follows that all the Arithmetick of mix'd and heterogeneous Numbers is to be perform'd by *Decimals*, with the same Ease, Expedition, and Pleasure as that of *whole Numbers*.

20. And by Consequence, That *Vulgar Arithmetick*, *Vulgar Fractions*, *Duodecimal* and *Sexagesimal Arithmetick*, (those Parts of the Science of Computation hitherto deem'd so hard and intricate, and therefore but little studied or known) are all by this noble Art of *Decimal Arithmetick* perform'd with the utmost Ease and Pleasure, that any Arithmetick is capable of, and which I have abundantly evinc'd in the Sequel of the ensuing Work.

21. The Figures of a *Decimal Number* are to be numerated as those of whole Numbers, *viz.* from the Right Hand to the Left; but they must be denominat'd of the Number of Parts the Integer is divided into. The following Table will make the Numeration and Denomination of Decimals very easy.

A TABLE of the Numeration and Denomination of
DECIMALS.

Integer
Tens
Hundreds
Thousands
Tens of Thousands
Hundreds of Thousands
Millions
Tens of Millions
Hundreds of Millions
Thousands of Millions

I.	0	0	0	0	0	0	0	0	0	=	Parts of the Integer.
O.	I	=	Parts of Ten.
O.	I	2	=	Parts of an Hundred.
O.	I	2	3	=	Parts of a Thousand.
O.	I	2	3	4	=	Parts of Ten Thousand.
O.	I	2	3	4	5	=	Parts of an Hundred Thousand.
O.	I	2	3	4	5	6	.	.	.	=	Parts of a Million.
O.	I	2	3	4	5	6	7	.	.	=	Parts of Ten Millions.
O.	I	2	3	4	5	6	7	8	.	=	Parts of an Hundred Millions.
O.	I	2	3	4	5	6	7	8	9	=	Parts of a Thousand Millions.

0.	0	0	0	0	0	0	0	0	9
0.	0	0	0	0	0	0	0	8	9
0.	0	0	0	0	0	0	7	8	9
0.	0	0	0	0	0	6	7	8	9
0.	0	0	0	0	5	6	7	8	9
0.	0	0	0	4	5	6	7	8	9
0.	0	0	3	4	5	6	7	8	9
0.	0	2	3	4	5	6	7	8	9
0.	1	2	3	4	5	6	7	8	9

These Numbers are all of them the respective Parts of a Thousand Millions, into which the Integer is divided as above.

Ад

An Explanation of the Characters and Abbreviatures used in the following Book.

It has been of late an Expedient to avoid Prolixity in Writing, to make use of some convenient and significant Characters to express those Words which most often occur, and occasion Tedioufness and Tautology in the Work, the most irksome Vices that can attend it; and accordingly I have here used them; which, with their Significations, are thus to be understood.

Figure.	Names.	Significations.
+	Plus, or more.	As $a + b$, is a more b ; in Addition.
—	Minus, or less.	As $a - b$, is a less b ; in Subtraction.
x	Multiplied into.	As $a \times b$, is a multipl. into b ; in Mult.
÷	Divided by.	As $a \div b$, is a divided by b ; in Divis.
=	Equal to.	As $a = b$, is a equal to b ; in Equat.
∴	Is to.	As $a : b :: c : d$; as a is to b , so is c to d ; in Proportions.
∴	So is.	
⊙	Involved.	As $2 \odot$, is the 2d involved.
ω	Evolved.	As 2ω , is the 2d evolved, or extracted.
√	Surd Root.	As \sqrt{ab} , $\sqrt[3]{ab}$, $\sqrt[4]{ab}$, &c. is the Root Square, cub'd, biquadrate, &c. of ab a Surd.

CHAP. I.

Addition of DECIMALS.

Addition of Decimal Parts admits of various Cases, according to their different Kinds; either as they are *terminate and compleat*, or *interminate*, and continually repeat either *one or more* Figures. I shall illustrate all the several Varieties by suitable Examples of Money, Weight, Measure, &c.

7

EXAMPLE I. $\left\{ \begin{array}{l} 56,7375 \\ 24,025 \\ 20,0125 \\ 19,4875 \\ 12,05 \end{array} \right. \quad \begin{array}{l} \text{Then} \\ l. \quad s. \quad d. \quad qr. \\ 25, = . - - - 2,4 \\ 31, = 6 - 2 - 1,6 \end{array}$

The Sum $132,3125 = 132 - 6 - 3 - 0$

l. Troy. Cap. 3. Suppose the
 457,825 Paying Dollars; if they
 570,065625 them all contained, that
 24,5375
Example II. l. oz. pwt. gr.
Add together 806,253125 43 = . — 1 — 0,768
 0,003125 73 = 8 — 15 — 4,8
 695,05 }

The Sum 2553,73.43.75 = 2553 — 8 — 16 — 6

Case 2. If you have a great many several Sums to add, and their Decimals run to a great Number of Places, it will not be necessary to add them all, but only so many Places, as are sufficient to give the Value in the aggregated Sum, which will require but 4 or 5 Places, or 6 at most; for so far only the largest Tables go.

Observe to make that Figure (at which you break off) more by a Unit, if the next rejected Figure be more than 5; but if the next Figure be less than 5, reject the Figures only.

Then add the several Sums, and the certain Places of the Decimal are generally fewer by one than the Decimal Places retained in any of the given Sums. I shall subjoin an Example at large, and the same thus contracted.

EXAMPLE

Addition of Decimals.

EXAMPLE.

47.982774354	{ Thus contracted }	47.98277
175.6732956		175.6733
2.43752432		2.43752
97.702006764		97.70201
276.92301762		276.92302
30.00420999		30.00421
630.722828648		630.72283

In this Example there are 4 Places of Decimals certain, or the same, in both Operations; and the Rule will scarce ever err *above an Unit* in the last Place. Now whatever you suppose the Integral Quantities to be, the *Tables* will shew you the Value of the Decimal Parts.

Case 3. Suppose the Numbers you are to add have *repeating Decimals*; if they are *single Repetends*, make them all *conterminous*, that is, end together; and then add as before, only to the last, or Right Hand Place of Decimals, add as many *Units* as there are *Nines* in it; and that *last Digit* shall be one of the *Repetends*.

N. B. It may be proper to give the first and last Places of all Repetends a *Dash* with the Pen, for Distinction.

EXAMPLE I.

l.		
124.233		
64.516		
0.833		
59.800	l. s. d. qr.	
3.833	{	,30 = . — . — 2,88
45.016		,73 = 14 — 7 — 08
Sum = 297.733		= 297 — 14 — 7 — 3,68

EXAMPLE

EXAMPLE II.

Feet.

5,91666

0.02083

2.56250

4.83333 *F. In. qrs.*

6.04166 { ,16 = . — .17

2.66666 { ,04 = . — 1.92

Sum = 22,04166 = 22 — 0 — 2

EXAMPLE III.

Oz. Troy Wt.

4,727083

2.583333

0.002083

9.029166

4.031250

7.035416 *oz. pwt. gr.*

10.758333 { ,33 = . — 1,58

6.916666 { ,08 = 1 — 14,4

45.083333 = 45 — 1 — 16

EXAMPLE IV.

Days.

275.252777

47.871111

436.027083

10.677777

101.235555

D. H.

127.769444 { ,20 = . — 2,8

243.958333 { ,81 = 19 — 26,4

Sum = 1242.812083 = 1242 — 19 — 29,2

Addition of Decimals.

Case 4. If your Decimal be a *compound Repetend*, that is, consists of several Places of Figures which *continually repeat or return*; the Sum or Aggregate of any given Number of such Decimals will also *repeat*; and the Number of Places, or of the Figures, in each repetend, will be equal to the least *common Multiple* of those several Numbers which represent the Places of Figures in the Repetends added. Hence (tho' it be scarce ever necessary to have above five or six Places of Decimals, yet) if any one be minded to see the *Repetend compleat*, he must observe this *Rule*;

From the Place where all the Repetends *begin together*, continue each Decimal to a Number of Places equal to the *Multiple aforesaid*; then add, and to the last Place add as many Units as there are 10's in the Place where the Repetends all begin together, and the Figures in those two Places are the *first* and the *last* of the *Repetend*. The Examples following will make all plain.

1.

		l.	s.	d.	qr.
E X A M. I.	{	13 . 0462	{	50 =	— 1 — 1,78
		2 . 6042		37 =	7 — 4 — 3,2
		5 . 7234			
		Sum		21 . 3760	=

C. Wt. Av.

		C.	Q.	lb.	oz.	dr.
E X A M. II.	{	14 . 472956	{	38 =	— . — . —	1,09
		12 . 307248		33 =	— . — . —	5 — 14,61
		9 . 020763		71 =	2 — 23 — 8 —	5,12
		11 . 912375				
		Sum =	47 . 713338	=	47 — 2 — 23 — 14 —	4,82

Rods.

		R.	Yd.	F.	In.	
E X A M. III.	{	121 . 47237	{	87 =	— . — . —	1,73
		80 . 27355		72 =	3 — 2 — 10,56	
		64 . 90834				
		89 . 07444				
		Sum =	355 . 72871	=	355 — 4 — 0 —	0,3

E X A M-

Years.

EXAM. IV. $\left\{ \begin{array}{l} 175.3724 \\ 84.2634 \\ 126.4526 \\ 79.3279 \\ 105.7105 \\ 28.4376 \end{array} \right.$ $\left. \begin{array}{l} Y. \\ M. \\ W. \\ D. \\ H. \end{array} \right\} \begin{array}{l} 46 = . - 1 - 16,3 \\ 56 = 7 - 1 - 1 - 9,6 \end{array}$

Sum = $\underline{599.5646} = \underline{599 - 7 - 1 - 3 - 2}$

Signs.

EXAM. V. $\left\{ \begin{array}{l} 4.218213 \\ 2.524756 \\ 6.030303 \\ 5.215215 \\ 7.070707 \\ 0.732612 \end{array} \right.$ $\left. \begin{array}{l} Sig. \\ 9 \\ ' \\ " \end{array} \right\} \begin{array}{l} 08 = . - 12 - 14 \\ 78 = 23 - 24 - 0 \end{array}$

Sum = $\underline{1.786807} = \underline{1 - 23 - 36 - 14}$

Feet Square.

EXAM. VI. $\left\{ \begin{array}{l} 2.954895439543 \\ 1.041041041041 \\ 3.737373737373 \\ 4.065826065826 \\ 4.731473147314 \end{array} \right.$ $\left. \begin{array}{l} Fq. \\ Iq. \\ Qq. \end{array} \right\} \begin{array}{l} 01 = . - 0,23 \\ 53 = 76 - 5,12 \end{array}$

Sum = $\underline{16.530109431099} = \underline{16 - 76 - 5,35}$

These Six Examples, I imagine, are sufficient to illustrate this last Case of *compound Repetends*; but if it chances to happen that a *complete* or *terminate* Decimal be to be added with them, you must affix *Cyphers* thereto, to esteem and deal with them as a *Repetend*.

C H A P. II.

S U B S T R A C T I O N.

Case I. **I**F your Decimals be *terminate* and *complete*, place as directed in this Case of *Addition*, and Subtract as in Whole Numbers; imagining all the *vacant Places* fill'd with Cyphers.

E X A M P L E I.

	<i>L.</i>	<i>l.</i>	<i>s.</i>	<i>d.</i>	<i>qr.</i>
From	729.4726	{	,56 = . — 1 — 1,38		
Subtract	634.927		,54 = 10 — 9 — 2,4		
Remains	<u>94.5456</u>		<u>= 94 : 10 : 10 : 3,78</u>		

E X A M P L E II.

	<i>C.</i>	<i>C.</i>	<i>Q.</i>	<i>lb.</i>	<i>oz.</i>	<i>dr.</i>
From	472,07	{	,46 = . — . — 8 — 3,88			
Subtract	392,4354		,63 = 2 — 14 — 8 — 5,36			
Remains	<u>79,6346</u>		<u>= 79 : 2 : 15 : 0 : 9,24</u>			

E X A M P L E III.

	<i>lb.</i>	<i>lb.</i>	<i>oz.</i>	<i>pwt.</i>	<i>gr.</i>
From	147,	{	,52 = . — 1 — 5,95		
Subtract	94,7248		,27 = 3 — 4 — 19,2		
Remains	<u>52,2752</u>		<u>52</u>	<u>— 3</u>	<u>— 6 — 1,15</u>

Case 2. If your Decimals run to *many Places* of Figures, do as directed in the Case of *Addition*; and *subtract* as in the *last Case*; and the last place of the Decimal Remainder will never *Err more than an Unit*.

E X A M-

EXAMPLE I

From 2,752804624 take 1,476937679.

	Miles,	M.	F.	P.	V.	F.	In.
Thus,	{ 2,752805	{	5,67 = . — . — . — 4,23				
	{ 1,476938	{	5,58 = . — 1 — 4 — 2 — 1,14				
			2,27 = 2 — 6 — 2 — 0 — 7,2				
Remains	<u>1,275867</u>	=	<u>1 — 2 — 8 — 6 — 1 — 6,57</u>				

In this Example the last Figure 7 is a Unit too much, but that is not to be regarded; for in this Case the Value of a Unit in that Place is but ,063 of an Inch.

Case 3. If your Decimal repeat Single Figures, proceed (as in this Case of Addition) to place them, and subtract as usual; except that when the Subtrahend is the greater Number, you must increase the upper Figure by 9 only, and in every such Case carry one to the next Place.

EXAMPLE I

	L.	l.	s.	d.	qr.
From	54,73333	{	91 = . — 2 = 0,73		
Subtract	17,95416	{	77 = 15 — 4 — 3,2		
Remains	<u>36,77916</u>	=	<u>36 — 15 — 6 — 3,93</u>		

EXAMPLE II

	L.	l.	s.	d.	qr.
From	57,5289	{	56 = . — 1 — 1,38		
Subtract	49,5833	{	94 = 18 — 9 — 2,4		
Remains	<u>7,9456</u>	=	<u>7 — 18 — 10 — 3,78</u>		

E X A M-

EXAMPLE III.

	Hogsheads.	H.	G.	P.
From	1672,4518	{	,16 = . — .8	
Subtract	879,3000		,15 = 9 — 3,6	
Remains	<u>793.1518</u>		<u>= 793 — 9 — 4,4</u>	

EXAMPLE IV.

	Loads.	L.	Q.	B.	G.
From	472,222	{	,60 = . — — 1,92		
Subtract	346,178		,04 = . — 1 — 4,8		
Remains	<u>126,048</u>		<u>= 126 — 0 — 1 — 6,72</u>		

EXAMPLE V.

	Years.	Y.	M.	W.	D.	H.
From	47,957200	{	,88 = . — . — 3 — 5,08			
Subtract	,008318		,94 = 12 — 1 — 0 — 2,4			
Remains	<u>47,948882</u>		<u>= 47 — 12 — 1 — 3 — 7,48</u>			

Case 4. If the Decimals be *compound Repetends*, order them as directed in the *Case of Addition*; then *subtract*; and look if you must borrow one in the Place where *both Repetends* begin together; if so, you must add one to the *Right-Hand* place of the *Subtrahend*; and the Remainder either *Whole* or in *Part*, will shew the *Repetend*.

EXAMPLE I.

	L.	l.	s.	d.	qr.
From	47,4178178	{	,21 = . — . — 2,01		
Subtract	15,8856565		,85 = 17 — 0 — 0		
Remains	<u>31,8321612</u>		<u>= 31 — 17 — 0 — 2,01</u>		

E X A M.

EXAMPLE II.

	Ounces.	Oz. pw.	gr.
From	153.92749	{,49=.	— 2,35
Subtract	142,83353	{,07=1—	9,6
Remains	11,07493	= 11—1—	11,95

EXAMPLE III.

	Rods Sq.	Rq.	Yq.	Fq.
From	75.5333	{,35=.	— . 94	
Subtract	42.7597	{,77=23—	2,44	
Remains	32.7733	= 32—23—	3,38	

EXAMPLE IV.

	Yards.	Y.	F.	In.	Q.
From	47.8340260	{,81=.	— . — 1,16		
Subtract	40.9239259	{,92=2—	9— 0,48		
Remains	6,9281001	= 6—2—	9— 1,64		

EXAMPLE V.

	Days.	D.	H.	M.
From	75.2758000	{,94=.	— 13,53	
Subtract	47,3563563	{,91=21—	50,4	
Remains	27,9194436	= 27—22—	3,93	

EXAMPLE VI.

	Degrees.	D.	'	"
From	49,5285285	{,49=.	— 17,64	
Subtract	38,4736000	{,05=3—	0	
Remains	11,0549285	= 11—3—	17,64	

C H A P. III.

M U L T I P L I C A T I O N.

Case I. **I**F your Decimals be *complete* and *terminate*, whether they be *pure* or join'd with *Integers*; Multiply them as if they were all *whole Numbers*; and cut off (to the Right-Hand) so many Places for Decimal Parts in the Product as there were in both the *Multipplier* and *Multiplicand* counted together. But if it so happen that there are not so many Places in the Product, supply the Defect by prefixing Cyphers.

E X A M P L E I.

$$\begin{array}{r}
 \text{Multiply } 32.12 \quad \} \text{ Feet.} \\
 \text{by } 24.3 \\
 \hline
 9636 \quad \text{Fq. Inq. Gr.} \\
 12848 \quad \} ,60 = . - 13,8 \\
 6424 \quad \} ,51 = 73 - 7,04 \\
 \hline
 \text{Product } 780,516 = 780 - 74 - 4,84
 \end{array}$$

E X A M P L E II.

$$\begin{array}{r}
 \text{Multiply } 42,51 \quad \} \text{ Yards.} \\
 \text{by } ,241 \\
 \hline
 4251 \quad \text{Yq. Fq. Inq.} \\
 17004 \quad \} ,99 = . - 12,84 \\
 8502 \quad \} ,24 = 2 - 23,04 \\
 \hline
 \text{Product } 10,24991 = 10 - 2 - 35,88
 \end{array}$$

E X A M-

Multiplication of Decimals.

17

EXAMPLE III.

Multiply 78,546 Miles,
by 436

471276
235638
314184

M. Fq. R. Y. F. In.
{,60=.— 1—5—0—1,8
{,05=.— 16—0—0—0

Product 34246,056 = 34246—0—17—5—0—1,8

EXAMPLE IV.

Multiply ,02365 } of a Mile.
by ,0435

11825
7095
9460

Mq. Aq. Rq. Pq.
{,28=.— .— 2,85
{,10=0—2—22,4

Product ,001028775 = 0—0—2—25,25

Case 2. When it happens that the Places of Decimals run far in both Factors, and consequently would make a very large Decimal in the Product, you may contract your Work, in such a Case, to as few Places of Decimals in the Product as you please, or is suitable to your Design, by the following *Rule*, viz. set the Units Place of the *Multiplier* directly under that Figure of the Decimal Part of the *Multiplicand* whose Place you would preserve in the Product.

Then *invert*, or place all the other Figures of the *Multiplier* in a contrary order to the common way.

Lastly, in Multiplying always begin at the Figure of the *Multiplicand* which stands over the Figure wherewith you are then a Multiplying, setting down the *first Figure* of each particular Product *directly under one another*. But withal take care to see what *Increase* would arise from the Multiplying of the *two* next Right-Hand Figures of the *Multiplicand*, which you must constantly *add* to the *first Figure* in every Product.

D

EXAM-

EXAMPLE I.

Suppose I would multiply 92.412031 Yards by 47.29195 Yards, and to have only *four Places* of Decimals in the Product.

Place them as before directed, and they will stand

Thus	{	92,412031	The Multiplicand as usual.
		59192,74	The Multiplier inversed.
		<hr/>	
		36964812	
		6468842	
		184824	
		83171	
		924	
		832	
		46	
		<hr/>	
		4370,3451	

The *Reason* of, and how *great a part* of the Work is saved by, this *Contraction*, will appear from the *Operation at large*.

Thus	{	92,412031	
		59192,74	
		<hr/>	
		46	2050155
		831	708279
		924	12031
		83170	8279
		184824	062
		6468842	17
		36964812	4
		<hr/>	
		4370,3451	4935045

Hence it appears, that *half* the Work is useless, *viz.* all those Figures included in the *Square*, whose Sum make indeed 7 places of Decimals, but are of *no value*, and therefore superfluous.

EXAM.

EXAMPLE II.

Multiply 14,794, by 12,123; and to have thereby reserv'd two Places of Decimals in the Product, place them

Thus	{	14,794		The common way	{	14,794
		321,21		at large.		12,123
		<hr/>				<hr/>
		14794				4 4382
		2959				29 588
		148				147 94
		30				2958 8
		4				14794
		<hr/>				<hr/>
		179,35				179,34 7662

EXAMPLE III.

Multiply 257,356 with 76,48, and for an Intire Product of Integers, place them as by the Rule

Thus	{	257,356		The same	{	257,356
		84,67		at large.		7648
		<hr/>				<hr/>
		18015				20 58848
		1544				102 9424
		103				1544 136
		20				18014 92
		<hr/>				<hr/>
		19682				19682, 58688

From these Examples, tis manifest how advantagous these compendious Contractions are to facilitate and shorten the Work of those long and operous Calculations and Computations, which the experienced Practitioner finds occur but too often in Arithmetick, Algebra, and Geometry.

Case 3. If the Multiplicand be a Repetend only, and the Multiplier a single Digit, Multiply as usual; only observe to add in the last place of the Product as many Units as it contains Nines, and that place is a Repetend.

Multiplication of Decimals.

EXAMPLE I.

Signs of the Zodiac.

$$\begin{array}{r} \text{Multiply } 10,7016 \\ \text{by } 5 \\ \hline \text{Product } 5,5082 \end{array} \quad \begin{array}{l} \{,83 = . - 15 - 0 \\ ,50 = 15 - 0 - 0 \end{array}$$

EXAMPLE II.

$$\begin{array}{r} \text{Multiply } 9,308 \\ \text{by } ,7 \\ \hline \text{Product } 65,138 \end{array} \quad \begin{array}{l} \text{Yards. } \{,80 = . - 10,33 \\ ,13 = 1 - 24,48 \end{array}$$

EXAMPLE III.

$$\begin{array}{r} \text{Multiply } 476,08 \\ \text{by } ,08 \\ \hline \text{Product } 38,0844 \end{array} \quad \begin{array}{l} \text{Poles. } \{,44 = . - 1,18 \\ ,08 = 2 - 3,76 \end{array}$$

But if the *Multiplier* consist of *several* Digits or Figures, then make each particular Product *conterminous*, by continuing the *single Repetend* of each towards the Right-Hand.

EXAMPLE I.

$$\begin{array}{r} \text{Multiply } 748,64 \\ \text{by } ,634 \\ \hline 299457 \\ 2245933 \\ 44918666 \\ \hline \text{Product } 474,63057 \end{array}$$

Multiplication of Decimals.

21

If the *Multiplier* be a *Repetend*, multiply as usual; but in the *Product*, cut off one place less for Decimals than usual (which is all one as multiplying by Ten) and divide by Nine; continue the Quotient till it becomes a *Single* or *Compound Repetend*; and this shall be the *true Result* or *Answer*.

EXAMPLE I.

Multiply 724,35
by .04

9)289,740

True Product 32,193

EXAMPLE II.

Multiply 26,54
by .03

9)7,963

True Product = ,8847

EXAMPLE III.

Multiply 251,43
by 8,74

9)100,473

111636

176001

201144

Product 2198,6046

EXAM

Multiplication of Decimals.

EXAMPLE IV.

$$\begin{array}{r}
 \text{Multiply } 48,754 \\
 \text{by } 2,13 \\
 \hline
 9)146263 \\
 1625148 \\
 4875444 \\
 97508888 \\
 \hline
 \text{Product } 104,009481
 \end{array}$$

Case 4. If the *Multiplicand* be a *Compound Repetend*, and the *Multiplier* but a *single Digit*, *Multiply* as in *Case the First*; but observe to add to the Right-Hand Place of the *Product* so many Units as there are *Tens* in the *Product* of the Left-Hand Place of the *Repetend*. And the *Product* shall contain a *Repetend* whose Places are equal to those in the *Multiplicand*.

EXAMPLE I.

$$\begin{array}{r}
 \text{Multiply } 582,347 \\
 \text{by } 8 \\
 \hline
 \text{Product } 4658,778
 \end{array}$$

EXAMPLE II.

$$\begin{array}{r}
 \text{Multiply } 5924,378 \\
 \text{by } ,03 \\
 \hline
 \text{Product } 177,78138
 \end{array}$$

EXAMPLE III.

$$\begin{array}{r}
 \text{Multiply } 3749,28 \\
 \text{by } ,007 \\
 \hline
 \text{Product } 26,24464
 \end{array}$$

Multiplication of Decimals.

23

If the *Multiplier* consists of Places more than one, make all the several Products *conterminous* towards the *Right-Hand*, as taught in the last Case.

EXAMPLE I.

Multiply 73,286
by 437

5128106

21977597

293184634

Product 3202,40838

EXAMPLE II.

Multiply 4027,3012
by 4370,2

80546028

2819108911

120819038190

1610920509205

Product 17600112,02332

But if not *only* the *Multiplicand*, but the *Multiplier* also be a *Compound Repetend*, Multiply (as has been before taught,) each Figure of the *Repetend*, and add the several Products together; Then add the Result to it self in this Manner, set the first Left-Hand Figure so many Places forward as *exceeds* the Number of Places in the *Repetend* by one; and the Rest of the Figures in order after it; and thus proceed till the Result *last added* be carried *beyond the first*; Lastly, add these several Results together, beginning under the Right-Hand Place of the first, and from thence dash as many Figures for a *Repetend*, as the *Repetend* of the *Multiplier* does consist of.

E X A M.

Multiplication of Decimals.

EXAMPLE I.

Multiply 235.01
by 3.28

$$\begin{array}{r}
 235.01 \\
 \times 3.28 \\
 \hline
 188008 \\
 47002 \\
 70503 \\
 \hline
 \text{First Product } 7871326 \\
 7871326 \\
 7871326 \\
 \hline
 \text{True Product } 787,9208
 \end{array}$$

EXAMPLE II.

Multiply 432067
by 0.2438

$$\begin{array}{r}
 432067 \\
 \times 0.2438 \\
 \hline
 1728268 \\
 884134 \\
 \hline
 \text{First Product } 1052515212 \\
 1052515 \text{ Gr.} \\
 105 \text{ Gr.} \\
 \hline
 \text{True Product } 10526,20474
 \end{array}$$

EXAMPLE III.

Multiply 42710.36
by 20403

$$\begin{array}{r}
 42710.36 \\
 \times 20403 \\
 \hline
 12813108 \\
 17084144 \\
 8542072 \\
 \hline
 \text{First Product } 8714,1957508 \\
 87141957 \text{ Gr.} \\
 871 \text{ Gr.} \\
 \hline
 \text{True Product } 8714,2828936
 \end{array}$$

Multiplication of Decimals.

25

If the *Multiplier* has any *terminate* Places join'd with the *Repetend*, and if the *Repetend* be *small* and these *many*, the best way will be to multiply and add the *Products* of the *Repetend* first; then after multiply by the *terminate Figures*, and add their *Products* to the *Sum* of the *Product* of the *Repetend*; and to this *last Result* add the said *Sum* of the *Repetend Products*, as in the last Examples.

EXAMPLE.

Multiply 432,43
by 23,414

172972

43243

The Sum 605402 of the *Product* of the *Repetends*.

172972

129729

86486

1012491602

605402

6054 3c.

60 3c.

10124,97717

But if the *terminate Figures* are *few*, and the *Places* of the *Repetend* are *many*; the shortest way will be to *subtract* the *terminate Figures* from those of the *Repetend*, and multiply by the *Remainder* as a *Repetend*.

E

EXAM-

Multiplication of Decimals.

E X A M P L E.

Multiply 1243,2701
by 428,438
From which subtract 42 The terminate Figures.
Remains a new }
Multiplier } 423394

49730804

111894309

37298103

37298103

24865402

49730804

First Product = 5263931007194

5263931007 Sc.

526393 Sc.

52 Sc.

True Product = 526445,7452939

But if both Factors are interminate, or have compound Repetends, the Places of the Repetend in the Product will be uncertain as to their Number, and can only be determined (in any Manner fit for Practice) by continuing and repeating the first Product, which will contain a certain Repetend, being equal in Places, to that of the Multiplier.

E X A M P L E I.

Multiply 3,148

by 4,297

Subtract 4

The terminate Part.

Remains 4293 a new Multiplier.

9438

283090

629090

12581818

First Product 135034383636 Sc.

135034363 Sc.

135034 Sc.

135 Sc.

True Product 13,5169533

If

If the Product runs far 'ere it begins to repeat, and your Curiosity leads you to investigate the Repetend, or, at least, the true Product to any given Number of Places, you may do it thus; continue the Repetends of the first Product at pleasure; then divide it into *Periods*, each Period to consist of as many Places as the Repetend of the Multiplier did; then add the first Period to the second, and the Result to the third Period, and this Result to the fourth; and so on till you arrive at the Repetend, or as many places in the true Product as you please; Lastly, in adding the last place to the Left-hand of every Period, if it increase to Ten or more, place One under the Right-hand Place of the former Period, and what is thus placed in one Period, must constantly be added to the Unit thus placed in the next, and so on. An Example will make the Rule *perspicuous*.

EXAMPLE.

$$\begin{array}{r}
 \text{Multiply} \quad 121,0303 \\
 \text{by} \quad 28,032x \\
 \hline
 1210303 \\
 24206106 \\
 363091391 \\
 96824424424 \\
 242061061061
 \end{array}$$

$$\begin{array}{r}
 \text{First Product} = 3392,72 \quad \begin{array}{|l} 993488 \\ 339278 \\ \hline 1 \quad 332761 \\ \hline \quad \quad 1 \end{array} \quad \begin{array}{|l} 488488 \\ 332761 \\ \hline 821249 \\ \hline \quad \quad 2 \end{array}
 \end{array}$$

$$\text{True Product} = 3392,74 \quad 332762 \quad 821251 \quad +$$

$$\begin{array}{r}
 \text{First Product continued} = \begin{array}{|l} 488488 \\ 821249 \\ \hline 309737 \\ \hline \quad \quad 2 \end{array} \quad \begin{array}{|l} 488488 \\ 309737 \\ \hline 868225 \\ \hline \quad \quad 3 \end{array} \quad \begin{array}{|l} 488488 \\ 868225 \\ \hline 356713 \\ \hline \quad \quad 3 \end{array}
 \end{array}$$

$$\text{Remainder of the true Product} \quad 309739 \quad 868228 \quad 356716 \quad \&c.$$

Thus you have thirty six Places of the true Product, which is indeed twenty six more than are necessary.

CHAP. IV.

DIVISION of DECIMALS.

DIVISION of Decimals is perform'd in the same manner as *Division of Integers*, both in regard of placing the Numbers, and the *Work* it self.

The *chiefest Difficulty*, in general, is to discover the *true Value* of the *Quotient Figures*, that is, how to separate *justly* the *Integers* and *Decimals* it contains. However the *Business* of *Valuing* the *Quotient* is render'd very plain and obvious, by a due *Observation* of either of the following *Rules*, viz.

Rule I. The *Quotient Figure* is always of the same *Value* with *that Figure* of the *Dividend*, under which the *Units Place* of its *Product* stands. Or thus,

Rule II. The *Decimal Parts* in the *Divisor* and *Quotient* must be always *equal in Number* to *those* of the *Dividend*.

Some Authors give *one* of these *Rules*, and some the other; but I have supplied you with both, that nothing may be wanting to render this *necessary* and *frequent Part* of the *Art* as *easy* and *ready* as possible.

From the second general Rule may be deduced these four *particular* and very *useful Directions*, viz.

1 *Direct.* When the *Decimal Places* in the *Divisor* and *Dividend* are equal, the *Quotient* will be whole Numbers.

2 *Direct.* When the *Places of Decimals* in the *Dividend* exceed those of the *Divisor*, the *Decimal Parts* in the *Quotient* must be equal to that *Excess*.

3 *Direct.* If the *Divisor* exceed the *Dividend* in *Decimal Places*, annex *Cyphers* to make them equal, then will the *Quotient* be *Integers*, by *Direct 1.*

4 *Direct.* If after you have finished *Division* and find not so many *Figures* in the *Quotient* as there ought to be *places of Decimal Parts* by the general Rule, supply that *Defect* by prefixing *Cyphers* to the *Quotient*.

The *Learner* being thus fraught with general *Rules* and particular *Directions*; cannot, I think, without *Impeachment* of his *Ingenuity*, require any thing farther to be said or done

Division of Decimals.

39

to make Division of Decimals evident and easy, except the Operations themselves in all the various Cases; to which I now proceed.

Case 1. When your Decimals are *complete* and *soon terminate*, place them and work as in *Whole Numbers*, having a strict regard to the Rules and Directions before given for *Valuing the Quotient*.

In *Division of Decimals* there may happen *Nine Varieties*, with respect to the Nature of the Numbers, which may be of *three Sorts*; viz.

First, *Integers*; or *Whole Numbers*.

Secondly, *Mixt*; consisting of *Integers* and *Decimals*.

Thirdly, *Pure Decimals*; without any *Whole Numbers*.

Now the Dividend being it self of *three Kinds*, and capable of a Divisor of three kinds, there follows of consequence, these *nine Diversities*, viz.

Any *Whole Number* may be divided by a $\left\{ \begin{array}{l} \text{Whole Number.} \\ \text{Mixt Number.} \\ \text{Decimal.} \end{array} \right.$

A *Mixt Number* may be divided by a $\left\{ \begin{array}{l} \text{Whole Number.} \\ \text{Mixt Number.} \\ \text{Decimal.} \end{array} \right.$

A *Pure Decimal* may be divided by a $\left\{ \begin{array}{l} \text{Whole Number.} \\ \text{Mixt Number.} \\ \text{Decimal.} \end{array} \right.$

I shall explain and exemplify this by one Example, viz. by 1722 divided by 12 thus, at large.

$$12) 1722.0 \quad (143.5$$

$$\underline{12}$$

$$\cdot 52$$

$$\underline{48}$$

$$\cdot 42$$

$$\underline{36}$$

$$\cdot 62$$

$$\underline{60}$$

$$\cdot \cdot$$

Here

Division of Decimals.

Here you see the *Divisor* and *Dividend* are both *whole Numbers*; and because there was a Remainder of 6, I borrow a Cypher in order to divide it off clean, which gives (by *Direct* 2.) one Place, to wit, 5 in the Quotient for a *Decimal*. I shall subjoin this one Example varied according to all the Varieties aforesaid.

Variety 1—	—12)1722.0	(143,5	by <i>Direct</i> . 2.
2—	—12)172,20	(14,5	by <i>Direct</i> . 2.
3—	—12)17220	(,01435	by <i>Direct</i> . 4.
4—	—1,2)1722.0	(1435	by <i>Direct</i> . 1.
5—	—1,2)172,20	(143,5	by <i>Direct</i> . 2.
6—	—1,2)17220	(,1435	by <i>Direct</i> . 2.
7—	—,12)1722.00	(14350	by <i>Direct</i> . 3.
8—	—,12)17,220	(143,5	by <i>Direct</i> . 2.
9—	—,12)17220	(1,435	by <i>Direct</i> . 2.

But notwithstanding I have given a *Specimen* of all the *Varieties* in the last Example, and pointed to the *Direction*, by which each Quotient was form'd; yet 'twill be necessary to illustrate the *general Rule* by Examples wrought at large, wherein the *immediate Use* of the particular *Directions* will more *obviously* appear.

Example 1. Wherein the *Places of Decimal Parts* in the *Divisor* and *Dividend* are equal.

8,45) 295,75 (35	,0074) 4884 (66
2535	444
—	—
4225	444
—	—
4225	444
—	—
....	...

Here because the *Decimals* in *Divisor* and *Dividend* are equal in *Number*, therefore the Quotients in both Instances are *whole Numbers*, by *Direct*. 1.

Example

Division of Decimals.

31

Example 2. When the *Decimal Parts* of the Dividend exceed those of the Divisor.

$$\begin{array}{r}
 24,3) 780,516 (32,12 \\
 \underline{729} \\
 515 \\
 \underline{486} \\
 291 \\
 \underline{243} \\
 486 \\
 \underline{486} \\
 0000
 \end{array}
 \qquad
 \begin{array}{r}
 ,0067) ,3953659 \\
 \underline{335} \\
 603 \\
 \underline{603} \\
 0000
 \end{array}$$

In this Case the *Excess* is cut off in both the Quotients for *Decimal Parts*; by *Direct. 2.*

Example 3. When there are not so many Places of Parts in the Dividend as in the Divisor.

$$\begin{array}{r}
 7,684) 192,100 (25 \\
 \underline{15368} \\
 38420 \\
 \underline{38420} \\
 0000
 \end{array}
 \qquad
 \begin{array}{r}
 ,7875) 441,0000 (560 \\
 \underline{39375} \\
 47250 \\
 \underline{47250} \\
 0000
 \end{array}$$

Here *Cyphers* are annexed to the Dividend, to answer the *Decimal Places* of the Divisor, that the Quote might be whole Numbers; as in *Direct. 1.* by *Direct. 3.*

Example 4. When, after *Division is finished*, there are not so many Figures in the Quotient as there should be *Decimal Parts* by the *General Rule.*

$$\begin{array}{r}
 957) 7,25406 (,00758 \\
 \underline{6699} \\
 5550 \\
 \underline{4785} \\
 7656 \\
 \underline{7656} \\
 0000
 \end{array}
 \qquad
 \begin{array}{r}
 ,575) ,0007475 (,0013 \\
 \underline{575} \\
 1725 \\
 \underline{1725} \\
 0000
 \end{array}$$

In

Division of Decimals.

In both these Instances, by *Direction 4.* I prefix Cyphers to the Quotients, that together with those in the Divisors they might be equal to the Decimal Places of the Dividend.

If any *Whole, Mixt, or Decimal Number* is given to be divided by 10, 100, 1000, &c. you only remove the separating Point towards the Left-hand *so many Places* as there are *Cyphers in the Divisor*; as on the contrary in *Multiplication*, the *separating Point* is moved to the Right-hand *so many Places* as there are Cyphers in the Multiplier.

E X A M P L E S in

<i>Multiplication.</i>	<i>Division.</i>
1,523 X 10 = 15,23	10)1523(152,3
1,523 X 100 = 152,3	100)1523(15,23
1,523 X 1000 = 1523	1000)1523(1,523
,0072 X 10 = ,072	10)72 (7,2
,0072 X 100 = ,72	100)72 (,72
,0072 X 1000 = 7,2	1000)72 (,072
,0072 X 10000 = 72	10000)72 (,0072

I shall next give a *Method* whereby you may work any Case of *Division* by *Multiplication*, and *vice versa*, any Case of *Multiplication* by *Division*; and this, in many Instances, will be found very excellent and useful.

P R O B L E M I.

Suppose I have any Number, 7315, to *multiply* by any other Number 125; but yet have a mind to *divide* the said Number, and to have a Quotient equal to the Product of those two Numbers; *Quere* the Divisor?

Rule. Divide a *Unit* with *Cyphers annexed* by the given *Multiplier*, and the *Quotient* is the *Divisor* sought.

E X A M P L E.

Given Multiplier 125) 1.000 (,008 = the Divisor sought.

1000

...

Then

Division of Decimals.

33

Then { Multiply 7315 ,008) 7315,000 (914375
 by 125 72
 36575 11
 14630 8
 7315 35
 914375 32
 30
 24
 60
 56
 40
 40
 ..

Thus I have obtain'd a Quotient *the same* as the Product.
 Q. E. I.

PROBLEM II.

Suppose I have any Number 7315 to be *divided* by any other Number ,008; but would *multiply* the said Number, and have a *Product* equal to the *Quotient* of the same Number divided by ,008; *Quere the Multiplier*?

Rule. Divide an *Unit*, with *Cyphers annexed*, by the *given Divisor*, and the *Quotient* will be the *Multiplier sought*.

,008) 1000 (125 = the Multiplier sought.
 8

20
 16
 40
 40
 40
 ..

Thus you see this, and the Remainder of the Work, is only the Reverse of the former; and therefore need not be repeated.

F

Case

Case 2. If your *Divisor* consist of many *Places* of *Decimal Parts*, the *Work* may be very much *contracted*, and yet a *just Quotient* obtained by the following *Rule*. Having determin'd the *Value* of the *Quotient Figures*, proceed in *multiplying the Divisor* with the first *Quotient Figure* as usual; but for *every Figure* after, in *multiplying*, omit or *prick off one* in the *Divisor*; still having a due regard to the *Increase*, which would arise from the *Figure* and *Figures* so omitted.

EXAMPLES.

Contracted.

$$7,9863 \overline{) 70,2300} (8,7938$$

$$\begin{array}{r}
 638904 \\
 \cdot 63396 \\
 55904 \\
 \cdot 7492 \\
 7187 \\
 \cdot 305 \\
 239 \\
 \cdot 66 \\
 63 \\
 \cdot 3
 \end{array}$$

At large.

$$7,9863 \overline{) 70,2300} (8,7938$$

$$\begin{array}{r}
 638904 \\
 63396 \ 0 \\
 55904 \ 1 \\
 7491 \ 90 \\
 7187 \ 67 \\
 304 \ 230 \\
 239 \ 589 \\
 64 \ 6410 \\
 63 \ 8904 \\
 0 \ 7506
 \end{array}$$

Tho' much Labour be this way saved, yet it is not proper to use it unless the *Decimals* in the *Quotient* be sure to four or six *Places*; since 'tis obvious, the next *Place*, or all the *Remainder* of the *Quotient* in the *contracted Work* would be three times more or greater than the same in the *Work* at large.

If the *Dividend* contain many *Places* of *Decimals*, there's no occasion for using but a *very few* of the *first*, as appears by this second *Example*.

E X A M-

EXAMPLE II.

$$\begin{array}{r|l}
 3,141592) 165,6995 & 001296 (52,7438 \\
 \underline{1570796} & \\
 86199 & \dots \\
 \underline{62831} & \dots \\
 23368 & \dots \\
 \underline{21991} & \dots \\
 1377 & \dots \\
 \underline{1256} & \dots \\
 121 & \dots \\
 \underline{94} & \dots \\
 27 & \dots \\
 \underline{25} & \dots \\
 2 & \dots
 \end{array}$$

Here you may observe, that of *Ten Decimal Places* in the Dividend, I have used only *Four*; and yet have a Quotient to *four Places* of Decimals true: Hence all the Figures which would have fill'd the *dotted Space*, had it been work'd at large, are *superfluous*; and those, tis evident, make *half* the Work.

Case 3. If your *Dividend* contain a *single Repetend*, and your *Divisor* be a *single terminate Digit*, divide as usual; and when you take down your *Repetend*, the Quotient will *begin to repeat*.

EXAM. I.

$$\begin{array}{r}
 4) 195,0\dot{2} (48,75 \\
 \underline{16} \\
 35 \\
 \underline{32} \\
 30 \\
 \underline{28} \\
 22 \\
 \underline{20} \\
 2
 \end{array}
 \left. \begin{array}{l} \\ \\ \\ \\ \end{array} \right\} \text{Ad Infinitum.}$$

EXAM. II.

$$\begin{array}{r}
 6) 3176,\dot{0} (5293 \\
 \underline{30} \\
 17 \\
 \underline{12} \\
 56 \\
 \underline{54} \\
 29 \\
 \underline{18} \\
 2
 \end{array}
 \left. \begin{array}{l} \\ \\ \\ \\ \end{array} \right\} \text{Ad Infinitum.}$$

F 2

If

Division of Decimals.

If the Divisor be any Number of *terminate Digits*, the Quotient will repeat a single Digit; but not always begin, when the Repetend is taken down.

E X A M. III.

,48)79,28(165,138

$$\begin{array}{r}
 48 \\
 \hline
 312 \\
 288 \\
 \hline
 248 \\
 240 \\
 \hline
 66 \\
 48 \\
 \hline
 186 \\
 144 \\
 \hline
 426 \\
 384 \\
 \hline
 \end{array}$$

} *ad infinitum.*

.42

E X A M. IV.

487,65)106036,788(217,4

$$\begin{array}{r}
 97530 \\
 \hline
 85067 \\
 48765 \\
 \hline
 363028 \\
 341355 \\
 \hline
 216733 \\
 195060 \\
 \hline
 21673
 \end{array}$$

} *ad infinitum.*

If your Divisor be only a *single Repetend*, and the Dividend a *terminate Number*, multiply the Dividend by 9, cutting off one more Right-hand Figure in the *Product*, which is now your *new Dividend*; then divide as usual, and the Quotient will be *just*.

EXAMPLE V. Divide 572,4 by ,8.

The Dividend = 572,4
Multiply by 9

Divisor = ,8) 515,16 (= the new Dividend.
643,95 = the true Quotient.

$$\begin{array}{r}
 48 \\
 \hline
 35 \\
 32 \\
 \hline
 31 \\
 24 \\
 \hline
 76 \\
 72 \\
 \hline
 40 \\
 40 \\
 \hline
 \end{array}$$

Other.

Division of Decimals.

37

Otherwise thus ; place the Dividend *under it self*, but *one Place forward to the Right-hand* ; and then subtract, the Remainder will be the new Dividend, the same as before.

The Dividend	572,40	as before.
The same placed	5724	one Figure forwards.
Remains a new	515,16	Dividend as before.

From hence also appears the *Reason* of cutting off *one more Figure* in the new Dividend for *Decimals*.

That *either* of these Ways will give the same Quotient you have seen, and that the Quotient this way produced is the *only true one* will appear from the Work of the last Example at large.

8,) 572,4	0000	Ec.	(643,95
5333	3333	Ec.	
390	6666	Ec.	
385	5555	Ec.	
35	1111	Ec.	
28	6666	Ec.	
8	4444	Ec.	
8	0000	Ec.	
.	4444	Ec.	

} *Ad Infinitum.*

In this Operation, tis manifest though the *Repetends* in every particular Step would proceed to *Infinity*, yet in the last Place you see there is an *infinite Product* equal to an *infinite Remainder* ; and consequently the Work must there cease, and the Quotient nevertheless be *true*.

If the *Divisor* consists of *terminate Numbers* join'd to the *Repetend*, and the *Dividend* be *complete* ; proceed thus : Subtract the *terminate Numbers* of the *Divisor* from the *Divisor it self*, and the *Remainder* shall be a *new Divisor* ; and deal with the *Dividend* as in the last Example, for a *new Dividend*.

E X A M-

EXAMPLE VI.

Suppose it required to divide 8569,88 by 4,86; Work as follows;

4,86) 8569,88 (1760,9 the Quotient if work'd at large.
 48 856988

4,38) 7712,892 (1760,9 the same.

438

3332

3066

2668

2628

4092

3942

50

If the *Divisor* and *Dividend* do each contain a *Repetend*, order them as before directed; and the *Quotient* will be either *terminate*, repeat a *single Digit*, or else a *compound Repetend*.

EXAMPLE VII.

Divide 134,26 by ,6

13426

,6) 120,84 (201,4 the true Quotient.

12

08

6

24

24

..

EXAM-

Division of Decimals.

39

EXAMPLE VIII.

Divide 450,98 by ,06

$$\begin{array}{r} 45098 \\ \underline{45095} \\ ,06)405,860(6764,3 \end{array}$$

36

45

42

38

36

26

24

20

18

Ad Infinitum.

2

EXAMPLE IX.

Divide 23,46 by 7.

Thus { $\begin{array}{r} 23,46 \\ \underline{2346} \end{array}$

7)21,120(3,01714285

21

12

7

50

49

10

7

30

28

20

14

60

56

40

35

5

Ad Infinitum.

Case

Division of Decimals.

Case. 4. If *Compound Repetends* are found in your *Divisor*, or *Dividend*, or *both*; then observe to set the *Divisor* and *Dividend* *under themselves* so many *Places forwards* to the *Right-hand*, as there are *Places* in the *Repetend* of the *Divisor* *exclusively*; next, *subtract* them, and the *Remainders* will be respectively a *new Divisor* and *Dividend*.

EXAMPLE I.

Divide 243306, by 11198.

$$\begin{array}{r}
 111,98 \overline{) 243,306} \\
 \underline{243} \\
 111,87 \overline{) 243,063} \quad (2,172 \\
 \underline{22374} \\
 19323 \\
 \underline{11187} \\
 81360 \\
 \underline{78309} \\
 30510 \\
 \underline{22374} \\
 8136
 \end{array}
 \left. \vphantom{\begin{array}{r} 19323 \\ 11187 \\ 81360 \\ 78309 \\ 30510 \\ 22374 \\ 8136 \end{array}} \right\} \text{Ad Infinitum.}$$

The Truth of the Work will appear as well by the *common* Rule of multiplying the *Divisor* and *Quotient*, as by the Work at large.

If there be no *terminate* Part of the *Divisor*, you *subtract* nothing from it.

EXAM-

Division of Decimals.

41

EXAMPLE II.

Divide 395,273.614 by .317
Then 395273

.317) 394,878.341 (1245,673

317

• 778

634

• 1447

1268

• 1798

1585

• 2133

1902

• 2314

2219

• 951

951

• • •

EXAMPLE III.

Divide 70005 by 1.48.

70005

70005

1,48) 69934,995 (47253,375

592

1073

1036

• 374

296

789

740

499

499

444

• 555

444

1110

1036

• 740

740

• • •

G

IF

Division of Decimals.

If there be no *Repetend* in the *Divisor*, whatever the *Dividend* may be, there's no *Subtraction* to be made of either *Divisor* or *Dividend*.

EXAMPLE IV.

Divide 1761,3840r, by 417,64.

417,64) 1761,3840r (4,217

167056

90804

83528

72760

41764

309961

292348

17613

} *Ad Infinitum.*

EXAMPLE V.

Divide 31928,007r12, by 76,45

76,45) 31928,007r12 (417,6323

30580

13480

7645

58350

53515

48357

45870

24871

22935

19361

15290

40712

38225

2487

} *Ad Infinitum.*

In *Division* it may often happen that the *Quotient* may not *repeat* so soon as is desired; in such Case the Value of the *Quotient* may be exprelled compleatly by a *Vulgar Fraction*.
But

But in order to understand this, 'twill be necessary to premise the following *Lemma's*.

Lemma 1.

A *Series* of Nines *infinitely* continued, is equal to *Unity*, or One, in the next Left-hand Place; thus 0,999, &c. is equal to 1; and ,0999 &c. = ,1; and ,00999 &c. = ,01; and 549,99 &c. = 55.

Demonstration. 'Tis evident that ,9 = $\frac{9}{10}$ wants, only $\frac{1}{10}$ of *Unity*; and ,99 wants $\frac{1}{100}$; ,999 wants $\frac{1}{1000}$ of *Unity*; so that if the *Series* were continued to *Infinity*, the *Difference* between that *Series* of Nines and an *Unit*, would be equal to *Unity* divided by *Infinity*, that is, *Nothing* at all. Q. E. D.

Lemma 2.

Any single *Repetend* multiplied by 10, and then subtracted from that *Product*; the *Remainder* will be the same Number *complete* or *terminate*, in the next superior Left-hand Place.

Demonstration. Let the given *Repetend* be ,6666, &c. this multiplied by 10 is 6,666 &c.

From which Subtract ,666 &c.

There will remain 6, ... a whole Number.

Thus 47,77 &c. will become 430. and ,0333 &c. will be ,3. Q. E. D.

Corollary 1.

Hence it follows that if any *Compound Repetend* be multiplied by an *Unit* with so many *Cyphers* annexed as are equal to the Places of the *Repetend*, and then subtracted from the *Product*, there will be left to the Left-hand the same Numbers *terminate* and *complete*, that constituted the *Repetend*; thus, 325 multiplied by 1000, will be 325,325 from which if you subtract ,325 there will remain the *terminate* Number 325; Thus 12,743 will be 12731; and ,000743 will be ,743, and 5275,3 will become 5270,1.

Corollary. 2.

Hence also if any *Repetend* be multiplied by so many *Nines* as it contains Places, the *Result* will be the same as be-

Division of Decimals.

45

The Reason why the Quotient is thus expressed will be evident if we consider.

First, That 314 is not the *entire* Remainder because the Dividend is a *Repetend*, perpetually supplying a *circulating Remainder*, which expressed in its proper Terms would be, where we leave off, wrote thus 31426586. But this *infinite Series* of Figures is truly expressed thus, $314 \frac{26586}{99999}$ by *Corol. 3.*

Secondly, It being plain that $314 \frac{26586}{99999}$ is the true Remainder, and 417,2 the Divisor, 'tis necessary they should be expressed in the Quotient as here you see them by the Rules of *common Division*.

If instead of $314 \frac{26586}{99999}$ you write its *Equivalent* 31426586, and from it subtract the *terminate* part 314, there will remain 31426272 a new Numerator. And if to 417,2 you *add* as many *Cyphers* as the *Repetend* consists of Places, thus 417,200000; and again subtract it, as a *terminate* part, there will remain 417195828 for a *new Denominator*; and then this new and more *simple Fraction* $\frac{31426272}{417195828}$ will be equal to *that* in the *Quotient*.

The *Reason* of reducing the *Fraction* of the Quotient in this Manner is obvious from *Corol. 1. of Lem. 2.*

For $31426586 \times 100000 = 3142658626586$ &c.

From which subtract it self $\underline{31426586}$ &c.

And there will remain the new Numerator 31426272 as before.

Then $417,2 \times 100000 = 417,200000$

From which subtract it self $\underline{4172}$

There remains the Denominator 417195828 as before.

Thus I have supplied you with Rules for managing the *whole Doctrine of Circulating Numbers*; and given the *Theory* and *Reasons* for the same; which you may search for in vain in any other Book (that I know of) so fully as here laid down.

C H A P.

CHAP. V.

REDUCTION of DECIMALS.

IT being sufficiently experienced that all *Arithmetical Operations* are with the greatest *Facility* and *Expedition* work'd in *whole Numbers*; that *Vulgar Fractions*, and *Numbers of diverse Denominations* in their Management require *great Art*, and are attended with *much Perplexity*; and that the *noble Art of Decimal Arithmetick* alone is susceptible of all the various kinds of *Numbers*, and at the same time hath all its Operations perform'd by the same *easy* and *common Rule*, and in the very same manner of *Integral Quantities*, or *whole Numbers*; This, I say, being well known to all versed in the *Science of Numbers*, hath justly rendered *Decimal Arithmetick* in the greatest *Esteem* among those who *understand* it; and is most *generally used* by them in almost all kinds of *Numerical Calculations*.

The Part we now treat of is *absolutely necessary* to the true *Understanding* and *Use* of this excellent Art; and teaches, *First*, To *reduce* or express any *Vulgar Fraction* in *Decimal Parts* of the *Integral Quantity*.

Secondly, To *reduce* such *Numbers* as consist of *various Parts* and *Denominations*, as those of *Money*, *Weight*, *Measure*, &c. into *Decimals* for more *easy* Operation.

Thirdly, To *reduce* *Decimal Parts*, into the *common* and *known Parts* of *Money*, *Measure*, &c.

Case 1. To *reduce* *Vulgar Fractions* into *Decimals*, the *common Rule* is, to divide the *Numerator* by the *Denominator*, and the *Quotient* will be the *Decimal required*; that is, equivalent to the *Vulgar Fraction* given.

EXAMPLE I.

What is the *Decimal equivalent* to the *Fraction* $\frac{3}{4}$?

4) 3,00 (,75 The *Decimal required*.

$$\begin{array}{r} 28 \\ \hline \cdot 20 \\ 20 \\ \hline .. \end{array}$$

E X.

EXAMPLE II.

Reduce $\frac{1}{4}$ of a Pound into *Decimal Parts* of a Pound.

L. s. d.
8) 3,000 (375 = 7 : 6 the Answer.

$$\begin{array}{r} 24 \\ \hline \cdot 60 \\ 56 \\ \hline 40 \\ 40 \\ \hline \dots \end{array}$$

EXAMPLE III.

Reduce $\frac{1}{16}$ of a Pound Troy into *Decimal Parts*.

oz. pwt.
16) 3,0000 (1875 = 2 : 5 the Answer.

$$\begin{array}{r} 16 \\ \hline 140 \\ 128 \\ \hline 120 \\ 112 \\ \hline \dots 80 \\ 80 \\ \hline \dots \end{array}$$

EXAMPLE IV.

Reduce $\frac{1}{17}$ of a Rod into *Decimals*.

Rod. F. In. Or.
17) 4,000 (235 = 2 : 5 : 1 Answer.

$$\begin{array}{r} 27 \\ \hline 130 \\ 108 \\ \hline 220 \\ 216 \\ \hline \dots 4 \end{array}$$

Ad Infinitum.

EXAM-

EXAMPLE V.

Reduce $5\frac{1}{17}$ of a Hoghead into Decimals:
Gal. Pts.

$$\begin{array}{r} 27) 11,000 \quad (407 = 25 : 5 \\ \underline{108} \\ \cdot\cdot 200 \\ \underline{189} \\ 11 \end{array} \left. \vphantom{\begin{array}{r} 27) 11,000 \\ \underline{108} \\ \cdot\cdot 200 \\ \underline{189} \\ 11 \end{array}} \right\} \text{Ad Infinitum.}$$

H. Gal. Pints.

Hence the Answer is $5,407 = 5 : 25 : 5$.

Reduction of Vulgar Fractions into Decimals is also commodiously perform'd by Logarithms, especially when the Fraction is large; thus, subtract the Logarithm of the Denominator from the Logarithm of the Numerator, the Remainder is the Logarithm of the Decimal Parts.

EXAMPLE I. By Logarithms.

Reduce the Fraction $\frac{127}{4123}$ into Decimal Parts.

Thus, the Logarithm of 127 is — — 2.1038037
the Logarithm of 4123 subtract, — 3.6152133

Remains the Logarithm or the Decim. ,0308 = 8.4885904

EXAMPLE II.

What is the Decimal of the mix'd Fraction $5\frac{91}{3479}$

From the Logarithm of the Numerator 91 1.9590414
Subtract the Logarithm of the Denominator 3.5414544

There Remains the Log. of the Dec. ,026157 — 8.4175870

To which prefix the *integral Quantity* 5, and the Answer will be compleat, thus 5.026157.

The same thing might as well have been done by reducing the *mixt* Fraction into an *improper* one, viz. $\frac{17486}{3479}$

Then,

Then, as in the first Example.

From the Log. of the Numerator 17486 — 4.2426904
Subtract the Log. of the Denominator 3479 — 3.5414544

Remains the Log. of the Answer 5,026157 — 0.7012360

Note, In the first and second Examples, and in all such Cases where you subtract a *greater* Index from a *less*, you borrow *Tens* and as *many Digits* as the *remaining Index* want; of *Nine*, so *many Cyphers* prefix to the *Decimal*.

Case 2. To reduce Numbers which express Quantities of various Kinds and Denominations, as *Money, Measure, &c.* into *Decimals*, there are *three Ways* or *Methods*, which are as follows.

Method 1. Reduce the *different Species* to *one*; that is, to the lowest Denomination they consist of: then reduce the *Integer* to the *same Denomination*; the *first* will be the *Numerator*, the *latter* the *Denominator* of a *Vulgar Fraction*; which Fraction reduced to a *Decimal* (by *Case 1.*) will be *that* required.

EXAMPLE I.

What Decimal Part of a Pound is 5 s. 7 d. $\frac{3}{4}$?

<p>• s. Multiply 5 By 12 d. 12</p> <hr/> <p>60</p> <p>Add the 7 d. 7</p> <hr/> <p>67 Pence.</p> <p>Mul. by 4 q 4</p> <hr/> <p>268</p> <p>Add the 3 q 3</p> <hr/> <p>271 Farthings.</p>	<p>l. Then reduce 1 the Integer.</p> <hr/> <p>20</p> <hr/> <p>20</p> <hr/> <p>12</p> <hr/> <p>240</p> <hr/> <p>4</p> <hr/> <p>960 Farthings in a Pound.</p>
--	---

That is $\left\{ \frac{271 \text{ the Numerator}}{960 \text{ the Denominator}} \right\}$ of the Vulgar Fraction.

Then 960) 271,0000 (.28229 &c. The Answer.

H

So

50 Reduction of different Denominations;

So that ,28229 is the *Decimal Part* of a Pound in one Denomination, equal to 5 s. 7 d. $\frac{1}{4}$, the Part of a Pound in *diverse Denominations*.

EXAMPLE II.

What *Decimal Part* of a *Hundred Weight* is 2 q. 21 lb. 12 oz.?

Q. lb. oz. C.

Reduce 2 : 21 : 12 to Ounces. And 1 reduce to Ounces.

$$\begin{array}{r} 28 \\ \hline 77 \\ 16 \end{array}$$

$$\begin{array}{r} 464 \\ 78 \end{array}$$

1244 Ounces

$$\begin{array}{r} 4 \\ \hline 4 \\ 28 \end{array}$$

$$\begin{array}{r} 112 \\ 16 \end{array}$$

1792 Ounces in C. W.

Then $\left\{ \frac{1244}{1792} \right\}$ is the Vulgar Fraction.

And 1792) 1244,0 (=,694196 the *Decimal Part* of an *Hundred Weight* (answering to 2 q. : 21 lb. : 12 oz. required.

EXAMPLE III.

What *Decimal Part* of a *Rod* or *Pole*, is 4 Y. : 2 F. : 8 In.?

Y. F. In.

4 : 2 : 8

$$\begin{array}{r} 3 \\ \hline \end{array}$$

14

12

176 Inches.

1 Pole.

$$\begin{array}{r} 5,5 \\ \hline \end{array}$$

5,5

3

16,5

12

198,0 Inches in a Pole.

But $\frac{176}{198} = \frac{88}{99} = ,8$ the (repeating) *Decimal* of a *Pole*, equal to the 4 *Yards*, 2 *Feet*, and 8 *Inches*.

In the same *Manner* proceed with any other given *Species*.
Method

Method 2. Find what *Decimal Part* the *least Denomination* of the given *Species* is of the next *superior*, to which prefix the given Part of the next superior Denomination; then see what *Decimal Part* this *mixt Number* is of the next *superior Denomination*, to which again prefix what is *given of it*; and thus proceed till you ascend to the *Integer* it self, and find what *Decimal Part* of it the last *mixed Number* is, which will be *that sought*.

EXAMPLE I.

What *Decimal Part* of a *Pound* is 12 s. 6 d. $\frac{1}{2}$?

First 4) 2,0 (5 the *Decimal* of one Penny for $\frac{1}{2}$.

Secondly 12) 6,5 (5418 the *Dec. Part* of a Shill. for 6 d. $\frac{1}{2}$.

Thirdly 20) 12,5418 (627083 the *Decimal Part* of a *Pound*, as was required, for 12 s. 6 d. $\frac{1}{2}$.

EXAMPLE II.

What *Decimal* of a *Pound Troy* is 2 oz. 18 pwt. 20 gr.?

First 24) 20,000 (875 the *Decimal* for 20 gr.

Secondly 20) 18,875 (94375 the *Decimal* for 18 pwt. 20 gr.

Thirdly, 12) 2,94375 (.2453125 the *Decimal Part* of a *Pound Troy* for 2 oz. 18 pwt. 20 gr. as was required.

EXAMPLE III.

What *Decimal Part* of a *Year* is 6 m. 3 w. 5 d. 6 h. 40' 50"?

First 60) 50,00 (.83 Decimals for 50" of 1'.

Secondly 60) 40,83 (.6808 Dec. for 40' : 50" of an *Hour*.

Thirdly, 24) 6,6808 (.2783148 = 6 H. 40' : 50" of a *Day*.

Fourthly, 7) 5,2783148 (.75404 Sc. Decimals of a *Week*.

Fifthly, 4) 3,75404 Sc. (.93851 Decimals of a *Month*.

Sixthly, 12) 6,93851 (.53373 Decimals of a *Year*.

So that we see the *six* different Parts of Time above specified are reduced to this small *Decimal* .53373; which expresseth the *same Part* of a *Year* as they do; which, by the way, may be an Instance of the great *Simplicity*, *Ease*, and *Excellency* of this *admirable Art*.

In these three Examples I have omitted the Work at large, setting down only the *Divisors*, *Dividends*, and *Quotients* as

52 *Reduction to Decimals by Tables.*

sufficient to give the *Learner* as good a Notion of the Method as the Operations at length, which he may make his *Exercise at pleasure* to good advantage.

Method 3. The *third Method* for finding the *Decimal* of any given *Part* of Quantity consisting of *diverse Denominations*, is by *Tables* ready calculated for that purpose.

This is not only the most *easy*, but the most *expeditious* Manner of working the Processes of *all Kind* of Computations in general; and is of particular service in this Case of *preparing* Numbers for *Decimal Operations*.

For that reason I have here inserted a *Set of Tables*, which, tho' some are *common* of this kind, are the most *complete* and *universal* of any I have seen extant; and in order to render them so, I have not only very much *enlarged* and *new* *vamped* the old ones, but also *added* other very useful ones; as those who are read in these Matters, will soon perceive.

By the *following Tables*, all the *Species* of *Money, Weight, Measure, &c.* consisting of *what ever* Denomination, and be the *Integer* what you please, are immediately turn'd into *Decimal Parts*; and are then work'd with the known *Facility* and *Pleasure* of Whole Numbers.

As to the *manner of using* those *Tables*, that is so *obvious* and *natural*, even by a *bare Inspection*, that I presume tis needless to say any thing to a Person of *Genius*, though a *Learner*, about that. The Scheme of *Examples* following being sufficient to testify the great *Use* and *Excellency* of such *Tables*, and are both *Precedents* and *Precepts* themselves.

E X A M P L E I.

What is the *Decimal Part* of a *Pound* for 13 s. 7 d. $\frac{3}{4}$?

In *Table I.* you find answering to

{	13 Shillings	}	—	—	,65
	7 Pence		—	—	,0322916
	3 Farthings		—	—	

The Answer is — — — — — ,6822916

E X A M P L E II.

What *Decimal Part* of a *Mark* is 11 s. 2 d. $\frac{1}{2}$?

In *Table I.* under a *Mark*, against

{	11 Shillings	}	—	—	,65
	2 Pence		—	—	,0125
	2 Farthings		—	—	,003125

The Answer — — — — — ,840625

E X A M-

EXAMPLE III.

What *Decimal Part* of a *Moidore* is 9 s. 3 d. $\frac{1}{4}$?

In *Table I.* under *Moi-* { 9 Shillings — — — ,333333
dore you find against { 3 Pence — — — ,00259
 { 1 Farthing — — — ,000771

The Answer is this *compound Repetend* ,343363492

EXAMPLE IV.

What *Decimal* of a *Pound Troy* answers to 5 oz. 17 pwt. 22 gr.?

In *Table II.* you { 5 Ounces — — — ,416666
 find against { 17 Penny Weight — — — ,070833
 { 22 Grains — — — ,003819

The *Sum* of which is the Answer — — — ,491319

EXAMPLE V.

What *Decimal Part* of an *Hundred Weight* is 21 lb. 14 oz.?

In *Table III.* you { 21 Pounds — — — ,1875
 find against { 14 Ounces — — — ,007812

The Answer is — — — ,195312

EXAMPLE VI.

What *Decimal Part* of a *Tun* is 3 qr. 6 bush. 7 gall.?

In *Table V. Dry* { 3 Quarters — — — ,6
Measure, against { 6 Bushels — — — ,15
 { 7 Gallons — — — ,021875

The Answer is — — — ,771875

EXAMPLE VII.

What *Decimal Part* of a *Hogshead* of Wine, is 2 $\frac{1}{2}$ Rund. 14 Gall.?

In *Table VI. Liquid Mea-* { 2 $\frac{1}{2}$ Rundlets ,71425
sure, you find against { 14 Gallons ,22222 &c.

The Answer (*repeating a single Figure* is ,936472 &c.

E X A M.

EXAMPLE VIII.

How is 27 Miles, 7 Furlongs, 25 Rod, and 4 Yards expressed in Decimals?

In Table VIII. Long Measure (one Mile the Integer) you find against

{	7 Furlongs	—	,	875
{	35 Rod, or Poles,	—	,	109375
{	4 Yards	—	,	002272

The 27 Miles prefixed, the Answer will be 27,986647

EXAMPLE IX.

What Decimal Part of a Year is 7 Months, 3 Weeks, and 2 Days?

In Table IX. you observe against

{	7 Months	—	,	538461
{	3 Weeks	—	,	057630
{	2 Days	—	,	005494

The Answer is ,601585

EXAMPLE X.

What Decimal Part of a Sign of the Zodiac is 25° 46' 8"?

In Table X. you find against

{	25 Degrees	—	,	833333
{	46 Minutes	—	,	025555
{	8 Seconds	—	,	000073

The Answer is a single Repetend — ,8589618

EXAMPLE XI.

What Decimal Part of a Degree, is 49' 57"?

In the same Table you see against

{	49 Minutes	—	,	816666
{	57 Seconds	—	,	015833

The Answer is — ,8325

Having thus so largely exemplified the Use of the Tables, the Tables themselves follow; wherein observe, 1. I have dash'd the first Figure of all single Repetends, and the first and last of the compound Repetends that come within the Table. 2. I have nevertheless continued each to six Places for their sakes who would be exact, but know not well how to manage Repetends.

Common Tables of Money, Weights, Measures, and Time.

Table I. Money.

Farth.

4 = 1 Penny.

48 = 12 = 1 Shilling.

960 = 240 = 20 = 1 Pound.

Table II. Apothecaries Weight.

Grains.

20 = 1 Scruple.

60 = 3 = 1 Dram.

480 = 24 = 8 = 1 Ounce.

5760 = 288 = 96 = 12 = 1 lb.

Table III. Troy Weight.

Grains.

24 = 1 Penny Weight.

480 = 20 = 1 Ounce.

7560 = 240 = 12 = 1 lb.

Table IV. Moneyers Weight.

Blanks.

24 = 1 Periot.

480 = 20 = 1 Droite.

11520 = 480 = 24 = 1 Mite.

230400 = 9600 = 480 = 20 = 1 Grain.

Table V. Averdupois Weight.

Drams.

16 = 1 Ounce.

256 = 16 = 1 Pound.

28672 = 1792 = 112 = 1 Hundred.

573440 = 35840 = 2240 = 20 = 1 Tun.

Table

Table VI: Wine Measure.

Cubic In.

231	=	1	Gallon.
9702	=	42	= 1 Terce.
14553	=	63	= 1½ = 1 Hogshead.
19404	=	84	= 2 = 1½ = 1 Punch.
29106	=	126	= 3 = 2 = 1½ = 1 Butt.
58212	=	252	= 6 = 4 = 3 = 2 = 1 Tun.

Table VII, Ale Measure.

Cubic In.

282	=	1	Gallon.
2256	=	8	= 1 Firkin.
4512	=	16	= 2 = 1 Kilderkin.
9024	=	32	= 4 = 2 = 1 Barrel.
13536	=	48	= 6 = 3 = 1½ = 1 Hogshead.

Table VIII. Beer Measure.

Cubic In.

282	=	1	Gallon.
2583	=	9	= 1 Firkin.
5076	=	18	= 2 = 1 Kilderkin.
10152	=	36	= 4 = 2 = 1 Barrel.
15228	=	54	= 6 = 3 = 1½ = 1 Hogshead.

Table IX. Dry Measure.

Cubic In.

268.8	=	1	Gallon.
537.6	=	2	= 1 Peck.
2150.4	=	8	= 4 = 1 Bushel.
8601.6	=	32	= 16 = 4 = 1 Coomb.
17203.2	=	64	= 32 = 8 = 2 = 1 Quarter.
68812.8	=	256	= 128 = 32 = 8 = 4 = 1 Chalder.
86016.0	=	350	= 160 = 40 = 10 = 5 = 1 Wey, or Load.
172032.0	=	640	= 320 = 80 = 20 = 10 = 2 = 1 Last.

Table

Table X. Of Time.

Seconds.

60 =	1 Minute.
3600 =	60 = 1 Hour.
86400 =	1440 = 24 = 1 Day.
604800 =	10080 = 168 = 7 = 1 Week.
2419200 =	40320 = 672 = 28 = 4 = 1 Month.
31556937 =	525949 = 8765 = 365 = 52 = 13 + 1 Day, + 5 Hours, + 48' + 57" = Year.

Table XI. Long Measure.

Barly Corns.

3 =	1 Inch.
36 =	12 = 1 Foot.
108 =	36 = 3 = 1 Yard.
594 =	198 = 16½ = 5½ = 1 Pole.
23760 =	7920 = 660 = 220 = 40 = 1 Furlong.
190080 =	63360 = 5280 = 1760 = 320 = 8 = 1 Mile.

Table XII. Square Measure.

Square In.

144 =	1 Feet sq.
1296 =	9 = 1 Yards sq.
3600 =	25 = 2½ = 1 Paces sq.
39204 =	372½ = 30½ = 10½ = 1 Poles sq.
1568160 =	10890 = 1210 = 435½ = 40 = 1 Rood sq.
6272640 =	43560 = 4840 = 1742½ = 160 = 4 = 1 Acres sq.

Table XIII. Scripture Measure.

Digits.

4 =	1 Palm.
12 =	3 = 1 Span.
24 =	6 = 2 = 1 Cubit.
96 =	24 = 8 = 4 = 1 Fathom.
144 =	36 = 12 = 6 = 1½ = 1 Ezekiels Reed.
192 =	48 = 16 = 8 = 2 = 1½ = 1 Arabian Pole.
1920 =	480 = 160 = 80 = 20 = 13½ = 10 = 1 Schœnus or measuring Line.

Table IV. Eastern Measure.

Cubits.

400 =	1 Stadium.
2000 =	5 = 1 Sabbath Days Journey.
4000 =	10 = 2 = 1 Eastern Miles.
12000 =	30 = 6 = 3 = 1 Parasang.
96000 =	240 = 48 = 24 = 8 = 1 Days Journey.

Table XV. Hebrew Measure.

Gachal.

20 =	1 Cab.
36 =	1½ = 1 Omer.
120 =	6 = 3½ = 1 Seab.
360 =	18 = 10 = 3 = 1 Epha.
1800 =	90 = 50 = 15 = 5 = 1 Lethch.
3600 =	180 = 100 = 30 = 10 = 2 = 1 Homer, or Coron.

Table XVI. Hebrew Measure.

Eaph.

1½ =	1 Log.
5½ =	4 = 1 Cab.
16 =	12 = 3 = 1 Hin.
32 =	24 = 6 = 2 = 1 Seab.
96 =	72 = 18 = 6 = 3 = 1 Bath Epha.
960 =	720 = 180 = 60 = 30 = 10 = 1 Coron Chomer.

Table XVII. Hebrew Money.

Gerahs.

10 =	1 Bekah.
20 =	2 = 1 Shekel.
1200 =	120 = 60 = 1 Maneh.
60000 =	6000 = 3000 = 50 = 1 Talent.

Decimal

Decimal Tables of Money, Weight, &c.

TABLE I. Of Money; one Pound the Integer.			P. q. Dec. Par.	P. q. Dec. Par.
S. Dec. Par.	P. q.	Dec. Par.		
1,05	0	$\frac{1}{8}$,0005208	8,0,033333	4,0,333333
2,1	0	$\frac{1}{4}$,0010416	8, $\frac{1}{4}$,034375	4, $\frac{1}{4}$,354166
3,15	0	$\frac{1}{2}$,002083	8, $\frac{1}{2}$,035416	4, $\frac{1}{2}$,375
4,2	0	$\frac{3}{4}$,003125	8, $\frac{3}{4}$,0364583	4, $\frac{3}{4}$,395833
5,25	1,0	0,00416	9,0,0375	5,0,418666
6,3	1	$\frac{1}{4}$,0052	9, $\frac{1}{4}$,0385416	5, $\frac{1}{4}$,4375
7,35	1	$\frac{1}{2}$,00625	9, $\frac{1}{2}$,039583	5, $\frac{1}{2}$,458333
8,4	1	$\frac{3}{4}$,00729	9, $\frac{3}{4}$,040625	5, $\frac{3}{4}$,479166
9,45	2,0	0,00833	10,0,041666	6,0,5
10,5	2	$\frac{1}{4}$,009375	10, $\frac{1}{4}$,0427083	6, $\frac{1}{4}$,520833
11,55	2	$\frac{1}{2}$,010416	10, $\frac{1}{2}$,04375	6, $\frac{1}{2}$,541666
12,6	2	$\frac{3}{4}$,0114583	10, $\frac{3}{4}$,0447916	6, $\frac{3}{4}$,5625
13,65	3,0	0,0125	11,0,045833	7,0,583333
14,7	3	$\frac{1}{4}$,0135416	11, $\frac{1}{4}$,046875	7, $\frac{1}{4}$,604166
15,75	3	$\frac{1}{2}$,014583	11, $\frac{1}{2}$,047916	7, $\frac{1}{2}$,625
16,8	3	$\frac{3}{4}$,015625	11, $\frac{3}{4}$,0489583	7, $\frac{3}{4}$,645833
17,85	4,0	0,016666	One Shilling the Integer.	
18,9	4	$\frac{1}{4}$,0177083		
19,95	4	$\frac{1}{2}$,01875	0, $\frac{1}{8}$,010416	8,0,666666
	4	$\frac{3}{4}$,0197916	0, $\frac{1}{4}$,020833	8, $\frac{1}{4}$,6875
	5,0	0,02083	0, $\frac{1}{2}$,041666	8, $\frac{1}{2}$,708333
	5	$\frac{1}{4}$,021875	0, $\frac{3}{4}$,0625	8, $\frac{3}{4}$,729166
	5	$\frac{1}{2}$,022916	0, $\frac{1}{4}$,083333	9,0,75
	5	$\frac{3}{4}$,0239583	1,0,083333	9, $\frac{1}{4}$,770833
	6,0	0,025	1, $\frac{1}{4}$,104166	9, $\frac{1}{2}$,791666
	6	$\frac{1}{4}$,0260416	1, $\frac{1}{2}$,125	9, $\frac{3}{4}$,8125
	6	$\frac{1}{2}$,0270083	1, $\frac{3}{4}$,145833	10,0,833333
	6	$\frac{3}{4}$,028125	2,0,16666	10, $\frac{1}{4}$,854166
	7,0	0,029166	2, $\frac{1}{4}$,1875	10, $\frac{1}{2}$,875
	7	$\frac{1}{4}$,0302083	2, $\frac{1}{2}$,208333	10, $\frac{3}{4}$,895833
	7	$\frac{1}{2}$,03125	2, $\frac{3}{4}$,229166	11,0,916666
	7	$\frac{3}{4}$,0322916	3,0,25	11, $\frac{1}{4}$,9375
			3, $\frac{1}{4}$,270833	11, $\frac{1}{2}$,958333
			3, $\frac{1}{2}$,291666	11, $\frac{3}{4}$,979166
			3, $\frac{3}{4}$,3125	

60 *Decimal Tables of Money, Weight,*

I Crown the In- teger.	P. D. Part.	P. D. Parts.	P. Dec. Parts.
	1,0125	6,0375	1,00396825
	2,025	7,04375	2,007936
S. D. Par.	3,0375	8,05	3,011904
	4,05	9,05625	4,015873
1,2	5,0625	10,0625	5,019841
2,4	6,075	11,06875	6,023809
3,6	7,0875		7,027777
4,8	8,1	qr. D. Parts.	8,031746
	9,1125	1,0015625	9,035714
P. D. Par.	10,125	2,003125	10,039682
	11,1375	3,0046875	11,043650
1,01666	qr. D. Part.		qr. Dr. Parts.
2,03333	1,003125		1,000992
3,05	2,00625	A Guinea the Integer.	2,001984
4,06666	3,009375		3,002976
5,08333		S. D. Parts.	
6,1	A Mark the Integer.	1,047619	
7,11666	S. D. Part.	2,095238	A Corol the Integer.
8,13333	1,075	3,142857	
9,15	2,15	4,190476	S. 043478
10,16666	3,225	5,238095	
11,18333	4,3	6,285714	1,043478
qr. D. Par.	5,375	7,333333	2,086957
	6,45	8,380952	3,130436
1,00416	7,525	9,428571	4,173914
2,00833	8,6	10,476190	5,217393
3,0125	9,675	11,523089	6,260872
	10,75	12,571428	7,304350
A Noble the In- teger.	11,825	13,619047	8,347829
	12,9	14,666666	9,391308
S. D. Par.	13,975	15,714285	10,434786
	P. D. Part.	16,761904	11,478265
1,15	1,00625	17,809523	12,521744
2,3	2,0125	18,857142	13,565222
3,45	3,01875	19,904761	14,608701
4,6	4,025	20,952380	15,652180
5,75	5,03125		16,695659
6,9			17,739137

S. D. Part.	S. D. Part.	A Moldore the Inter- ger.	S. D. Parts.	S. D. Parts.
18,782616	9,36		21,777777	
19,826095	10,4		22,814814	
20,869573	11,44		23,851851	
21,913052	12,48		24,888888	
22,956531	13,52		25,925925	
P. D. Part.	14,56		26,962962	
1,003623	15,6		P. D. Parts.	
2,007246	16,64		1,003086	
3,010969	17,68		2,006172	
4,014592	18,72		3,009259	
5,018215	19,76		4,012345	
6,021938	20,8		5,015432	
7,025561	21,84		6,0185185	
8,029184	22,86		7,021604	
9,032707	23,92		8,024691	
10,036232	24,96		9,027777	
11,039855	P. D. Part.		10,030864	
qr. D. Part.	1,003333		11,033950	
1,000905	2,006666		qr. D. Parts.	
2,001811	3,01		1,000771	
3,002717	4,013333		2,001543	
	5,016666		3,002314	
	6,02			
	7,023333			
	8,026666			
A Jacobus the Inter- ger.	9,03			
	10,033333			
	11,036666			
S. D. Part.	qr. D. Part.			
1,04	1,000833			
2,08	2,001666			
3,12	3,0025			
4,16				
5,2				
6,24				
7,28				
8,32				

TABLE II.

Troy Weight; one Pound
the Integer.

Oz.	D. Par.	Oz.	D. Par.
1	,08333	7	,58333
2	,16666	8	,66666
3	,25	9	,75
4	,33333	10	,83333
5	,41666	11	,91666
6	,5		

Pwt.

62 *Decimal Tables of Money, Weight,*

Pw.	D. Part.	Pw.	D. Part.	Gr.	D. Part.	Gr.	D. Part.
1	,004166	19	,003298	8	,018666	16	,033333
2	,008833	20	,003472	9	,01875	17	,035416
3	,0125	21	,003646	10	,020833	18	,0375
4	,016666	22	,003819	11	,022916	19	,039583
5	,020833	23	,003993	12	,025	20	,041666
6	,025			13	,027083	21	,04375
7	,029166			14	,029166	22	,045833
8	,033333			15	,03125	23	,047916
9	,0375						
10	,041666						
11	,045833						
12	,05						
13	,054166						
14	,058333						
15	,0625						
16	,066666						
17	,070833						
18	,075						
19	,079166						
Gr.	D. Part.						
1	,000173						
2	,000347						
3	,000521						
4	,000694						
5	,000868						
6	,001042						
7	,001215						
8	,001389						
9	,001562						
10	,001736						
11	,00191						
12	,002083						
13	,002257						
14	,002431						
15	,002604						
16	,002778						
17	,002951						
18	,003125						
						</	

Oz.	D. Part.	Oz.	D. Part.	Oz.	D. Part.	Sc.	D. Part.
7	,003906	10	,625	11	,91866	1	,041866
8	,004464	11	,6875	Dr.	D. Part.	2	,083333
9	,005022	12	,75	1	,010418	Gr.	D. Part.
10	,00558	13	,8125	2	,020833	1	,002083
11	,006138	14	,875	3	,031250	2	,004166
12	,006696	15	,9375	4	,041866	3	,00625
13	,007254	Dr.	D. Part.	5	,052083	4	,008333
14	,007812	1	,003906	6	,0625	5	,010418
15	,00837	2	,007812	7	,072916	6	,0125
One Pound the Integer.		3	,011718	Sc.	D. Part.	7	,014583
Oz.	D. Part.	4	,015625	1	,003472	8	,018666
1	,0625	5	,019531	2	,006944	9	,01875
2	,125	6	,023437	One Ounce the Integer.		10	,020833
3	,1875	7	,027343	Dr.	D. Part.	11	,022916
4	,25	8	,03125	1	,125	12	,025
5	,3125	9	,035156	2	,25	13	,027083
6	,375	10	,039062	3	,375	14	,029166
7	,4375	11	,042968	4	,5	15	,03125
8	,5	12	,046875	5	,625	16	,033333
9	,5625	13	,050781	6	,75	17	,035416
		14	,054687	7	,875	18	,375
		15	,058593			19	,039583

TABLE IV.

Apothecary's Weight,
one Pound the Integer.

Oz.	D. Part.	Oz.	D. Part.
1	,08333	6	,5
2	,16666	7	,58333
3	,25	8	,86666
4	,33333	9	,75
5	,41666	10	,83333

TABLE V.

Dry Measure; one Tun or
Load the Integer.

Qr.	D. Part.	Qr.	D. Part.
1	,2	3	,6
2	,4	4	,8

64 *Decimal Tables of Money, Weight,*

B. D. Part.		G. D. Part.		TABLE VI.			
1	,025	1	,015625	Liquid Measure. For			
2	,05	2	,03125	Wine, &c. One Tun			
3	,075	3	,046875	the Integer.			
4	,1	4	,0625				
5	,125	5	,078125				
6	,15	6	,09375				
7	,175	7	,109375				
G. D. Part.				H. D. Part.		One Hogsh.	
1	,003125	One Bushel		1	,25	the Integer.	
2	,00625	the Integer.		2	,5	R.	D. Part.
3	,009375			3	,75		
4	,0125	G.	D. Part.	R.	D. Part.	1	,14285
5	,015625	1	,125	1	,035714	1	,28571
6	,01875	2	,25	1	,071428	1	,42856
7	,021875	3	,375	1	,107143	2	,57142
		4	,5	2	,142857	2	,71425
		5	,625	2	,178572	3	,85713
		6	,75	3	,214285	G.	D. Part.
		7	,875	G.	D. Part.	1	,015873
One Quarter		P.	D. Part.	1	,003968	2	,031746
the Integer.		1	,015625	2	,007936	3	,047619
B.	D. Part.	2	,03125	3	,011904	4	,063492
1	,125	3	,046875	4	,015872	5	,079365
2	,25	4	,0625	5	,019841	6	,095239
3	,375	5	,078125	6	,023808	7	,111111
4	,5	6	,09375	7	,027776	8	,126984
5	,625	7	,109375	8	,031744	9	,142857
6	,75			9	,035714	10	,15873
7	,875			10	,039682	11	,174603
				11	,04365	12	,190476
				12	,047618	13	,206349
				13	,051586	14	,222222
				14	,055555	15	,238095
				15	,059523	16	,253968
				16	,063491	17	,269841
				17	,06746		

One Rundlet the Integer.		P.	D. Part.	G.	D. Part.	G.	D. Part.
G.	D. Part.						
1	,095555	1	,006944	1	,018518	5	,092892
2	,x11111	2	,013888	2	,037037	6	,x11111
3	,186666	3	,020833	3	,055555	7	,129829
4	,222222	4	,027777	4	,074074	8	,x48148
5	,277777	5	,034722				
6	,333333	6	,041666				
7	,388888	7	,048611				
8	,444444						
9	,5						
10	,555555						
11	,6x1111						
12	,666666						
13	,722222						
14	,777777						
15	,833333						
16	,888888						
17	,944444						

One Gallon
the Integer.

P.	D. Part.
1	,125
2	,25
3	,375
4	,5
5	,625
6	,75
7	,875

TABLE VII.

Of Ale and Beer Mea-
sure ; one Hoghead
the Integer.

Fr.	D. Part.	G.	D. Part.
1	,18666	2	,041666
2	,33333	3	,0625
3	,5	4	,083333
4	,66666	5	,104166
5	,83333	6	,125
G.	D. Part.	7	,145833
1	,020833		

TABLE VIII.

Long Measure ; one Mile
the Integer.

F.	D. Part.	P.	D. Part.
1	,125	17	,053125
2	,25	18	,05625
3	,375	19	,059375
4	,5	20	,0625
5	,625	21	,065625
6	,75	22	,06875
7	,875	23	,071875
P.	D. Part.	24	,075
1	,003125	25	,078125
2	,00625	26	,08125
3	,009375	27	,084375
4	,0125	28	,0875
5	,015625	29	,090625
6	,01875	30	,09375
7	,021875	31	,096875
8	,025	32	,1
9	,028125	33	,103125
10	,03125	34	,10625
11	,034375	35	,109375
12	,0375	36	,1125
13	,040625	37	,115625
14	,04375	38	,11875
15	,046875	39	,121875
16	,05		

Y.	D. Part.	P.	D. Part.	In.	D. Part.	In.	D. Part.
$\frac{1}{2}$,000284	28	,7	1	,005050	1	,0277
1	,000568	29	,725	2	,010101	2	,0555
2	,001136	30	,75	3	,01515	3	,0833
3	,001704	31	,775	4	,020202	4	,1111
4	,002272	32	,8	5	,02525	5	,1388
5	,002841	33	,825	6	,030303	6	,1666
		34	,85	7	,035353	7	,1944
		35	,875	8	,040404	8	,2222
		36	,9	9	,045454	9	,25
		37	,925	10	,050505	10	,2777
		38	,95	11	,055555	11	,3055
		39	,975				
<i>One Furlong the Integer.</i>		Y.	D. Part	<i>One Yard the Integer.</i>			
P.	D. Part.	1	,004545	F.	D. Part.		
1	,025	2	,009090	1	,3333		
2	,05	3	,013636	2	,6666		
3	,075	4	,018181				
4	,1	5	,022727				
5	,125	F.	D. Part.				
6	,15	1	,001515				
7	,175	2	,003030				
8	,2						
9	,225						
10	,25						
11	,275						
12	,3						
13	,325						
14	,35						
15	,375						
16	,4						
17	,425						
18	,45						
19	,475						
20	,5						
21	,525						
22	,55						
23	,575						
24	,6						
25	,625						
26	,65						
27	,675						

D. D. Part.		D. D. Part.		T A B L E X.			
				Of Motion, A Sign of the Zodiac the Integer.			
				D ^o . D. Part.		M'. D. Part.	
1	,002747	2	,285714	1	,033333	3	,001866
2	,005494	3	,438571	2	,066006	4	,002222
3	,008241	4	,571428	3	,1	5	,002777
4	,010988	5	,714285	4	,133333	6	,003333
5	,013735	6	,857142	5	,166666	7	,003888
6	,016482			6	,2	8	,004444
		H. D. Part.					
One Month the Integer.		1	,005952	7	,233333	9	,005
		2	,011904	8	,266666	10	,005555
		3	,017856	9	,3	11	,006111
		4	,023808	10	,333333	12	,006666
		5	,02976	11	,366666	13	,007222
W. D. Part.							
1	,25	6	,035712	12	,4	14	,007777
2	,5	7	,041664	13	,433333	15	,008333
3	,75	8	,047616	14	,466666	16	,008888
D. D. Part.							
1	,035714	9	,053568	15	,5	17	,009444
2	,071428	10	,05952	16	,533333	18	,01
3	,107142	11	,065472	17	,566666	19	,010555
4	,142856	12	,071424	18	,6	20	,011111
5	,17857	13	,077376	19	,633333	21	,011666
6	,214284	14	,083328	20	,666666	22	,012222
One Week the Integer.		15	,08928	21	,7	23	,012777
		16	,095232	22	,733333	24	,013333
		17	,101184	23	,766666	25	,013888
		18	,107136	24	,8	26	,014444
		19	,113088	25	,833333	27	,015
D. D. Part.							
1	,142857	20	,11904	26	,866666	28	,015555

M'. D. Part.	S". D. Part.	S". D. Part.	S". D. Part.	M'. D. Part.	M'. D. Part.
37 ,020355	15 ,000137	54 ,000496	28 ,486666		
38 ,021111	16 ,000146	55 ,000506	29 ,483333		
39 ,021866	17 ,000155	56 ,000515	30 ,5		
40 ,022222	18 ,000164	57 ,000524	31 ,518666		
41 ,022777	19 ,000173	58 ,000533	32 ,533333		
42 ,023333	20 ,000184	59 ,000542	33 ,55		
43 ,023888	21 ,000193		34 ,586666		
44 ,024444	22 ,000202		35 ,583333		
45 ,025	23 ,000211		36 ,6		
46 ,025555	24 ,00022		37 ,618666		
47 ,026111	25 ,000229		38 ,633333		
48 ,026666	26 ,000238		39 ,65		
49 ,027222	27 ,000247		40 ,866666		
50 ,027777	28 ,000256		41 ,683333		
51 ,028333	29 ,000265		42 ,7		
52 ,028888	30 ,000276		43 ,718666		
53 ,029444	31 ,000285		44 ,733333		
54 ,03	32 ,000294		45 ,75		
55 ,030555	33 ,000303		46 ,786666		
56 ,031111	34 ,000312		47 ,783333		
57 ,031866	35 ,000322		48 ,8		
58 ,032222	36 ,000331		49 ,818666		
59 ,032777	37 ,00034		50 ,833333		
	38 ,000349		51 ,85		
S". D. Part.	39 ,000358		52 ,886666		
1 ,000009	40 ,000368		53 ,883333		
2 ,000018	41 ,000377		54 ,9		
3 ,000027	42 ,000386		55 ,918666		
4 ,000036	43 ,000395		56 ,933333		
5 ,000046	44 ,000404		57 ,95		
6 ,000055	45 ,000414		58 ,986666		
7 ,000064	46 ,000423		59 ,983333		
8 ,000073	47 ,000432		S". D. Part.		
9 ,000083	48 ,000441		1 ,000277		
10 ,000092	49 ,00045		2 ,000555		
11 ,000101	50 ,00046		3 ,000833		
12 ,00011	51 ,000469		4 ,001111		
13 ,000119	52 ,000478		5 ,001388		
14 ,000128	53 ,000487				

S'. D. Part.	S'. D. Part.	S'. D. Part.	S'. D. Part.
6 ,001866	20 ,005555	34 ,009444	48 ,013333
7 ,001944	21 ,005833	35 ,009722	49 ,013611
8 ,002222	22 ,006111	36 ,01	50 ,013888
9 ,0025	23 ,006388	37 ,010277	51 ,014166
10 ,002777	24 ,006666	38 ,010555	52 ,014444
11 ,003055	25 ,006944	39 ,010833	53 ,014722
12 ,003333	26 ,007222	40 ,011111	54 ,015
13 ,003611	27 ,0075	41 ,011388	55 ,015277
14 ,003888	28 ,007777	42 ,011666	56 ,015555
15 ,004166	29 ,008055	43 ,011944	57 ,015833
16 ,004444	30 ,008333	44 ,012222	58 ,016111
17 ,004722	31 ,008611	45 ,0125	59 ,016388
18 ,005	32 ,008888	46 ,012777	
19 ,005277	33 ,009166	47 ,013055	

A General Decimal Table.

A General Decimal Table for all Divisions of Parts under Twenty.

[illegible]

The Explanation and Use of the foregoing general Decimal Table.

1. The Figures at *Top*, which run to 20, shew the *Number of Parts* any *Quantity* or *Integer* is *divided* into; and the Figures in the *side Column* are those *Parts* themselves; The Figures forming the *triangular Space*, and disposed into proper *Columns*, are the *Decimal Parts* of the *Integral Quantity* answering thereto. Hence any *Quantity* divided into any *Number* of *Parts* under 20, the *Decimals* answering to each of those *Parts* are seen in *one view* in their proper *Column*.

Example. Suppose a *Quantity* divided into *Eight equal Parts*, and you would know the *Decimal Part* equivalent to *each*: Look at *top* for 8, *under* which are disposed the *Decimals*, viz. ,125,25,375 &c. answering in order to the *Parts* in the *side Column*.

2. The Figures in the *side Column* may be taken for the *Numerator*, and those at *top*, for the *Denominator* of a *Vulgar Fraction*.

Then the *Decimal* corresponding to those two *Numbers* respectively, is equal to the fore said *Fraction*: Thus the *Decimal* 5714 answering to 4 in the *side*, and 7 at *top*, is equal to the *Fraction* $\frac{4}{7}$. So $\frac{9}{13} = ,6922$. $\frac{5}{6} = ,8\bar{3}$. $\frac{6}{11} = ,54$.

$\frac{13}{16} = ,8125$. $\frac{16}{19} = ,8418$. $\frac{17}{20} = ,85$ &c.

Also any *larger Fraction* whose *Parts* are an *Equimultiple* of any of these *tabular Fractions*, or may be reduced to them, are equally answer'd in *Decimals* by this *Table*; see

this *Example* $\frac{588}{1512} = \frac{84}{210} = \frac{21}{54} = \frac{7}{18} = ,38$ *Decimals* in the *Table*.

Case 3. To *reduce* any *Decimal* into the *equivalent known Parts* of *Coin*, *Weights*, *Measure*, *Motion*, &c. observe this

Rule. Multiply the given *Decimal* by the *Number* of *Units* contain'd in the next *lower Denomination* of that *Species* of *Quantity*, which your *Decimal* is of; and thus proceed,
till

till you have converted your Decimals, or come to the lowest Part; and the several Products will be the several Parts of the Quantity required. See the following Examples.

EXAMPLE I.

What common Parts of a Pound (*viz.* Shillings, Pence, &c.) are contain'd in 0,73825 Decimal Parts of a Pound?

First, Multiply by 20 Shillings, the next lower Dem.

14,76500 Shillings.

Then Multiply by 12 the next lower De. to the last.

9,18000 Pence.

Lastly, Multiply by 4 the lowest Denomin. of all.

0,72000 Farthings.

Hence the Answer is 14 Shillings, 9 Pence, and 7 Tenths, or 72 Hundredths of a Farthing.

EXAMPLE II.

Reduce 0,72083 to the known Parts of a Crown.

5

3,60416

12

7,24999

4

Answer 3 s. 7 d. 4.

0,99999

EXAMPLE III.

Reduce 5,890625 into known Parts of a Mark.

13,3

9) 2,671875

,2968750

2671875

890625

11,8750000

12

10,5000000

4

2,00000000

Thus the Answer is exact without any Remainder, *viz.*

Marks s. d.
5 : 11 : 10 : 1/2.

EXAM-

EXAMPLE IV.

Reduce ,727564 into the known Parts of a Pound Troy.

$$\begin{array}{r}
 12 \\
 \hline
 8,730768 \\
 20 \\
 \hline
 14,615360 \\
 24 \\
 \hline
 2461440 \\
 1230720 \\
 \hline
 14,768640
 \end{array}$$

The Answer is 8 oz. 14 penny wt. 14 gr. $\frac{1}{4}$.

EXAMPLE V.

Reduce ,49723 into the known Parts of an C. Weight.

$$\begin{array}{r}
 4 \\
 \hline
 1,98892 \\
 28 \\
 \hline
 791136 \\
 197784 \\
 \hline
 27,68976 \\
 16 \\
 \hline
 413856 \\
 68976 \\
 \hline
 11,03616 \\
 16 \\
 \hline
 21696 \\
 3616 \\
 \hline
 0,57856
 \end{array}$$

The Answer will stand thus, 1 qr. 27 lb. 11 oz. 0 $\frac{1}{2}$ dr.

L

EXAM-

EXAMPLE VI.

Reduce .578 of a Rod into its known Parts.

$$\begin{array}{r}
 .578 \\
 \underline{.55} \\
 2892 \\
 \underline{28928} \\
 31828 \\
 \underline{3} \\
 05468 \\
 \underline{12} \\
 65588
 \end{array}$$

The Answer is 3 yds. 0 Ft. 6 $\frac{1}{2}$ In.; and this repeating Decimal .0588 over.

These six Examples I imagine sufficient to shew the *common* Method of *reducing Decimal Parts* into the common and known Parts of any Species of Quantity.

But as those *Operations of Reduction* are for the most part very *laborious, tedious*, and require *abundance of Figures*, I have sometimes wondred that a *Set of Tables* have not been compos'd to *facilitate* this Part of *Decimal Arithmetick*, as well as for the *contrary* Operations of *reducing different Species* into Decimals; especially since *one* is as *necessary* as the *other*. *Tables* for that purpose have long since been contriv'd, but none for the *reverse*; to turn *into*, but not to turn *out* of Decimals.

'Tis true, some *Decimals* (as those of *Money*) have the *first* and sometimes the *second* Figure pretty easily *valued* by a small Application of thought; but even this is for the *Skilful* to do, not for any that are but *Tyrō's*, or *rude* in the Art. Yet how much *Tables* for *Reduction* of *Decimals* to *vulgar* Parts are wanting, may appear from the great Industry many have used to lay down *Rules* for that purpose. which being so *prolix, verbose, obscure*, and consequently *impertinent*, that a Person wou'd sooner and with more ease and pleasure work out his Answer by the *ordinary* Method than by those *unintelligible* and *insignificant* Rules; and according to the *old Saw*, wou'd find the farthest way about, the nearest way home.

But

Description and Use of New Tables. 75

But having for Reasons already render'd determin'd to write a *complete Treatise of Decimal Arithmetick*, I thought it could by no means be *worthy of*, or *answer that Title*, unless with many other *Improvements*, I could make one more to render this Part of the Art *most easy and expeditious*; and having imploy'd my Thoughts a little on this *Topic*, I soon perceived an *Expedient* that wou'd do the business, which was this.

Viz. To divide the Figures of *Decimals* into *Pairs*, from the left Hand to the right; then to *tabulate* the *Digits* of every *Pair* from *Units* to an *Hundred*, in proper Columns; and lastly, to *affix the true Value* of every Place of Figures in the Column *appositely* answering thereto.

So that by this means the *Value* of any two *Places* of Figures (so far as they are valuable) in any *Decimal*, is seen by *Inspection* only; and the Value of four or six Places are as it were placed in one view, and with the greatest *ease and readiness* are obtain'd in any common Species of *Coins, Measures, and Weight*. But it being a Contrivance of *my own*, I shall not on that Account say any thing as to their Merit; but only give a short *Description* of the *Tables* and the *Manner* of using them by an Example of each.

A Description of the following new Decimal Tables.

For every different Kind of *Decimals* in common Use, I have compos'd a proper Set of *Tables*, and according as the *Decimal* is more or less valuable, it is divided into two or three Pairs; and to each of these Pairs is a *Table* exhibiting the *true value* of the *Figures* in each *Pair* from *Unit* to 100. The *Tables* of every sort are seen at the top of the Page; thus represented *Table 1. Table 2. &c.* Each *Table* consists of several Columns; the first Column has at Top *N^{o.}* to signify the *Numbers* of the *Decimal Pairs*. The other Columns (all but the last) represent the *Value* of the *Numbers* in the first Column in the various *Denominations* of the Parts the *Integer* are vulgarly known by. The last Column contains the *Decimal Parts* of the last Denomination, and in every Table is marked with *P.ts.*

Having given a *brief Explanation* of the *Tables* in general, I shall now shew their Use in discovering the *Value* of

76 Description and Use of New Tables, &c.

any *Decimal* given, which is thus. Suppose you would know the *Value* of the *Decimal* ,689457 of a Tun *Averdupois Weight*, proceed thus; first divide those Numbers into *Pairs* as here, 68,94,57; then take the *first* right *Hand Pair*, viz. 57, and seek in *Table 1. Averdupois Weight*, and you will

find against $\left\{ \begin{array}{l} 7 - 0 - ,25 \\ 50 - 1 - ,79 \end{array} \right\}$ that is, 2,04; then for the second Pair, 94, look in *Table 2.* and against 94 you see 21 *lb.* 0 *oz.* 52 *pwt.*; and for the third Pair, 68, seek in *Table 3.* and against 68 you observe 13 *C.* 2 *qr.* 11 *lb.* 3,2 *oz.* So that these several Numbers added in a proper manner will stand thus,

		<i>C.</i>	<i>qr.</i>	<i>lb.</i>	<i>oz.</i>
Against the	First Pair ,57	— 00	: 0	: 00	: 02,04
	Second Pair ,94	— 00	: 0	: 21	: 00,52
	Third Pair ,68	— 13	: 2	: 11	: 03,2
Hence the value of ,689457 is		13	: 3	: 04	: 05,76

Now that the Reader may see at once both the *Advantage* and *Exactness* of the *Tables*; I shall shew the Work of finding the value of the said *Decimal*, at large in the *common way* of *Reduction*, which is as follows.

$$\begin{array}{r}
 ,689457 \text{ of a Tun.} \\
 \hline
 20 \\
 13,789140 = \text{Hundred.} \\
 \hline
 4 \\
 3,156560 = \text{Quarter of C.} \\
 \hline
 28 \\
 1252480 \\
 313120 \\
 \hline
 4,383680 = \text{Pounds.} \\
 \hline
 16 \\
 2302080 \\
 383680 \\
 \hline
 6,138880 = \text{Ounces.}
 \end{array}$$

The

	C.	qr.	lb.	oz.
The Value this way is	13	3	04	06,1388 &c.
The Value by the Table is	13	3	04	05,76
The Difference only	00	0	00	00,3788

Hence it appears how *exact*, and yet how *easy* and *expeditions* these *Tables* are in the business of *Reducing Decimals* to their *proper Value* in the *common* and *vulgar Parts* of their *proper Integer*. I shall next give *Examples* of all kinds of *Decimal Parts*, in order to render the *Use* and *Emoluments* of these *new invented Tables*, as *plain* and *obvious* as may be; tho' they are of themselves as *easy* to be understood as any *Arithmetical Tables* whatsoever.

EXAMPLE I.

What is the Value of ,4725 of a *Pound Sterling*?

		s.	d.
In { Table 1. }	against { ,25 is	—	00 : 0 : $\frac{1}{2}$,4
{ Table 2. }	{ ,47 is	—	9 : 4 : $\frac{1}{4}$,2
	Answer		9 : 5 : $\frac{1}{4}$,6

EXAMPLE II.

What is the Value of ,147 of a *Shilling* or *Foot*?

		d.	qr.
In { Table 1. }	against { ,70 is	—	00 : 0,336
{ Table 2. }	{ ,14 is	—	01 : 2,72
The Answer in d, and f, or In. and qr. is			01 : 3,056

EXAMPLE III.

What is the Value of ,7347 of a *Pound Troy*?

		oz.	pwt.	gr.
In { Table 1. }	against { ,47 is	—	00	01 : 3,07
{ Table 2. }	{ ,73 is	—	08	15 : 4,8
Answer			08	16 : 7,87

EXAM.

EXAMPLE IV.

What is the Value of ,91249 of a *C. Weight Averdupois*?

		<i>Qr. lb. oz. dr.</i>	
In {	Table 1.	} against {	,90 is — 0 : 00 : 00 : 02,58
	Table 2.		,24 is — 0 : 00 : 04 : 04,8
	Table 3.		,91 is — 3 : 17 : 14 : 11,52
		Answer	<u>3 : 18 : 03 : 02,9</u>

EXAMPLE V.

What is the Value of ,7777 of a *Pound Apothecaries Weight*?

		<i>gr.</i>	
In {	Table 1.	} against {	,77 is — 00 : 0 : 2 : 04,88
	Table 2.		,77 is — 09 : 1 : 2 : 15,2
		Answer	<u>09 : 2 : 2 : 00,08</u>

EXAMPLE VI.

What is the Value of ,8754 of a *Tun, Wine Measure*.

		<i>P. Hg. T. G.</i>	
In {	Table 1.	} against {	,54 is — 0 : 0 : 0 : 01,36
	Table 2.		,87 is — 1 : 1 : 0 : 30,24
		Answer	<u>1 : 1 : 0 : 31 6</u>

EXAMPLE VII.

What is the Value of ,7509 of a *Load of Corn*?

		<i>2 B. G.</i>	
In {	Table 1.	} against {	,09 is — 0 : 0 : 0,28
	Table 2.		,75 is — 3 : 6 : 0
		Answer	<u>3 : 6 : 0,28</u>

EXAM-

EXAMPLE VIII.

What is the Value of ,8495 of a Year?

			M.	W.	D.	H.
In {	Table 1.	} against {	,95 is —	—	00 : 0 : 3 :	11,22
	Table 2.					
			,84 is —	—	10 : 3 : 5 :	14,4
		Answer			11 : 0 : 2 :	01,62

EXAMPLE IX.

What is the Value of ,889 of an Hour or Degree?

In {	Table 1.	} against {	,90 is —	—	00 : 32,4
	Table 2.				
			,88 is —	—	52 : 48,
		Answer			53 : 20,4

EXAMPLE X.

What is the Value of ,0596 of a Sign of the Zodiac?

In {	Table 1.	} against {	,96 is —	—	00 : 17 : 16
	Table 2.				
			,05 is —	—	01 : 30 : 00
		Answer			01 : 47 : 16

EXAMPLE XI.

What is the Value of ,976305 of a Mile?

				Fr.	R.	Yd.	Ft.	In.
In {	Table 1.	} against {	,05 is	0	:	00	:	0 : 00,31
	Table 2.		,63 is	0	:	02	:	0 : 02,79
	Table 3.		,97 is	7	:	30	:	2 : 07,2
			Answer	7	:	32	:	2 : 10,3

EXAM.

30 The Use of the new Tables, &c.

EXAMPLE XII.

What is the Value of ,278 of a *Yard square*?

		<i>F.q. In.q.</i>	
In {	Table 1. }	against { ,80 is	— 0 : 10,33
	Table 2. }	,27 is	— 2 : 61,92
		Answer	<u>2 : 72,25</u>

EXAMPLE XIII.

What is the Value of ,795007 of a *Mile square*?

		<i>A. R. P.</i>	
In {	Table 1. }	against { ,07 is	— 0 : 0 : 00,71
	Table 2. }	,50 is	— 3 : 0 : 32
	Table 3. }	,79 is	— 505 : 2 : 16
		Answer	<u>508 : 3 : 08,71</u>

EXAMPLE XIV.

What is the Value of ,974 of an *Acre*?

		<i>R. P. Y.</i>	
In {	Table 1. }	against { ,40	— 0 : 00 : 19,36
	Table 2. }	,97	— 3 : 35 : 06,05
		Answer	<u>3 : 35 : 25,41</u>

EXAMPLE XV.

What is the Value of ,629 of a *Yard solid*?

		<i>F. In.</i>	
In {	Table 1. }	against { ,90 is	— 00 : 419,94
	Table 2. }	,62 is	— 16 : 1278,72
		Answer	<u>16 : 1698,66</u>

TABLE

A Set of NEW DECIMAL TABLES expressing the Value of any Decimal in the Known or Vulgar Denominations of the Integral Quantity, whether Money, Weight, Time, Motion, or Measure of every Kind.

Table I. Of Money, One Pound the Integer.

N ^o .	d.	f.	Pt.	N ^o .	d.	f.	Pt.	N ^o .	d.	f.	Pt.
1	—	—	,09	29	—	2	,78	57	1	1	,48
2	—	—	,19	30	—	2	,88	58	1	1	,58
3	—	—	,28	31	—	2	,98	59	1	1	,68
4	—	—	,38	32	—	3	,08	60	1	1	,78
5	—	—	,48	33	—	3	,17	61	1	1	,87
6	—	—	,57	34	—	3	,27	62	1	1	,97
7	—	—	,67	35	—	3	,37	63	1	2	,06
8	—	—	,76	36	—	3	,46	64	1	2	,16
9	—	—	,86	37	—	3	,56	65	1	2	,26
10	—	—	,96	38	—	3	,65	66	1	2	,35
11	—	1	,05	39	—	3	,75	67	1	2	,45
12	—	1	,15	40	—	3	,84	68	1	2	,54
13	—	1	,24	41	—	3	,94	69	1	2	,64
14	—	1	,34	42	1	0	,03	70	1	2	,74
15	—	1	,44	43	1	0	,12	71	1	2	,83
16	—	1	,53	44	1	0	,22	72	1	2	,93
17	—	1	,63	45	1	0	,32	73	1	3	,02
18	—	1	,72	46	1	0	,41	74	1	3	,12
19	—	1	,82	47	1	0	,51	75	1	3	,22
20	—	1	,92	48	1	0	,6	76	1	3	,31
21	—	2	,01	49	1	0	,7	77	1	3	,41
22	—	2	,11	50	1	0	,8	78	1	3	,5
23	—	2	,20	51	1	0	,89	79	1	3	,6
24	—	2	,3	52	1	0	,99	80	1	3	,7
25	—	2	,4	53	1	1	,09	81	1	3	,79
26	—	2	,49	54	1	1	,19	82	1	3	,89
27	—	2	,59	55	1	1	,29	83	1	3	,98
28	—	2	,68	56	1	1	,38	84	2	0	,06

82 *A Set of New Decimal Tables, &c.*

N ^o .	d.	f.	Pt.	N ^o .	d.	f.	Pt.	N ^o .	d.	f.	Pt.
85	2	0	,16	90	2	0	,64	95	2	1	,12
86	2	0	,25	91	2	0	,73	96	2	1	,21
87	2	0	,35	92	2	0	,83	97	2	1	,31
88	2	0	,44	93	2	0	,92	98	2	1	,4
89	2	0	,54	94	2	1	,02	99	2	1	,5

Table II. *Of Money, one Pound the Integer.*

N ^o .	f.	d.	f.	Pt.	N ^o .	f.	d.	f.	Pt.	N ^o .	f.	d.	f.	Pt.
1	—	2	1	,6	29	5	9	2	,4	57	11	4	3	,2
2	—	4	3	,2	30	6	0	0	,—	58	11	7	0	,8
3	—	7	0	,8	31	6	2	1	,6	59	11	9	2	,4
4	—	9	2	,4	32	6	4	3	,2	60	12	0	0	,—
5	1	0	0	,—	33	6	7	0	,8	61	12	2	1	,6
6	1	2	1	,6	34	6	9	2	,4	62	12	4	3	,2
7	1	4	3	,2	35	7	0	0	,—	63	12	7	0	,8
8	1	7	0	,8	36	7	2	1	,6	64	12	9	2	,4
9	1	9	2	,4	37	7	4	3	,2	65	13	0	0	,—
10	2	0	0	,—	38	7	7	0	,8	66	13	2	1	,6
11	2	2	1	,6	39	7	9	2	,4	67	13	4	3	,2
12	2	4	3	,2	40	8	0	0	,—	68	13	7	0	,8
13	2	7	0	,8	41	8	2	1	,6	69	13	9	2	,4
14	2	9	2	,4	42	8	4	3	,2	70	14	0	0	,—
15	3	0	0	,—	43	8	7	0	,8	71	14	2	1	,6
16	3	2	1	,6	44	8	9	2	,4	72	14	4	3	,2
17	3	4	3	,2	45	9	0	0	,—	73	14	7	0	,8
18	3	7	0	,8	46	9	2	1	,6	74	14	9	2	,4
19	3	9	2	,4	47	9	4	3	,2	75	15	0	0	,—
20	4	0	0	,—	48	9	7	0	,8	76	15	2	1	,6
21	4	2	1	,6	49	9	9	2	,4	77	15	4	3	,2
22	4	4	3	,2	50	10	0	0	,—	78	15	7	0	,8
23	4	7	0	,8	51	10	2	1	,6	79	15	9	2	,4
24	4	9	2	,4	52	10	4	3	,2	80	16	0	0	,—
25	5	0	0	,—	53	10	7	0	,8	81	16	2	1	,6
26	5	2	1	,6	54	10	9	2	,4	82	16	4	3	,2
27	5	4	3	,2	55	11	0	0	,—	83	16	7	0	,8
28	5	7	0	,8	56	11	2	1	,6	84	16	9	2	,4

N ^o .	l.	d.	f.	Pt.	N ^o .	l.	d.	f.	Pt.	N ^o .	l.	d.	f.	Pt.
85	17	0	0	—	90	18	0	0	—	95	19	0	0	—
86	17	2	1	,6	91	18	2	1	,6	96	19	2	1	,6
87	17	4	3	,2	92	18	4	3	,2	97	19	4	3	,2
88	17	7	0	,8	93	18	7	0	,8	98	19	7	0	,8
89	17	9	2	,4	94	18	9	2	,4	99	19	9	2	,4

Table I. Troy Weight, *one Pound the Integer.*

N ^o .	Pwt.	gr.	Pt.	N ^o .	Pwt.	gr.	Pt.	N ^o .	Pwt.	gr.	Pt.
1	—	—	,57	29	—	16	,7	57	1	8	,84
2	—	1	,15	30	—	17	,28	58	1	9	,41
3	—	1	,72	31	—	17	,85	59	1	9	,99
4	—	2	,3	32	—	18	,43	60	1	10	,57
5	—	2	,88	33	—	19	—	61	1	11	,14
6	—	3	,45	34	—	19	,58	62	1	11	,72
7	—	4	,03	35	—	20	,16	63	1	12	,29
8	—	4	,6	36	—	20	,73	64	1	12	,87
9	—	5	,19	37	—	21	,31	65	1	13	,45
10	—	5	,76	38	—	21	,88	66	1	14	,02
11	—	6	,34	39	—	22	,46	67	1	14	,6
12	—	6	,91	40	—	23	,04	68	1	15	,17
13	—	7	,48	41	—	23	,61	69	1	15	,75
14	—	8	,06	42	1	0	,19	70	1	16	,33
15	—	8	,64	43	1	0	,76	71	1	16	,9
16	—	9	,21	44	1	1	,34	72	1	17	,48
17	—	9	,79	45	1	1	,92	73	1	18	,05
18	—	10	,36	46	1	2	,49	74	1	18	,63
19	—	10	,94	47	1	3	,07	75	1	19	,2
20	—	11	,52	48	1	3	,64	76	1	19	,77
21	—	12	,09	49	1	4	,22	77	1	20	,35
22	—	12	,67	50	1	4	,8	78	1	20	,92
23	—	13	,24	51	1	5	,37	79	1	21	,5
24	—	13	,82	52	1	5	,95	80	1	22	,08
25	—	14	,4	53	1	6	,52	81	1	22	,65
26	—	14	,97	54	1	7	,1	82	1	23	,23
27	—	15	,55	55	1	7	,69	83	1	23	,8
28	—	16	,12	56	1	8	,26	84	2	0	,38

84 *A Set of New Decimal Tables, &c.*

N ^o .	Pwt.	gr.	Pt.	N ^o .	Pwt.	gr.	Pt.	N ^o .	Pwt.	gr.	Pt.
85	2	0	,96	90	2	3	,84	95	2	6	,72
86	2	1	,53	91	2	4	,41	96	2	7	,29
87	2	2	,11	92	2	4	,99	97	2	7	,87
88	2	2	,68	93	2	5	,56	98	2	8	,44
89	2	3	,26	94	2	6	,14	99	2	9	,02

Table II. Troy Weight, *one Pound the Integer.*

N ^o .	oz.	pwt.	gr.	pt.	N ^o .	oz.	pwt.	gr.	pt.	N ^o .	oz.	pwt.	gr.	pt.
1	—	2	9	,6	29	3	9	14	,4	57	6	16	19	,2
2	—	4	19	,2	30	3	12	0	,—	58	6	19	4	,8
3	—	7	4	,8	31	3	14	9	,6	59	7	1	14	,4
4	—	9	14	,4	32	3	16	19	,2	60	7	4	0	,—
5	—	12	0	,—	33	3	19	4	,8	61	7	6	9	,6
6	—	14	9	,6	34	4	1	14	,4	62	7	8	19	,2
7	—	16	19	,2	35	4	4	0	,—	63	7	11	4	,8
8	—	19	4	,8	36	4	6	9	,6	64	7	13	14	,4
9	1	1	14	,4	37	4	8	19	,2	65	7	16	0	,—
10	1	4	0	,—	38	4	11	4	,8	66	7	18	9	,6
11	1	6	9	,6	39	4	13	14	,4	67	8	0	19	,2
12	1	8	19	,2	40	4	16	0	,—	68	8	3	4	,8
13	1	11	4	,8	41	4	18	9	,6	69	8	5	14	,4
14	1	13	14	,4	42	5	0	19	,2	70	8	8	0	,—
15	1	16	0	,—	43	5	3	4	,8	71	8	10	9	,6
16	1	18	9	,6	44	5	5	14	,4	72	8	12	19	,2
17	2	0	19	,2	45	5	8	0	,—	73	8	15	4	,8
18	2	3	4	,8	46	5	10	9	,6	74	8	17	14	,4
19	2	5	14	,4	47	5	12	19	,2	75	9	0	0	,—
20	2	8	0	,—	48	5	15	4	,8	76	9	2	9	,6
21	2	10	9	,6	49	5	17	14	,4	77	9	4	19	,2
22	2	12	19	,2	50	6	0	0	,—	78	9	7	4	,8
23	2	15	4	,8	51	6	2	9	,6	79	9	9	14	,4
24	2	17	14	,4	52	6	4	19	,2	80	9	12	0	,—
25	3	0	0	,—	53	6	7	4	,8	81	9	14	9	,6
26	3	2	9	,6	54	6	9	14	,4	82	9	16	19	,2
27	3	4	19	,2	55	6	12	0	,—	83	9	19	4	,8
28	3	7	4	,8	56	6	14	2	,6	84	10	1	14	,4

N^o.

N ^o .	oz.	pw.	gr.	pt.	N ^o .	oz.	pw.	gr.	pt.	N ^o .	oz.	pw.	gr.	pt.
85	10	4	0	—	90	10	16	0	—	95	11	8	0	—
86	10	6	9	,6	91	10	18	9	,6	96	11	10	9	,6
87	10	8	19	,2	92	11	0	19	,2	97	11	12	19	,2
88	10	11	4	,8	93	11	3	4	,8	98	11	15	4	,8
89	10	13	14	,4	94	11	5	14	,4	99	11	17	14	,4

Table I. | Table II. { *Troy Weight, one Ounce the Integer.*

N ^o .	Pwt.	gr.	Pts.	N ^o .	Pwt.	gr.	Pts.	N ^o .	Pwt.	gr.	Pts.
1	—	—	,048	1	—	4	,8	29	5	19	,2
2	—	—	,096	2	—	9	,6	30	6	0	—
3	—	—	,144	3	—	14	,4	31	6	4	,8
4	—	—	,192	4	—	19	,2	32	6	9	,6
5	—	—	,24	5	1	0	—	33	6	14	,4
6	—	—	,288	6	1	4	,8	34	6	19	,2
7	—	—	,336	7	1	9	,6	35	7	0	—
8	—	—	,384	8	1	14	,4	36	7	4	,8
9	—	—	,432	9	1	19	,2	37	7	9	,6
10	—	—	,48	10	2	0	—	38	7	14	,4
20	—	—	,96	11	2	4	,8	39	7	19	,2
30	—	1	,44	12	2	9	,6	40	8	0	—
40	—	1	,92	13	2	14	,4	41	8	4	,8
50	—	2	,4	14	2	19	,2	42	8	9	,6
60	—	2	,88	15	3	0	—	43	8	14	,4
70	—	3	,36	16	3	4	,8	44	8	19	,2
80	—	3	,84	17	3	9	,6	45	9	0	—
90	—	4	,32	18	3	14	,4	46	9	4	,8
				19	3	19	,2	47	9	9	,6
				20	4	0	—	48	9	14	,4
				21	4	4	,8	49	9	19	,2
				22	4	9	,6	50	10	0	—
				23	4	14	,4	51	10	4	,8
				24	4	19	,2	52	10	9	,6
				25	5	0	—	53	10	14	,4
				26	5	4	,8	54	10	19	,2
				27	5	9	,6	55	11	0	—
				28	5	14	,4	56	11	4	,8

N ^o .	Pwt.	gr.	Pt.	N ^o .	Pwt.	gr.	Pt.	N ^o .	Pwt.	gr.	Pt.
57	11	9	,6	72	14	9	,6	87	17	9	,6
58	11	14	,4	73	14	14	,4	88	17	14	,4
59	11	19	,2	74	14	19	,2	89	17	19	,2
60	12	0	,—	75	15	0	,—	90	18	0	,—
61	12	4	,8	76	15	4	,8	91	18	4	,8
62	12	9	,6	77	15	9	,6	92	18	9	,6
63	12	14	,4	78	15	14	,4	93	18	14	,4
64	12	19	,2	79	15	19	,2	94	18	19	,2
65	13	0	,—	80	16	0	,—	95	19	0	,—
66	13	4	,8	81	16	4	,8	96	19	4	,8
67	13	9	,6	82	16	9	,6	97	19	9	,6
68	13	14	,4	83	16	14	,4	98	19	14	,4
69	13	19	,2	84	16	19	,2	99	19	19	,2
70	14	0	,—	85	17	0	,—				
71	14	4	,8	86	17	4	,8				

Table I.

Table II. { *Averdupois Weight,*
one Pound the Integer.

N ^o .	dr.	Pts.	N ^o .	oz.	dr.	Pt.	N ^o .	oz.	dr.	Pt.
1	--	,025	1	--	2	,56	19	3	0	,64
2	--	,051	2	--	5	,12	20	3	3	,2
3	--	,076	3	--	7	,68	21	3	5	,76
4	--	,102	4	--	10	,24	22	3	8	,32
5	--	,128	5	--	12	,8	23	3	10	,88
6	--	,153	6	--	15	,36	24	3	13	,44
7	--	,179	7	I	1	,92	25	4	0	,—
8	--	,204	8	I	4	,48	26	4	2	,56
9	--	,23	9	I	7	,04	27	4	5	,12
10	--	,256	10	I	9	,6	28	4	7	,68
20	--	,512	11	I	12	,16	29	4	10	,24
30	--	,768	12	I	14	,72	30	4	12	,8
40	I	,024	13	2	1	,28	31	4	15	,36
50	I	,28	14	2	3	,84	32	5	1	,92
60	I	,536	15	2	6	,4	33	5	4	,48
70	I	,792	16	2	8	,96	34	5	7	,04
80	2	,048	17	2	11	,52	35	5	9	,6
90	2	,304	18	2	14	,08	36	5	12	,16

N^o.

N ^o .	oz.	dr.	Pt.	N ^o .	oz.	dr.	Pt.	N ^o .	oz.	dr.	Pt.
37	5	14	,72	58	9	4	,48	79	12	10	,24
38	6	1	,28	59	9	7	,04	80	12	12	,8
39	6	3	,84	60	9	9	,6	81	12	15	,36
40	6	6	,4	61	9	12	,16	82	13	1	,92
41	6	8	,96	62	9	14	,72	83	13	4	,48
42	6	11	,52	63	10	1	,28	84	13	7	,04
43	6	14	,08	64	10	3	,84	85	13	9	,6
44	7	0	,64	65	10	6	,4	86	13	12	,16
45	7	3	,2	66	10	8	,96	87	13	14	,72
46	7	5	,76	67	10	11	,52	88	14	1	,28
47	7	8	,32	68	10	14	,08	89	14	3	,84
48	7	10	,88	69	11	0	,64	90	14	6	,4
49	7	13	,44	70	11	3	,2	91	14	8	,96
50	8	0	,—	71	11	5	,76	92	14	11	,52
51	8	2	,56	72	11	8	,32	93	14	14	,08
52	8	5	,12	73	11	10	,88	94	15	0	,64
53	8	7	,68	74	11	13	,44	95	15	3	,2
54	8	10	,24	75	12	0	,—	96	15	5	,76
55	8	12	,8	76	12	2	,56	97	15	8	,32
56	8	15	,36	77	12	5	,12	98	15	10	,88
57	9	1	,92	78	12	7	,68	99	15	13	,44

Table I. { Table II. { Averdupois Weight,
one C. or 112 lb. the Int.

N ^o .	dr.	Pt.	N ^o .	oz.	dr.	Pt.	N ^o .	oz.	dr.	Pt.
1	--	,02	1	--	2	,86	12	2	2	,4
2	--	,05	2	--	5	,73	13	2	5	,27
3	--	,08	3	--	8	,6	14	2	8	,13
4	--	,11	4	--	11	,46	15	2	11	,0
5	--	,14	5	--	14	,33	16	2	13	,87
6	--	,17	6	1	1	,2	17	3	0	,73
7	--	,2	7	1	4	,06	18	3	3	,6
8	--	,22	8	1	6	,93	19	3	6	,47
9	--	,25	9	1	9	,80	20	3	9	,34
10	--	,28	10	1	12	,67	21	3	12	,2
20	--	,57	11	1	15	,53	22	3	15	,7

N^o.

Table I.

Table II. { *Averdupois Weight,*
one C. or 112 lb. the Int.

N ^o .	dr.	Pt.	N ^o .	oz.	dr.	Pt.	N ^o .	oz.	dr.	Pt.
30	--	,86	23	4	1	,94	50	8	15	,35
40	1	,14	24	4	4	,8	51	9	2	,21
50	1	,43	25	4	7	,67	52	9	5	,08
60	1	,72	26	4	10	,54	53	9	7	,95
70	2	,0	27	4	13	,4	54	9	10	,81
80	2	,29	28	5	0	,27	55	9	13	,68
90	2	,58	29	5	3	,14	56	10	0	,55
			30	5	6	,01	57	10	3	,41
			31	5	8	,87	58	10	6	,28
			32	5	11	,74	59	10	9	,15
			33	5	14	,61	60	10	12	,02
			34	6	1	,47	61	10	14	,88
			35	6	4	,34	62	11	1	,75
			36	6	7	,21	63	11	4	,62
			37	6	10	,07	64	11	7	,48
			38	6	12	,94	65	11	10	,35
			39	6	15	,81	66	11	13	,22
			40	7	2	,68	67	12	0	,08
			41	7	5	,54	68	12	2	,95
			42	7	8	,41	69	12	5	,82
			43	7	11	,28	70	12	8	,69
			44	7	14	,14	71	12	11	,55
			45	8	1	,01	72	12	14	,42
			46	8	3	,88	73	13	1	,29
			47	8	6	,74	74	13	4	,15
			48	8	9	,61	75	13	7	,02
			49	8	12	,48				

Table

Table I.

Table II. { *Averdupois Weight.*
one C. Wt. the Integer.

N ^o .	lb.	oz.	dr.	Pt.	Q.	0	1	2	3	lb.	oz.	dr.	Pt.
76	—	13	9	,89		N ^o	25	50	75	c	0	0	—
77	—	13	12	,75		1	26	51	76	1	1	14	,72
78	—	13	15	,62		2	27	52	77	2	3	13	,44
79	—	14	2	,49		3	28	53	78	3	5	12	,16
80	—	14	5	,36		4	29	54	79	4	7	10	,88
81	—	14	8	,22		5	30	55	80	5	9	9	,6
82	—	14	11	,09		6	31	56	81	6	11	8	,32
83	—	14	13	,96		7	32	57	82	7	13	7	,04
84	—	15	0	,82		8	33	58	83	8	15	5	,76
85	—	15	3	,69		9	34	59	84	10	1	4	,48
86	—	15	6	,56		10	35	60	85	11	3	3	,2
87	—	15	9	,41		11	36	61	86	12	15	1	,92
88	—	15	12	,28		12	37	62	87	13	7	c	,64
89	—	15	15	,15		13	38	63	88	14	8	5	,36
90	I	0	2	,03		14	39	64	89	15	10	14	,08
91	I	0	4	,89		15	40	65	90	16	12	12	,8
92	I	0	7	,76		16	41	66	91	17	14	11	,52
93	I	0	10	,63		17	42	67	92	19	0	10	,24
94	I	0	13	,49		18	43	68	93	20	2	8	,96
95	I	I	0	,36		19	44	69	94	21	4	7	,68
96	I	I	3	,23		20	45	70	95	22	6	6	,4
97	I	I	6	,09		21	46	71	96	23	8	5	,12
98	I	I	8	,96		22	47	72	97	24	10	3	,84
99	I	I	11	,83		23	48	73	98	25	12	2	,5c
						24	49	74	99	26	14	1	,28

Table I.

Table II. { *Averdupois Weight,*
one Tun or Load the Int.

N ^o .	lb.	Pt.	N ^o .	lb.	oz.	Pt.	N ^o .	lb.	oz.	Pt.
1	—	,03	1	0	3	,58	5	1	1	,9
2	—	,07	2	0	7	,16	6	1	5	,48
3	—	,1	3	0	10	,74	7	1	9	,06
4	—	,14	4	0	14	,32	8	1	12	,64

N

N^o.

90 *A Set of New Decimal Tables, &c.*

N ^o	oz.	Pt.	N ^o	lb.	oz.	Pt.	N ^o	lb.	oz.	Pt.
5	--	,18	9	2	0	,22	48	10	11	,84
6	--	,21	10	2	3	,8	49	10	15	,42
7	--	,25	11	2	7	,38	50	11	3	,—
8	--	,28	12	2	10	,96	51	11	6	,58
9	--	,32	13	2	14	,54	52	11	10	,16
10	--	,35	14	3	2	,12	53	11	13	,74
20	--	,41	15	3	5	,7	54	12	1	,32
30	1	,07	16	3	9	,28	55	12	4	,9
40	1	,43	17	3	12	,86	56	12	8	,48
50	1	,79	18	4	0	,44	57	12	12	,06
60	2	,14	19	4	4	,02	58	12	15	,64
70	2	,5	20	4	7	,6	59	13	3	,22
80	2	,86	21	4	11	,18	60	13	6	,8
90	3	,22	22	4	14	,76	61	13	10	,38
			23	5	2	,34	62	13	13	,96
			24	5	5	,92	63	14	1	,54
			25	5	9	,5	64	14	5	,12
			26	5	13	,08	65	14	8	,7
			27	6	0	,66	66	14	12	,28
			28	6	4	,24	67	14	15	,86
			29	6	7	,82	68	15	3	,44
			30	6	11	,4	69	15	7	,02
			31	6	14	,98	70	15	10	,6
			32	7	2	,56	71	15	14	,18
			33	7	6	,14	72	16	1	,76
			34	7	9	,72	73	16	5	,34
			35	7	13	,3	74	16	8	,92
			36	8	0	,88	75	16	12	,5
			37	8	4	,46	76	17	0	,08
			38	8	8	,04	77	17	3	,66
			39	8	11	,62	78	17	7	,34
			40	8	15	,2	79	17	10	,82
			41	9	2	,78	80	17	14	,4
			42	9	7	,36	81	18	1	,98
			43	9	9	,94	82	18	5	,56
			44	9	13	,52	83	18	9	,14
			45	10	1	,1	84	18	12	,72
			46	10	4	,68	85	19	0	,3
			47	10	8	,26	86	19	3	,88

N^o.

N ^o .	lb.	oz.	Pt.	N ^o .	lb.	oz.	Pt.	N ^o .	lb.	oz.	Pt.
87	19	7	,46	92	20	9	,36	97	21	11	,26
88	19	11	,04	93	20	12	,94	98	21	14	,84
89	19	14	,62	94	21	0	,52	99	22	2	,42
90	20	2	,2	95	21	4	,1				
91	20	5	,78	96	21	7	,68				

Table III. Averdupois Weight, one Tun the Integer.

N ^o .	C.	Q.	lb.	oz.	Pt.	N ^o .	C.	Q.	lb.	oz.	Pt.
1	0	0	22	6	,4	29	5	3	5	9	,6
2	0	1	16	12	,8	30	6	0	0	0	,—
3	0	2	11	3	,2	31	6	0	22	6	,4
4	0	3	5	9	,6	32	6	1	16	12	,8
5	1	0	0	0	,—	33	6	2	11	3	,2
6	1	0	22	6	,4	34	6	3	5	9	,6
7	1	1	16	12	,8	35	7	0	0	0	,—
8	1	2	11	3	,2	36	7	0	22	6	,4
9	1	3	5	9	,6	37	7	1	16	12	,8
10	2	0	0	0	,—	38	7	2	11	3	,2
11	2	0	22	6	,4	39	7	3	5	9	,6
12	2	1	16	12	,8	40	8	0	0	0	,—
13	2	2	11	3	,2	41	8	0	22	6	,4
14	2	3	5	9	,6	42	8	1	16	12	,8
15	3	0	0	0	,—	43	8	2	11	3	,2
16	3	0	22	6	,4	44	8	3	5	9	,6
17	3	1	16	12	,8	45	9	0	0	0	,—
18	3	2	11	3	,2	46	9	0	22	6	,4
19	3	3	5	9	,6	47	9	1	16	12	,8
20	4	0	0	0	,—	48	9	2	11	3	,2
21	4	0	22	6	,4	49	9	3	5	9	,6
22	4	1	16	12	,8	50	10	0	0	0	,—
23	4	2	11	3	,2	51	10	0	22	6	,4
24	4	3	5	9	,6	52	10	1	16	12	,8
25	5	0	0	0	,—	53	10	2	11	3	,2
26	5	0	22	6	,4	54	10	3	5	9	,6
27	5	1	16	12	,8	55	11	0	0	0	,—
28	5	2	11	3	,2	56	11	0	22	6	,4

92 *A Set of New Decimal Tables, &c.*

N ^o .	C.	Q.	lb.	oz.	Pt.	N ^o .	C.	Q.	lb.	oz.	Pt.
57	11	1	16	12	,8	79	15	3	5	9	,6
58	11	2	11	3	,2	80	16	0	0	0	,0
59	11	3	5	9	,6	81	16	0	22	6	,4
60	12	0	0	0	,—	82	16	1	16	12	,8
61	12	0	22	6	,4	83	16	2	11	3	,2
62	12	1	16	12	,8	84	16	3	5	9	,6
63	12	2	11	3	,2	85	17	0	0	0	,0
64	12	3	5	9	,6	86	17	0	22	6	,4
65	13	0	0	0	,—	87	17	1	16	12	,8
66	13	0	22	6	,4	88	17	2	11	3	,2
67	13	1	16	12	,8	89	17	3	5	9	,6
68	13	2	11	3	,2	90	18	0	0	0	,0
69	13	3	5	9	,6	91	18	0	22	6	,4
70	14	0	0	0	,—	92	18	1	16	12	,8
71	14	0	22	6	,4	93	18	2	11	3	,2
72	14	1	16	12	,8	94	18	3	5	9	,6
73	14	2	11	3	,2	95	19	0	0	0	,—
74	14	3	5	9	,6	96	19	0	22	6	,4
75	15	0	0	0	,—	97	19	1	16	12	,8
76	15	0	22	6	,4	98	19	2	11	3	,2
77	15	1	16	12	,8	99	19	3	5	9	,6
78	15	2	11	3	,2						

Table I.

Table II. { Apothecaries Weight,
one lb the Integer.

N ^o .	gr.	Pt.	N ^o .	gr.	Pt.	N ^o .	gr.	Pt.
1	—	,56	1	2	17,6	11	1	13,6
2	1	,15	2	1	215,2	12	1	11,2
3	1	,72	3	2	212,8	13	1	8,8
4	2	,3	4	3	210,4	14	1	6,4
5	2	,8	5	4	28,—	15	1	4,—
6	3	,45	6	5	25,6	16	1	1,6
7	4	,03	7	6	23,2	17	2	019,2
8	4	,6	8	7	20,8	18	2	016,8
9	5	,19	9	1	018,4	19	2	014,4
10	5	,76	10	1	016,—	20	2	012,—

N^o.

N ^o	E	gr.	Pt.	N ^o .	3	3	3	gr.	Pt.	N ^o .	3	3	3	gr.	Pt.
20	—	11	,52	21	2	4	0	9,6		61	7	2	1	13,6	
30	—	17	,28	22	2	5	0	7,2		62	7	3	1	11,2	
40	I	3	,04	23	2	6	C	4,8		63	7	4	1	8,8	
50	I	8	,8	24	2	7	0	2,4		64	7	5	1	6,4	
60	I	14	,56	25	3	C	C	C, —		65	7	6	1	4, —	
70	2	0	,28	26	3	C	2	17,6		66	7	7	1	1,6	
80	2	6	,02	27	3	1	2	15,2		67	8	0	C	19,2	
90	2	11	,75	28	3	2	2	12,8		68	8	1	C	16,8	
				29	3	3	2	10,4		69	8	2	C	14,4	
				30	3	4	2	8, —		70	8	3	C	12, —	
				31	3	5	2	5,6		71	8	4	C	9,6	
				32	3	6	2	3,2		72	8	5	C	7,2	
				33	3	7	2	0,8		73	8	6	C	4,8	
				34	4	0	1	18,4		74	8	7	C	2,4	
				35	4	1	1	16, —		75	9	0	C	0, —	
				36	4	2	1	13,6		76	9	0	2	17,6	
				37	4	3	1	11,2		77	9	1	2	15,2	
				38	4	4	1	8,8		78	9	2	2	12,8	
				39	4	5	1	6,4		79	9	3	2	10,4	
				40	4	6	1	4, —		80	9	4	2	8, —	
				41	4	7	1	1,5		81	9	5	2	5,6	
				42	5	0	0	19,2		82	9	6	2	3,2	
				43	5	1	0	16,8		83	9	7	2	0,8	
				44	5	2	C	14,4		84	10	0	1	18,4	
				45	5	3	0	12, —		85	10	1	1	16, —	
				46	5	4	0	9,6		86	10	2	1	13,6	
				47	5	5	0	7,2		87	10	3	1	11,2	
				48	5	6	0	4,8		88	10	4	1	8,8	
				49	5	7	0	2,4		89	10	5	1	6,4	
				50	6	0	0	C, —		90	10	6	1	4, —	
				51	6	0	2	17,6		91	10	7	1	1,6	
				52	6	1	2	15,2		92	11	0	C	19,2	
				53	6	2	2	12,8		93	11	1	C	16,8	
				54	6	3	2	10,4		94	11	2	C	14,4	
				55	6	4	2	8, —		95	11	3	C	12, —	
				56	6	5	2	5,6		96	11	4	C	9,6	
				57	6	6	2	3,2		97	11	5	C	7,2	
				58	6	7	2	0,8		98	11	6	C	4,8	
				59	7	0	1	18,4		99	11	7	C	2,4	
				60	7	1	1	16, —							

Table

Table I. | Table II. { *Wine Measure, one Hog-*
head the Integer.

N ^o .	G.	P.	Pt.	N ^o .	G.	P.	Pt.	N ^o .	G.	P.	Pt.
1	—	—	,05	1	—	5	,04	36	22	5	,44
2	—	—	,1	2	1	2	,08	37	23	2	,48
3	—	—	,15	3	1	7	,12	38	23	7	,52
4	—	—	,2	4	2	4	,16	39	24	4	,56
5	—	—	,25	5	3	1	,2	40	25	1	,6
6	—	—	,3	6	3	6	,24	41	25	6	,64
7	—	—	,35	7	4	3	,28	42	26	3	,68
8	—	—	,4	8	5	0	,32	43	27	0	,72
9	—	—	,45	9	5	5	,36	44	27	5	,76
10	—	—	,5	10	6	2	,4	45	28	2	,8
20	1	—	,—	11	6	7	,44	46	28	7	,84
30	1	,51		12	7	4	,48	47	29	4	,88
40	2	,01		13	8	1	,52	48	30	1	,92
50	2	,52		14	8	6	,56	49	30	6	,96
60	3	,02		15	9	3	,6	50	31	4	,—
70	3	,52		16	10	0	,64	51	32	1	,04
80	4	,02		17	10	5	,68	52	32	6	,08
90	4	,53		18	11	2	,72	53	33	3	,12
				19	11	7	,76	54	34	0	,16
				20	12	4	,8	55	34	5	,2
				21	13	1	,84	56	35	2	,24
				22	13	6	,88	57	35	7	,28
				23	14	3	,92	58	36	4	,32
				24	15	0	,96	59	37	1	,36
				25	15	6	,—	60	37	6	,4
				26	16	3	,04	61	38	3	,44
				27	17	0	,08	62	39	0	,48
				28	17	5	,12	63	39	5	,52
				29	18	2	,16	64	40	2	,56
				30	18	7	,2	65	40	7	,6
				31	19	4	,24	66	41	4	,64
				32	20	1	,28	67	42	1	,68
				33	20	6	,32	68	42	6	,72
				34	21	3	,36	69	43	3	,76
				35	22	0	,4	70	44	0	,8

N ^o .	G.	P.	Pt.	N ^o .	G.	P.	Pt.	N ^o .	G.	P.	Pt.
71	44	5	,84	81	51	0	,24	91	57	2	,64
72	45	2	,88	82	51	5	,28	92	57	7	,68
73	45	7	,92	83	52	2	,32	93	58	4	,72
74	46	4	,96	84	52	7	,36	94	59	1	,76
75	47	2	,—	85	53	4	,4	95	59	6	,8
76	47	7	,04	86	54	1	,44	96	60	3	,84
77	48	4	,08	87	54	6	,48	97	61	0	,88
78	49	1	,12	88	55	3	,52	98	61	5	,92
79	49	6	,16	89	56	0	,56	99	62	2	,96
80	50	3	,2	90	56	5	,6				,—

Table I.

Table II.

{ Beer Measure, one
Hogshead the Integer.

N ^o .	P.	Pt.	N ^o .	G.	P.	Pt.	N ^o .	G.	P.	Pt.
1	—	,04	1	0	4	,32	23	12	3	,36
2	—	,08	2	1	0	,64	24	12	7	,68
3	—	,12	3	1	4	,96	25	13	4	,—
4	—	,17	4	2	1	,28	26	14	0	,32
5	—	,21	5	2	5	,6	27	14	4	,64
6	—	,25	6	3	1	,92	28	15	0	,96
7	—	,3	7	3	6	,24	29	15	5	,28
8	—	,34	8	4	2	,56	30	16	1	,6
9	—	,38	9	4	6	,88	31	16	5	,92
10	—	,43	10	5	3	,2	32	17	2	,24
20	—	,86	11	5	7	,52	33	17	6	,56
30	1	,29	12	6	3	,84	34	18	2	,88
40	1	,72	13	7	0	,16	35	18	7	,2
50	2	,16	14	7	4	,48	36	19	3	,52
60	2	,59	15	8	0	,8	37	19	7	,84
70	3	,02	16	8	5	,12	38	20	4	,16
80	3	,45	17	9	1	,44	39	21	0	,48
90	3	,88	18	9	5	,76	40	21	4	,8
			19	10	2	,08	41	22	0	,12
			20	10	6	,4	42	22	5	,44
			21	11	2	,72	43	23	1	,76
			22	11	7	,04	44	23	6	,08

N^o.

N ^o .	G.	P.	Pt.	N ^o .	G.	P.	Pt.	N ^o .	G.	P.	Pt.
45	24	2	,4	64	34	4	,48	83	44	6	,56
46	24	6	,72	65	35	0	,8	84	45	2	,88
47	25	3	,04	66	35	5	,12	85	45	7	,2
48	25	7	,36	67	36	1	,44	86	46	3	,52
49	26	3	,68	68	36	5	,76	87	46	7	,84
50	27	0	,—	69	37	2	,08	88	47	4	,16
51	27	4	,32	70	37	6	,4	89	48	0	,48
52	28	0	,64	71	38	2	,72	90	48	4	,8
53	28	4	,96	72	38	7	,04	91	49	0	,12
54	29	1	,28	73	39	3	,36	92	49	5	,44
55	29	5	,6	74	39	7	,68	93	50	1	,76
56	30	1	,92	75	40	4	,—	94	50	6	,08
57	30	6	,24	76	41	0	,32	95	51	2	,4
58	31	2	,56	77	41	4	,64	96	51	6	,72
59	31	6	,88	78	42	0	,96	97	52	3	,04
60	32	3	,2	79	42	5	,28	98	52	7	,36
61	32	7	,52	80	43	1	,6	99	53	3	,68
62	33	3	,64	81	43	5	,92				
63	34	0	,16	82	44	2	,24				

Table I. Corn Measure, one Bushel the Integer.

N ^o .	G.	P.	Pt.	N ^o .	G.	P.	Pt.	N ^o .	G.	P.	Pt.
1	—	—	,64	16	1	2	,24	31	2	3	,84
2	—	1	,28	17	1	2	,88	32	2	4	,48
3	—	1	,92	18	1	3	,52	33	2	5	,12
4	—	2	,56	19	1	4	,16	34	2	5	,76
5	—	3	,2	20	1	4	,8	35	2	6	,4
6	—	3	,84	21	1	5	,44	36	2	7	,04
7	—	4	,48	22	1	6	,08	37	2	7	,68
8	—	5	,12	23	1	6	,72	38	3	0	,32
9	—	5	,76	24	1	7	,36	39	3	0	,96
10	—	6	,4	25	2	0	,—	40	3	1	,6
11	—	7	,04	26	2	0	,64	41	3	2	,24
12	—	7	,68	27	2	1	,28	42	3	2	,88
13	1	0	,32	28	2	1	,92	43	3	3	,52
14	1	0	,96	29	2	2	,56	44	3	4	,16
15	1	1	,6	30	2	3	,2	45	3	4	,8

N^o.

N ^o .	G.	P.	Pt.	N ^o .	G.	P.	Pt.	N ^o .	G.	P.	Pt.
46	3	5	,44	64	5	0	,96	82	6	4	,48
47	3	6	,08	65	5	1	,6	83	6	5	,12
48	3	6	,72	66	5	2	,24	84	6	5	,76
49	3	7	,36	67	5	2	,88	85	6	6	,4
50	4	0	,—	68	5	3	,52	86	6	7	,04
51	4	0	,64	69	5	4	,16	87	6	7	,68
52	4	1	,28	70	5	4	,8	88	7	0	,32
53	4	1	,92	71	5	5	,44	89	7	0	,96
54	4	2	,56	72	5	6	,08	90	7	1	,6
55	4	3	,2	73	5	6	,72	91	7	2	,24
56	4	3	,84	74	5	7	,36	92	7	2	,88
57	4	4	,48	75	6	0	,—	93	7	3	,52
58	4	5	,12	76	6	0	,64	94	7	4	,16
59	4	5	,76	77	6	1	,28	95	7	4	,8
60	4	6	,4	78	6	1	,92	96	7	5	,44
61	4	7	,04	79	6	2	,56	97	7	6	,08
62	4	7	,68	80	6	3	,2	98	7	6	,72
63	5	0	,32	81	6	3	,84	99	7	7	,36

Table I.

Table II.

{ Corn Measure, one
Quarter the Integer.

N ^o .	P.	Pt.	N ^o .	B.	G.	P.	Pt.	N ^o .	B.	G.	P.	Pt.
1	—	,05	1	0	0	5	,12	14	1	0	7	,68
2	—	,1	2	0	1	2	,24	15	1	1	4	,8
3	—	,15	3	0	1	7	,36	16	1	2	1	,92
4	—	,2	4	0	2	4	,48	17	1	2	7	,04
5	—	,25	5	0	3	1	,6	18	1	3	4	,16
6	—	,3	6	0	3	6	,72	19	1	4	1	,28
7	—	,35	7	0	4	3	,84	20	1	4	6	,4
8	—	,4	8	0	5	0	,96	21	1	5	3	,52
9	—	,45	9	0	5	5	,08	22	1	6	0	,64
10	—	,51	10	0	6	3	,2	23	1	6	5	,76
20	1	,02	11	0	7	0	,32	24	1	7	2	,88
30	1	,53	12	0	7	5	,44	25	2	0	0	,—
40	2	,04	13	1	0	2	,56	26	2	0	5	,12

N ^o .	P.	Pt.	N ^o .	B.	G.	P.	Pt.	N ^o .	B.	G.	P.	Pt.
50	2	,55	27	2	1	2	,24	64	5	0	7	,68
60	3	,07	28	2	1	7	,36	65	5	1	4	,8
70	3	,58	29	2	2	4	,48	66	5	2	1	,92
80	4	,09	30	2	3	1	,6	67	5	2	7	,04
90	4	,6	31	2	3	6	,72	68	5	3	4	,16
			32	2	4	3	,84	69	5	4	1	,28
			33	2	5	0	,96	70	5	4	6	,4
			34	2	5	5	,08	71	5	5	3	,52
			35	2	6	3	,2	72	5	6	0	,64
			36	2	7	0	,32	73	5	6	5	,76
			37	2	7	5	,44	74	5	7	2	,88
			38	3	0	2	,56	75	6	0	0	,—
			39	3	0	7	,68	76	6	0	5	,12
			40	3	1	4	,8	77	6	1	2	,24
			41	3	2	1	,92	78	6	1	7	,36
			42	3	2	7	,04	79	6	2	4	,48
			43	3	3	4	,16	80	6	3	1	,6
			44	3	4	1	,28	81	6	3	6	,72
			45	3	4	6	,4	82	6	4	3	,84
			46	3	5	3	,52	83	6	5	0	,96
			47	3	6	0	,64	84	6	5	5	,08
			48	3	6	5	,76	85	6	6	3	,2
			49	3	7	2	,88	86	6	7	0	,32
			50	4	0	0	,—	87	6	7	5	,44
			51	4	0	5	,12	88	7	0	2	,56
			52	4	1	2	,24	89	7	0	7	,68
			53	4	1	7	,36	90	7	1	4	,8
			54	4	2	4	,48	91	7	2	1	,92
			55	4	3	1	,6	92	7	2	7	,04
			56	4	3	6	,72	93	7	3	4	,16
			57	4	4	3	,84	94	7	4	1	,28
			58	4	5	0	,96	95	7	4	6	,4
			59	4	5	5	,08	96	7	5	3	,52
			60	4	6	3	,2	97	7	6	0	,64
			61	4	7	0	,32	98	7	6	5	,76
			62	4	7	5	,44	99	7	7	2	,88
			63	5	0	2	,56					

Table I. | Table II. { Wine Measure, one Tun
the Integer.

N ^o .	G.	Pts.	N ^o .	P.	H.	T.	G.	Pt.	N ^o .	P.	H.	T.	G.	Pt.
1	--	,025	1	--	--	--	2	,52	36	--	I	0	27	,72
2	--	,05	2	--	--	--	5	,04	37	--	I	0	30	,24
3	--	,075	3	--	--	--	7	,56	38	--	I	0	32	,76
4	--	,1	4	--	--	--	10	,08	39	--	I	0	35	,28
5	--	,126	5	--	--	--	12	,6	40	--	I	0	37	,8
6	--	,151	6	--	--	--	15	,12	41	--	I	0	40	,32
7	--	,176	7	--	--	--	17	,64	42	--	I	I	0	,84
8	--	,201	8	--	--	--	20	,16	43	--	I	I	3	,36
9	--	,226	9	--	--	--	22	,68	44	--	I	I	5	,88
10	--	,252	10	--	--	--	25	,2	45	--	I	I	8	,4
20	--	,504	11	--	--	--	27	,72	46	--	I	I	10	,92
30	--	,756	12	--	--	--	30	,24	47	--	I	I	13	,44
40	I	,008	13	--	--	--	32	,76	48	--	I	I	15	,96
50	I	,26	14	--	--	--	35	,28	49	--	I	I	18	,48
60	I	,512	15	--	--	--	37	,8	50	I	0	0	0	--
70	I	,764	16	--	--	--	40	,32	51	I	0	0	2	,52
80	2	,016	17	--	--	I	0	,84	52	I	0	0	5	,04
90	2	,268	18	--	--	I	3	,36	53	I	0	0	7	,56
			19	--	--	I	5	,88	54	I	0	0	10	,08
			20	--	--	I	8	,4	55	I	0	0	12	,6
			21	--	--	I	10	,92	56	I	0	0	15	,12
			22	--	--	I	13	,44	57	I	0	0	17	,64
			23	--	--	I	15	,96	58	I	0	0	20	,16
			24	--	--	I	18	,48	59	I	0	0	22	,68
			25	--	I	0	0	,	60	I	0	0	25	,2
			26	--	I	0	2	,52	61	I	0	0	27	,72
			27	--	I	0	5	,04	62	I	0	0	30	,24
			28	--	I	0	7	,56	63	I	0	0	32	,76
			29	--	I	0	10	,08	64	I	0	0	35	,28
			30	--	I	0	12	,6	65	I	0	0	37	,8
			31	--	I	0	15	,12	66	I	0	0	40	,32
			32	--	I	0	17	,64	67	I	0	I	0	,84
			33	--	I	0	20	,16	68	I	0	I	3	,36
			34	--	I	0	22	,68	69	I	0	I	5	,88
			35	--	I	0	25	,2	70	I	0	I	8	,4

N ^o .	P.	H.	T.	G.	Pt.	N ^o .	P.	H.	T.	G.	Pt.	N ^o .	P.	H.	T.	G.	Pt.
71	I	O	I	10	92	81	I	I	O	15	12	91	I	I	O	40	32
72	I	O	I	13	44	82	I	I	O	17	64	92	I	I	I	O	84
73	I	O	I	15	96	83	I	I	O	20	16	93	I	I	I	3	36
74	I	O	I	18	48	84	I	I	O	22	68	94	I	I	I	5	88
75	I	I	O	O	—	85	I	I	O	25	2	95	I	I	I	8	4
76	I	I	O	2	52	86	I	I	O	27	72	96	I	I	I	10	92
77	I	I	O	5	04	87	I	I	O	30	24	97	I	I	I	13	44
78	I	I	O	7	56	88	I	I	O	32	76	98	I	I	I	15	96
79	I	I	O	10	08	89	I	I	O	35	28	99	I	I	I	18	48
80	I	I	O	12	6	90	I	I	O	37	8						

Table I. | Table II. { *One Foot, or one Shilling the Integer.*

N ^o .	Q.	Pts.	N ^o .	In.	P.	Q.	Pt.	N ^o .	In.	P.	Q.	Pt.
1	--	,004	1	—	—	—	,48	22	2	2	2	,56
2	--	,009	2	—	—	—	,96	23	2	3	3	,04
3	--	,014	3	—	I	—	,44	24	2	3	3	,52
4	--	,019	4	—	I	—	,92	25	3	O	—	—
5	--	,024	5	—	2	—	,4	26	3	O	—	,48
6	--	,028	6	—	2	—	,88	27	3	O	—	,96
7	--	,033	7	—	3	—	,36	28	3	I	—	,44
8	--	,038	8	—	3	—	,84	29	3	I	—	,92
9	--	,043	9	I	O	—	,32	30	3	2	—	,4
10	--	,048	10	I	O	—	,8	31	3	2	—	,88
20	--	,096	11	I	I	—	,28	32	3	3	—	,36
30	--	,144	12	I	I	—	,76	33	3	3	—	,84
40	--	,192	13	I	2	—	,24	34	4	O	—	,32
50	--	,24	14	I	2	—	,72	35	4	O	—	,8
60	--	,288	15	I	3	—	,2	36	4	I	—	,28
70	--	,336	16	I	3	—	,68	37	4	I	—	,76
80	--	,384	17	2	O	—	,16	38	4	2	—	,24
90	--	,432	18	2	O	—	,64	39	4	2	—	,72
			19	2	I	—	,12	40	4	3	—	,2
			20	2	I	—	,6	41	4	3	—	,68
			21	2	2	—	,08	42	5	O	—	,16

N^o.

N ^o .	In. P.	Q.	Pt.	N ^o .	In. P.	Q.	Pt.	N ^o .	In. P.	Q.	Pt.
43	5	0	,64	62	7	1	,76	81	9	2	,88
44	5	1	,12	63	7	2	,24	82	9	3	,36
45	5	1	,6	64	7	2	,72	83	9	3	,84
46	5	2	,08	65	7	3	,2	84	10	0	,32
47	5	2	,56	66	7	3	,68	85	10	0	,8
48	5	3	,04	67	8	0	,16	86	10	1	,28
49	5	3	,52	68	8	0	,64	87	10	1	,76
50	6	0	,—	69	8	1	,12	88	10	2	,24
51	6	0	,48	70	8	1	,6	89	10	2	,72
52	6	0	,96	71	8	2	,08	90	10	3	,2
53	6	1	,44	72	8	2	,56	91	10	3	,68
54	6	1	,92	73	8	3	,04	92	11	0	,16
55	6	2	,4	74	8	3	,52	93	11	0	,64
56	6	2	,88	75	9	0	,—	94	11	1	,12
57	6	3	,36	76	9	0	,48	95	11	1	,6
58	6	3	,84	77	9	0	,96	96	11	2	,08
59	7	0	,32	78	9	1	,44	97	11	2	,56
60	7	0	,8	79	9	1	,92	98	11	3	,04
61	7	1	,28	80	9	2	,4	99	11	3	,52

Table I. Table II. { Liquid common Meas.
 { a Hogsb. of 51 Gall. Int.

N ^o .	P.	Pt.	N ^o .	G.	P.	Pt.	N ^o .	G.	P.	Pt.
1	—	,03	1	—	3	,84	14	6	5	,76
2	—	,07	2	—	7	,68	15	7	1	,6
3	—	,11	3	1	3	,52	16	7	5	,44
4	—	,15	4	1	7	,36	17	8	1	,28
5	—	,19	5	2	3	,2	18	8	5	,12
6	—	,22	6	2	7	,04	19	9	0	,96
7	—	,26	7	3	2	,88	20	9	4	,8
8	—	,3	8	3	6	,72	21	10	0	,64
9	—	,34	9	4	2	,56	22	10	4	,48
10	—	,38	10	4	6	,4	23	11	0	,32
20	—	,76	11	5	2	,24	24	11	4	,16
30	1	,15	12	5	6	,08	25	12	0	,—
40	1	,53	13	6	1	,92	26	12	3	,84

N^o.

102 *A Set of New Decimal Tables, &c.*

N ^o .	P.	Pt.	N ^o .	G.	P.	Pt.	N ^o .	G.	P.	Pt.
50	1	,92	27	12	7	,68	64	30	5	,76
60	2	,3	28	13	3	,52	65	31	1	,6
70	2	,68	29	13	7	,36	66	31	5	,44
80	3	,07	30	14	3	,2	67	32	1	,28
90	3	,45	31	14	7	,04	68	32	5	,12
			32	15	2	,88	69	33	0	,96
			33	15	6	,72	70	33	4	,8
			34	16	2	,56	71	34	0	,64
			35	16	6	,4	72	34	4	,48
			36	17	2	,24	73	35	0	,32
			37	17	6	,08	74	35	4	,16
			38	18	1	,92	75	36	0	,—
			39	18	5	,76	76	36	3	,84
			40	19	1	,6	77	36	7	,68
			41	19	5	,44	78	37	3	,52
			42	20	1	,28	79	37	7	,36
			43	20	5	,12	80	38	3	,2
			44	21	0	,96	81	38	7	,04
			45	21	4	,8	82	39	2	,88
			46	22	0	,64	83	39	6	,72
			47	22	4	,48	84	40	2	,56
			48	23	0	,32	85	40	6	,4
			49	23	4	,16	86	41	2	,24
			50	24	0	,—	87	41	6	,08
			51	24	3	,84	88	42	1	,92
			52	24	7	,68	89	42	5	,76
			53	25	3	,52	90	43	1	,6
			54	25	7	,36	91	43	5	,44
			55	26	3	,2	92	44	1	,28
			56	26	7	,04	93	44	5	,12
			57	27	2	,88	94	45	0	,96
			58	27	6	,72	95	45	4	,8
			59	28	2	,56	96	46	0	,64
			60	28	6	,4	97	46	4	,48
			61	29	2	,24	98	47	0	,32
			62	29	6	,08	99	47	4	,16
			63	30	1	,92				

Table I. | Table II. { Corn Measure, a Tun
or Load the Integer.

N ^o .	G.	Pt.	N ^o .	Q.	B.	G.	Pt.	N ^o .	Q.	B.	G.	Pt.
1	—	,03	1	—	—	3	,2	35	1	6	0	,—
2	—	,06	2	—	—	6	,4	36	1	6	3	,2
3	—	,09	3	—	1	1	,6	37	1	6	6	,4
4	—	,12	4	—	1	4	,8	38	1	7	1	,6
5	—	,16	5	—	2	0	,—	39	1	7	4	,8
6	—	,19	6	—	2	3	,2	40	2	0	0	,—
7	—	,22	7	—	2	6	,4	41	2	0	3	,2
8	—	,25	8	—	3	1	,6	42	2	0	6	,4
9	—	,28	9	—	3	4	,8	43	2	1	1	,6
10	—	,32	10	—	4	0	,—	44	2	1	4	,8
20	—	,64	11	—	4	3	,2	45	2	2	0	,—
30	—	,96	12	—	4	6	,4	46	2	2	3	,2
40	1	,28	13	—	5	1	,6	47	2	2	6	,4
50	1	,6	14	—	5	4	,8	48	2	3	1	,6
60	1	,92	15	—	6	0	,—	49	2	3	4	,8
70	2	,24	16	—	6	3	,2	50	2	4	0	,—
80	2	,56	17	—	6	6	,4	51	2	4	3	,2
90	2	,88	18	—	7	1	,6	52	2	4	6	,4
			19	—	7	4	,8	53	2	5	1	,6
			20	1	0	0	,—	54	2	5	4	,8
			21	1	0	3	,2	55	2	6	0	,—
			22	1	0	6	,4	56	2	6	3	,2
			23	1	1	1	,6	57	2	6	6	,4
			24	1	1	4	,8	58	2	7	1	,6
			25	1	2	0	,—	59	2	7	4	,8
			26	1	2	3	,2	60	3	0	0	,—
			27	1	2	6	,4	61	3	0	3	,2
			28	1	3	1	,6	62	3	0	6	,4
			29	1	3	4	,8	63	3	1	1	,6
			30	1	4	0	,—	64	3	1	4	,8
			31	1	4	3	,2	65	3	2	0	,—
			32	1	4	6	,4	66	3	2	3	,2
			33	1	5	1	,6	67	3	2	6	,4
			34	1	5	4	,8	68	3	3	1	,6

104 *A Set of New Decimal Tables, &c.*

N ^o .	Q.	B.	G.	Pt.	N ^o .	Q.	B.	G.	Pt.	N ^o .	Q.	B.	G.	Pt.
69	3	3	4	,8	80	4	0	0	,—	91	4	4	3	,2
70	3	4	0	,—	81	4	0	3	,2	92	4	4	6	,4
71	3	4	3	,2	82	4	0	6	,4	93	4	5	1	,6
72	3	4	6	,4	83	4	1	1	,6	94	4	5	4	,8
73	3	5	1	,6	84	4	1	4	,8	95	4	6	0	,—
74	3	5	4	,8	85	4	2	0	,—	96	4	6	3	,2
75	3	6	0	,—	86	4	2	3	,2	97	4	6	6	,4
76	3	6	3	,2	87	4	2	6	,4	98	4	7	1	,6
77	3	6	6	,4	88	4	3	1	,6	99	4	7	4	,8
78	3	7	1	,6	89	4	3	4	,8					
79	3	7	4	,8	90	4	4	0	,—					

Table I. | Table II. { Long Measure, one Pole
or Rod the Integer.

N ^o .	In.	Pts.	N ^o .	Yd.	Ft.	In.	Pt.	N ^o .	Yd.	Ft.	In.	Pt.
1	—	,019	1	—	—	1	,98	23	1	0	9	,54
2	—	,039	2	—	—	3	,96	24	1	0	11	,52
3	—	,059	3	—	—	5	,94	25	1	1	1	,5
4	—	,079	4	—	—	7	,92	26	1	1	3	,48
5	—	,099	5	—	—	9	,9	27	1	1	5	,46
6	—	,118	6	—	—	11	,88	28	1	1	7	,44
7	—	,138	7	—	1	1	,86	29	1	1	9	,42
8	—	,158	8	—	1	3	,84	30	1	1	11	,4
9	—	,178	9	—	1	5	,82	31	1	2	1	,38
10	—	,198	10	—	1	7	,8	32	1	2	3	,36
20	—	,396	11	—	1	9	,78	33	1	2	5	,34
30	—	,594	12	—	1	11	,76	34	1	2	7	,32
40	—	,792	13	—	2	1	,74	35	1	2	9	,3
50	—	,99	14	—	2	3	,72	36	1	2	11	,28
60	1	,188	15	—	2	5	,7	37	2	0	1	,26
70	1	,386	16	—	2	7	,68	38	2	0	3	,24
80	1	,584	17	—	2	9	,66	39	2	0	5	,22
90	1	,782	18	—	2	11	,64	40	2	0	7	,2
			19	1	0	1	,62	41	2	0	9	,18
			20	1	0	3	,6	42	2	0	11	,16
			21	1	0	5	,58	43	2	1	1	,14
			22	1	0	7	,56	44	2	1	3	,12

N^o.

N ^o	Yd	Ft	In	Pt.	N ^o	Yd	Ft	In	Pt.	N ^o	Yd	Ft	In	Pt.
45	2	I	5	,I	64	3	I	6	,72	83	4	I	8	,34
46	2	I	7	,08	65	3	I	8	,7	84	4	I	10	,32
47	2	I	9	,06	66	3	I	10	,68	85	4	2	0	,3
48	2	I	11	,04	67	3	2	0	,66	86	4	2	2	,26
49	2	2	I	,02	68	3	2	2	,64	87	4	2	4	,26
50	2	2	3	,—	69	3	2	4	,62	88	4	2	6	,24
51	2	2	4	,98	70	3	2	6	,6	89	4	2	8	,22
52	2	2	6	,96	71	3	2	8	,58	90	4	2	10	,2
53	2	2	8	,94	72	3	2	10	,56	91	5	0	0	,18
54	2	2	10	,92	73	4	0	0	,54	92	5	0	2	,16
55	3	0	0	,9	74	4	0	2	,52	93	5	0	4	,14
56	3	0	2	,88	75	4	0	4	,5	94	5	0	6	,12
57	3	0	4	,86	76	4	0	6	,48	95	5	0	8	,I
58	3	0	6	,84	77	4	0	8	,46	96	5	0	10	,08
59	3	0	8	,82	78	4	0	10	,44	97	5	I	0	,06
60	3	0	10	,8	79	4	I	0	,42	98	5	I	2	,04
61	3	I	0	,78	80	4	I	2	,4	99	5	I	4	,02
62	3	I	2	,76	81	4	I	4	,38					
63	3	I	4	,74	82	4	I	6	,36					

Table I.

Table II.

{ Long Measure, one
Yard the Integer.

N ^o	Q.	Pt.	N ^o	Ft	In	Q.	Pt.	N ^o	Ft	In	Q.	Pt.
I	—	,014	I	—	—	I	,44	14	—	5	0	,16
2	—	,028	2	—	—	2	,88	15	—	5	I	,6
3	—	,042	3	—	I	0	,32	16	—	5	3	,04
4	—	,057	4	—	I	I	,76	17	—	6	0	,48
5	—	,072	5	—	I	3	,2	18	—	6	I	,92
6	—	,086	6	—	2	0	,64	19	—	6	3	,36
7	—	,I	7	—	2	2	,08	20	—	7	0	,8
8	—	,115	8	—	2	3	,52	21	—	7	2	,24
9	—	,129	9	—	3	0	,96	22	—	7	3	,68
10	—	,144	10	—	3	2	,4	23	—	8	I	,12
20	—	,288	11	—	3	3	,84	24	—	8	2	,56
30	—	,432	12	—	4	I	,28	25	—	9	0	,—
40	—	,576	13	—	4	2	,72	26	—	9	I	,44

P

N^o.

106 *A Set of New Decimal Tables, &c.*

N ^o .	Q.	Pt.	N ^o .	Fr.	In.	Q.	Pt.	N ^o .	Fr.	In.	Q.	Pt.
50	—	,72	27	—	9	2	,88	64	1	11	0	,16
60	—	,864	28	—	10	0	,32	65	1	11	1	,6
70	—	,908	29	—	10	1	,76	66	1	11	3	,04
80	1	,152	30	—	10	3	,2	67	2	0	0	,48
90	1	,296	31	—	11	0	,64	68	2	0	1	,92
			32	—	11	2	,08	69	2	0	3	,36
			33	—	11	3	,52	70	2	1	0	,8
			34	1	0	0	,96	71	2	1	2	,24
			35	1	0	2	,4	72	2	1	3	,68
			36	1	0	3	,84	73	2	2	1	,12
			37	1	1	1	,28	74	2	2	2	,56
			38	1	1	2	,72	75	2	3	0	—
			39	1	2	0	,16	76	2	3	1	,44
			40	1	2	1	,6	77	2	3	2	,88
			41	1	2	3	,04	78	2	4	0	,32
			42	1	3	0	,48	79	2	4	1	,76
			43	1	3	1	,92	80	2	4	3	,2
			44	1	3	3	,36	81	2	5	0	,64
			45	1	4	0	,8	82	2	5	2	,08
			46	1	4	2	,24	83	2	5	3	,52
			47	1	4	3	,68	84	2	6	0	,96
			48	1	5	1	,12	85	2	6	2	,4
			49	1	5	2	,56	86	2	6	3	,84
			50	1	6	0	—	87	2	7	1	,28
			51	1	6	1	,44	88	2	7	2	,72
			52	1	6	2	,88	89	2	8	0	,16
			53	1	7	0	,32	90	2	8	1	,6
			54	1	7	1	,76	91	2	8	3	,04
			55	1	7	3	,2	92	2	9	0	,48
			56	1	8	0	,64	93	2	9	1	,92
			57	1	8	2	,08	94	2	9	3	,36
			58	1	8	3	,52	95	2	10	0	,8
			59	1	9	0	,96	96	2	10	2	,24
			60	1	9	2	,4	97	2	10	3	,68
			61	1	9	3	,84	98	2	11	1	,12
			62	1	10	1	,28	99	2	11	2	,56
			63	1	10	2	,72					

N^o.

Table I. | Table II. { Long Measure, *one Mile*
the Integer.

N ^o .	In.	Pts.	N ^o .	R.	Yd	Ft	In	Pt.	N ^o .	R.	Yd	Ft	In	Pt.
1	--	,063	1	—	—	—	6	,33	36	1	C	2	5	,88
2	--	,126	2	—	—	1	0	,66	37	1	1	0	C	,21
3	--	,189	3	—	—	1	6	,99	38	1	1	0	6	,54
4	--	,252	4	—	—	2	1	,32	39	1	1	1	C	,87
5	--	,315	5	—	—	2	7	,65	40	1	1	1	7	,2
6	--	,378	6	—	1	0	1	,98	41	1	1	2	1	,53
7	--	,441	7	—	1	0	8	,31	42	1	1	2	7	,86
8	--	,504	8	—	1	1	2	,64	43	1	2	0	2	,19
9	--	,567	9	—	1	1	8	,97	44	1	2	0	8	,52
10	--	,633	10	—	1	2	3	,3	45	1	2	1	2	,85
20	1	,266	11	—	1	2	9	,63	46	1	2	1	9	,18
30	1	,899	12	—	2	0	3	,96	47	1	2	2	3	,51
40	2	,532	13	—	2	0	10	,29	48	1	2	2	9	,84
50	3	,165	14	—	2	1	4	,62	49	1	3	0	4	,17
60	3	,789	15	—	2	1	10	,95	50	1	3	0	10	,5
70	4	,431	16	—	2	2	5	,28	51	1	3	1	4	,83
80	5	,064	17	—	2	2	11	,61	52	1	3	1	11	,16
90	5	,697	18	—	3	0	5	,94	53	1	3	2	5	,49
			19	—	3	1	C	,27	54	1	3	2	11	,82
			20	—	3	1	6	,6	55	1	4	C	6	,15
			21	—	3	2	0	,93	56	1	4	1	0	,48
			22	—	3	2	7	,26	57	1	4	1	6	,81
			23	—	4	0	1	,59	58	1	4	2	1	,14
			24	—	4	C	7	,92	59	1	4	2	7	,47
			25	—	4	1	2	,25	60	1	5	C	1	,8
			26	—	4	1	8	,58	61	1	5	C	8	,13
			27	—	4	2	2	,91	62	1	5	1	2	,46
			28	—	4	2	9	,24	63	2	0	C	2	,79
			29	—	5	C	3	,57	64	2	0	C	9	,12
			30	—	5	C	9	,9	65	2	0	1	3	,45
			31	C	5	1	4	,23	66	2	0	1	9	,78
			32	1	0	C	4	,56	67	2	0	2	4	,11
			33	1	0	C	10	,89	68	2	0	2	10	,44
			34	1	0	1	5	,22	69	2	1	C	4	,77
			35	1	0	1	11	,55	70	2	1	C	11	,1

N ^o .	R.	Yd	Ft	In	Pt.	N ^o .	R.	Yd	Ft	In	Pt.	N ^o .	R.	Yd	Ft	In	Pt.
71	2	1	1	5	43	81	2	3	0	8	73	91	2	5	0	0	03
72	2	1	1	11	76	82	2	3	1	3	06	92	2	5	0	6	36
73	2	1	2	6	09	83	2	3	1	9	39	93	2	5	1	0	69
74	2	2	0	0	42	84	2	3	2	3	72	94	3	0	0	1	02
75	2	2	0	6	75	85	2	3	2	10	05	95	3	0	0	7	35
76	2	2	1	1	08	86	2	4	0	4	38	96	3	0	1	1	68
77	2	2	1	7	41	87	2	4	0	10	71	97	3	0	1	8	01
78	2	2	2	1	74	88	2	4	1	5	04	98	3	0	2	2	34
79	2	2	2	8	07	89	2	4	1	11	37	99	3	0	2	8	67
80	2	3	0	2	4	90	2	4	2	5	7						

Table III. Long Measure, *one Mile the Integer.*

N ^o .	Fg	R.	Yd	Ft	In.	Pt.	N ^o .	Fg	R.	Yd	Ft	In.	Pt.
1	0	3	1	0	3	,6	23	1	33	3	0	10	,8
2	--	6	2	0	7	,2	24	1	36	4	1	2	,4
3	--	9	3	0	10	,8	25	2	0	0	0	0	,—
4	--	12	4	1	2	,4	26	2	3	1	0	3	,6
5	--	16	0	0	0	,—	27	2	6	2	0	7	,2
6	--	19	1	0	3	,6	28	2	9	3	0	10	,8
7	--	22	2	0	7	,2	29	2	12	4	1	2	,4
8	--	25	3	0	10	,8	30	2	16	0	0	0	,—
9	--	28	4	1	2	,4	31	2	19	1	0	3	,6
10	--	32	0	0	0	,—	32	2	22	2	0	7	,2
11	--	35	1	0	3	,6	33	2	25	3	0	10	,8
12	--	38	2	0	7	,2	34	2	28	4	1	2	,4
13	I	1	3	0	10	,8	35	2	32	0	0	0	,—
14	I	4	4	1	2	,4	36	2	35	1	0	3	,6
15	I	8	0	0	0	,—	37	2	38	2	0	7	,2
16	I	11	1	0	3	,6	38	3	1	3	0	10	,8
17	I	14	2	0	7	,2	39	3	4	4	1	2	,4
18	I	17	3	0	10	,8	40	3	8	0	0	0	,—
19	I	20	4	1	2	,4	41	3	11	1	0	3	,6
20	I	24	0	0	0	,—	42	3	14	2	0	7	,2
21	I	27	1	0	3	,6	43	3	17	3	0	10	,8
22	I	30	2	0	7	,2	44	3	20	4	1	2	,4

N ^o .	Fg	R.	Yd	Ft	In.	Pt.	N ^o .	Fg	R.	Yd	Ft	In.	Pt.
45	3	24	0	0	0	,—	73	5	33	5	0	10	,8
46	3	27	1	0	3	,6	74	5	36	4	1	2	,4
47	3	30	2	0	7	,2	75	6	0	0	0	0	,—
48	3	33	3	0	10	,8	76	6	3	1	0	3	,6
49	3	36	4	1	2	,4	77	6	6	2	0	7	,2
50	4	0	0	0	0	,—	78	6	9	3	0	10	,8
51	4	3	1	0	3	,6	79	6	12	4	1	2	,4
52	4	6	2	0	7	,2	80	6	16	0	0	0	,—
53	4	9	3	0	10	,8	81	6	19	1	0	3	,6
54	4	12	4	1	2	,4	82	6	22	2	0	7	,2
55	4	16	0	0	0	,—	83	6	25	3	0	10	,8
56	4	19	1	0	3	,6	84	6	28	4	1	2	,4
57	4	22	2	0	7	,2	85	6	32	0	0	0	,—
58	4	25	3	0	10	,8	86	6	35	1	0	3	,6
59	4	28	4	1	2	,4	87	6	38	2	0	7	,2
60	4	32	0	0	0	,—	88	7	1	3	0	10	,8
61	4	35	1	0	3	,6	89	7	4	4	1	2	,4
62	4	38	2	0	7	,2	90	7	8	0	0	0	,—
63	5	1	3	0	10	,8	91	7	11	1	0	3	,6
64	5	4	4	1	2	,4	92	7	14	2	0	7	,2
65	5	8	0	0	0	,—	93	7	17	3	0	10	,8
66	5	11	1	0	3	,6	94	7	20	4	1	2	,4
67	5	14	2	0	7	,2	95	7	24	0	0	0	,—
68	5	17	3	0	10	,8	96	7	27	1	0	3	,6
69	5	20	4	1	2	,4	97	7	30	2	0	7	,2
70	5	24	0	0	0	,—	98	7	33	3	0	10	,8
71	5	27	1	0	3	,6	99	7	36	4	1	2	,4
72	5	30	2	0	7	,2							

Table I. | Table II. { Square Measure, one
Mile Square the Integer.

N ^o .	Pq	Pts.	N ^o .	Aq	Rq	Pq	Pt.	N ^o .	Aq	Rq	Pq	Pt.
1	—	,102	1	—	—	10	,24	6	—	1	21	,44
2	—	,204	2	—	—	20	,48	7	—	1	31	,68
3	—	,307	3	—	—	30	,72	8	—	2	1	,92
4	—	,409	4	—	1	0	,96	9	—	2	12	,16
5	—	,512	5	—	1	11	,2	10	—	2	22	,4

N^o.

110 *A Set of New Decimal Tables, &c.*

N ^o .	Pq	Pts.	N ^o .	Aq	Rq	Pq	Pt.	N ^o .	Aq	Rq	Pq	Pt.
6	--	,614	11	--	2	32	,64	51	3	1	2	,24
7	--	,716	12	--	3	2	,88	52	3	1	12	,48
8	--	,819	13	--	3	13	,12	53	3	1	22	,72
9	--	,921	14	--	3	23	,36	54	3	1	32	,96
10	1	,02	15	--	3	33	,6	55	3	2	3	,2
20	2	,04	16	1	0	3	,84	56	3	2	13	,44
30	3	,07	17	1	0	14	,08	57	3	2	23	,68
40	4	,09	18	1	0	24	,32	58	3	2	33	,92
50	5	,12	19	1	0	34	,56	59	3	3	4	,16
60	6	,14	20	1	1	4	,8	60	3	3	14	,4
70	7	,16	21	1	1	15	,04	61	3	3	24	,64
80	8	,19	22	1	1	25	,28	62	3	3	34	,88
90	9	,21	23	1	1	35	,52	63	4	0	5	,12
			24	1	2	5	,76	64	4	0	15	,36
			25	1	2	16	,—	65	4	0	25	,6
			26	1	2	26	,24	66	4	0	35	,84
			27	1	2	36	,48	67	4	1	6	,08
			28	1	3	6	,72	68	4	1	16	,32
			29	1	3	16	,96	69	4	1	26	,56
			30	1	3	27	,2	70	4	1	36	,8
			31	1	3	37	,44	71	4	2	7	,04
			32	2	0	7	,68	72	4	2	17	,28
			33	2	0	17	,92	73	4	2	27	,52
			34	2	0	28	,16	74	4	2	37	,76
			35	2	0	38	,4	75	4	3	8	,—
			36	2	1	8	,64	76	4	3	18	,24
			37	2	1	19	,88	77	4	3	28	,48
			38	2	1	29	,12	78	4	3	38	,72
			39	2	1	39	,36	79	5	0	8	,96
			40	2	2	9	,6	80	5	0	19	,2
			41	2	2	19	,84	81	5	0	29	,44
			42	2	2	29	,08	82	5	0	39	,68
			43	2	2	39	,32	83	5	1	9	,92
			44	2	3	10	,56	84	5	1	20	,16
			45	2	3	20	,8	85	5	1	30	,4
			46	2	3	31	,04	86	5	2	0	,64
			47	3	0	1	,28	87	5	2	10	,88
			48	3	0	11	,52	88	5	2	21	,12
			49	3	0	21	,76	89	5	2	31	,36
			50	3	0	32	,—	90	5	3	1	,6

N ^o .	Aq	Rq	Pq	Pt.	N ^o .	Aq	Rq	Pq	Pt.	N ^o .	Aq	Rq	Pq	Pt.
91	5	3	11	84	94	6	0	12	56	97	6	0	33	28
92	5	3	22	08	95	6	0	12	8	98	6	1	3	52
93	5	3	32	32	96	6	0	23	04	99	6	1	13	76

Table III. Square Measure, *one Mile Square the Integ.*

N ^o .	Aq.	Rq	Pq	N ^o .	Aq.	Rq	Pq	N ^o .	Aq.	Rq	Pq
1	6	1	24	34	217	2	16	67	429	3	8
2	12	3	8	35	224	0	—	68	435	0	32
3	19	0	32	36	230	1	24	69	441	2	16
4	25	2	16	37	236	3	8	70	448	0	—
5	32	0	—	38	243	0	32	71	454	1	24
6	38	1	24	39	249	2	16	72	460	3	8
7	44	3	8	40	256	0	—	73	467	0	32
8	51	0	32	41	262	1	24	74	473	2	16
9	57	2	16	42	268	3	8	75	480	0	—
10	64	0	—	43	275	0	32	76	486	1	24
11	70	1	24	44	281	2	16	77	492	3	8
12	76	3	8	45	288	0	—	78	499	0	32
13	83	0	32	46	294	1	24	79	505	2	16
14	89	2	16	47	300	3	8	80	512	0	—
15	96	0	—	48	307	0	32	81	518	1	24
16	102	1	24	49	313	2	16	82	524	3	8
17	108	3	8	50	320	0	—	83	531	0	32
18	115	0	32	51	326	1	24	84	537	2	16
19	121	2	16	52	332	3	8	85	544	0	—
20	128	0	—	53	339	0	32	86	550	1	24
21	134	1	24	54	345	2	16	87	556	3	8
22	140	3	8	55	352	0	—	88	563	0	32
23	147	0	32	56	358	1	24	89	569	2	16
24	153	2	16	57	364	3	8	90	576	0	—
25	160	0	—	58	371	0	32	91	583	1	24
26	166	1	24	59	377	2	16	92	588	3	8
27	172	3	8	60	384	0	—	93	595	0	32
28	179	0	32	61	390	1	24	94	601	2	16
29	185	2	16	62	396	3	8	95	60	0	—
30	192	0	—	63	403	0	32	96	614	1	24
31	298	1	24	64	409	2	16	97	620	3	8
32	204	3	8	65	416	0	—	98	627	0	32
33	211	0	32	66	422	1	24	99	633	2	16

N^o.

Table I.

Table II.

Square Measure, *one*
Rod square the Integer.

N ^o .	Fq	Pt.	N ^o .	Yq.	Fq	Pt.	N ^o .	Yq	Fq	Pt.
1	—	,02	1	—	2	,72	36	10	7	,92
2	—	,05	2	—	5	,44	37	11	1	,64
3	—	,08	3	—	8	,16	38	11	4	,36
4	—	,1	4	1	1	,88	39	11	7	,08
5	—	,13	5	1	4	,6	40	12	0	,8
6	—	,16	6	1	7	,32	41	12	3	,52
7	—	,18	7	2	1	,04	42	12	6	,24
8	—	,21	8	2	3	,76	43	12	8	,96
9	—	,24	9	2	6	,48	44	13	2	,68
10	—	,27	10	3	0	,2	45	13	5	,4
20	—	,54	11	3	2	,92	46	13	8	,12
30	—	,81	12	3	5	,64	47	14	1	,84
40	1	,08	13	3	8	,36	48	14	4	,56
50	1	,35	14	4	2	,08	49	14	7	,28
60	1	,62	15	4	4	,8	50	15	1	,—
70	1	,89	16	4	7	,52	51	15	3	,72
80	2	,16	17	5	1	,24	52	15	6	,44
90	2	,43	18	5	3	,96	53	16	0	,16
			19	5	6	,68	54	16	2	,88
			20	6	0	,4	55	16	5	,6
			21	6	3	,12	56	16	8	,32
			22	6	5	,84	57	17	2	,04
			23	6	8	,56	58	17	4	,76
			24	7	2	,28	59	17	7	,48
			25	7	5	,—	60	18	1	,2
			26	7	7	,72	61	18	3	,92
			27	8	1	,44	62	18	6	,64
			28	8	4	,16	63	19	0	,36
			29	8	6	,88	64	19	3	,08
			30	9	0	,6	65	19	5	,8
			31	9	3	,32	66	19	8	,52
			32	9	6	,04	67	20	2	,24
			33	9	8	,76	68	20	4	,96
			34	10	2	,48	69	20	7	,68
			35	10	5	,2	70	21	1	,4

N^o.

N ^o .	Yq	Fq	Pt.	N ^o .	Yq	Fq	Pt.	N ^o .	Yq	Fq	Pt.
71	21	4	,12	81	24	4	,32	91	27	4	,52
72	21	6	,84	82	24	7	,04	92	27	7	,24
73	22	0	,56	83	25	0	,76	93	28	0	,96
74	22	3	,28	84	25	3	,48	94	28	3	,68
75	22	6	,—	85	25	6	,2	95	28	6	,4
76	22	8	,72	86	25	8	,92	96	29	0	,12
77	23	2	,44	87	26	2	,64	97	29	2	,84
78	23	5	,16	88	26	5	,36	98	29	5	,56
79	23	7	,88	89	26	8	,08	99	29	8	,28
80	23	1	,6	90	27	1	,8				

Table I. } Table II. { Square Measure, one
Yard Square the Integer.

N ^o .	lq.	Pt.	N ^o .	Fq	lq.	Pt.	N ^o .	Fq	lq.	Pt.
1	--	,12	1	--	12	,96	23	2	10	,08
2	--	,25	2	--	25	,92	24	2	23	,04
3	--	,38	3	--	38	,88	25	2	36	,—
4	--	,51	4	--	51	,84	26	2	48	,96
5	--	,64	5	--	64	,8	27	2	61	,92
6	--	,77	6	--	77	,76	28	2	74	,88
7	--	,9	7	--	90	,72	29	2	87	,84
8	1	,03	8	--	103	,68	30	2	100	,8
9	1	,16	9	--	116	,64	31	2	113	,76
10	1	,29	10	--	129	,6	32	2	126	,72
20	2	,59	11	--	142	,56	33	2	139	,68
30	3	,88	12	1	11	,52	34	3	8	,64
40	5	,17	13	1	24	,48	35	3	21	,6
50	6	,46	14	1	37	,44	36	3	34	,56
60	7	,75	15	1	50	,4	37	3	47	,52
70	9	,04	16	1	63	,36	38	3	60	,48
80	10	,33	17	1	76	,32	39	3	73	,44
90	11	,62	18	1	89	,28	40	3	86	,4
			19	1	102	,24	41	3	99	,36
			20	1	115	,2	42	3	112	,32
			21	1	128	,16	43	3	125	,28
			22	1	141	,12	44	3	138	,24

N ^o .	Fq.	Iq.	Pt.	N ^o .	Fq.	Iq.	Pt.	N ^o .	Fq.	Iq.	Pt.
45	4	7	,2	64	5	109	,44	83	7	67	,68
46	4	20	,16	65	5	122	,4	84	7	80	,64
47	4	33	,12	66	5	135	,36	85	7	93	,6
48	4	46	,08	67	6	4	,32	86	7	106	,56
49	4	59	,04	68	6	17	,28	87	7	119	,52
50	4	72	—	69	6	30	,24	88	7	132	,48
51	4	84	,96	70	6	43	,2	89	8	1	,44
52	4	97	,92	71	6	56	,16	90	8	14	,4
53	4	110	,88	72	6	69	,12	91	8	27	,36
54	4	123	,84	73	6	82	,08	92	8	40	,32
55	4	136	,8	74	6	95	,04	93	8	53	,28
56	5	5	,76	75	6	108	—	94	8	66	,24
57	5	18	,72	76	6	120	,96	95	8	79	,2
58	5	31	,68	77	6	133	,92	96	8	92	,16
59	5	44	,64	78	7	2	,88	97	8	105	,12
60	5	57	,6	79	7	15	,84	98	8	118	,08
61	5	70	,56	80	7	28	,8	99	8	131	,04
62	5	83	,52	81	7	41	,76				
63	5	96	,48	82	7	54	,72				

Table I.

Table II.

{ Square Measure, one
Foot Square the Integer.

N ^o .	Iq.	q.q.	Pt.	N ^o .	Iq.	q.q.	Pt.	N ^o .	Iq.	q.q.	Pt.
1	—	—	,23	1	1	7	,04	14	20	2	,56
2	—	—	,46	2	2	14	,08	15	21	9	,6
3	—	—	,69	3	4	5	,12	16	23	0	,64
4	—	—	,92	4	5	12	,16	17	24	7	,68
5	—	1	,15	5	7	3	,2	18	25	14	,72
6	—	1	,38	6	8	10	,24	19	27	5	,76
7	—	1	,61	7	10	1	,28	20	28	12	,8
8	—	1	,84	8	11	8	,32	21	30	3	,84
9	—	2	,07	9	12	15	,36	22	31	10	,88
10	—	2	,3	10	14	6	,4	23	33	1	,92
20	—	4	,6	11	15	13	,44	24	34	8	,96
30	—	6	,9	12	17	4	,48	25	36	0	—
40	—	9	2,	13	18	11	,52	26	37	7	,84

N^o.

A Set of New Decimal Tables, &c. 115

N ^o .	Iq.	q.q	Pt.	N ^o .	Iq.	q.q	Pt.	N ^o .	Iq.	q.q	Pt.
50	—	11	,5	27	38	14	,08	64	92	2	,56
60	—	13	,8	28	40	5	,12	65	93	9	,6
70	1	0	,1	29	41	12	,16	66	95	0	,64
80	1	2	,4	30	43	3	,2	67	96	7	,68
90	1	4	,7	31	44	10	,24	68	97	14	,72
				32	46	1	,28	69	99	5	,76
				33	47	8	,32	70	100	12	,8
				34	48	15	,36	71	102	3	,84
				35	50	6	,4	72	103	10	,88
				36	51	13	,44	73	105	1	,92
				37	53	4	,48	74	106	8	,96
				38	54	11	,52	75	108	0	,—
				39	56	2	,56	76	109	7	,04
				40	57	9	,6	77	110	14	,08
				41	59	0	,64	78	112	5	,12
				42	60	7	,68	79	113	12	,16
				43	61	14	,72	80	115	3	,2
				44	63	5	,76	81	116	10	,24
				45	64	12	,8	82	118	1	,28
				46	66	3	,84	83	119	8	,32
				47	67	10	,88	84	120	15	,36
				48	69	1	,92	85	122	6	,4
				49	70	8	,96	86	123	13	,44
				50	72	0	,—	87	125	4	,48
				51	73	7	,04	88	126	11	,52
				52	74	14	,08	89	128	2	,56
				53	76	5	,12	90	129	9	,6
				54	77	12	,16	91	131	0	,64
				55	79	3	,2	92	132	7	,68
				56	80	10	,24	93	133	14	,72
				57	82	1	,28	94	135	5	,76
				58	83	8	,32	95	136	12	,8
				59	84	15	,36	96	138	3	,84
				60	86	6	,4	97	139	10	,88
				61	87	13	,44	98	141	1	,92
				62	89	4	,48	99	142	8	,96
				63	90	11	,52				

Table I. | Table II. { Cubic or Solid Measure,
one Solid Yard the Integer.

N ^o .	Fc.	Pc.	N ^o .	Fc.	L. c.	Pt.	N ^o .	Fc.	L. c.	Pt.
1	4	,66	1	—	466	,56	36	9	1244	,16
2	9	,32	2	—	933	,12	37	9	1710	,72
3	13	,98	3	—	1399	,68	38	10	449	,28
4	18	,65	4	1	138	,24	39	10	915	,84
5	23	,31	5	1	604	,8	40	10	1382	,4
6	27	,98	6	1	1071	,36	41	11	120	,96
7	32	,64	7	1	1537	,92	42	11	587	,52
8	37	,31	8	2	276	,48	43	11	1054	,08
9	41	,97	9	2	743	,04	44	11	1520	,64
10	46	,65	10	2	1209	,6	45	12	259	,2
20	93	,3	11	2	1676	,16	46	12	725	,76
30	139	,98	12	3	414	,72	47	12	1192	,32
40	186	,6	13	3	881	,28	48	12	1658	,88
50	233	,25	14	3	1347	,84	49	13	397	,44
60	279	,9	15	4	86	,4	50	13	864	,—
70	326	,62	16	4	552	,96	51	13	1330	,56
80	373	,2	17	4	1019	,52	52	14	69	,12
90	419	,94	18	4	1486	,08	53	14	535	,68
			19	5	224	,64	54	14	1002	,24
			20	5	691	,2	55	14	1468	,8
			21	5	1157	,76	56	15	207	,36
			22	5	1624	,32	57	15	673	,92
			23	6	362	,88	58	15	1140	,48
			24	6	829	,44	59	15	1607	,04
			25	6	1296	,—	60	16	345	,6
			26	7	34	,56	61	16	812	,16
			27	7	501	,12	62	16	1278	,72
			28	7	967	,68	63	17	17	,28
			29	7	1434	,24	64	17	483	,84
			30	8	172	,8	65	17	950	,4
			31	8	639	,36	66	17	1416	,96
			32	8	1105	,92	67	18	155	,52
			33	8	1572	,48	68	18	622	,08
			34	9	311	,04	69	18	1088	,64
			35	9	777	,6	70	18	1555	,2

N ^o .	Fc	<u>l. c.</u>	<u>Pt.</u>	N ^o .	Fc	<u>l. c.</u>	<u>Pt.</u>	N ^o .	Fc	<u>l. c.</u>	<u>Pt.</u>
71	19	293	,76	81	21	1503	,36	91	24	984	,96
72	19	760	,32	82	22	241	,92	92	24	1451	,52
73	19	1226	,88	83	22	708	,48	93	25	190	,08
74	19	1693	,44	84	22	1175	,04	94	25	656	,64
75	20	432	,—	85	22	1641	,6	95	25	1123	,2
76	20	898	,56	86	23	380	,16	96	25	1589	,76
77	20	1365	,12	87	23	846	,72	97	26	328	,32
78	21	103	,68	88	23	1313	,28	98	26	794	,88
79	21	570	,24	89	24	51	,84	99	26	1261	,44
80	21	1036	,8	90	24	518	,4				

Table I. | Table II. { Cubic or Solid Measure,
one Solid Foot the Integer.

N ^o .	I. c.	Qc	Pt.	N ^o .	I. c.	Qc	Pt.	N ^o .	I. c.	Qc	Pt.
1	—	11	,05	1	17	17	,92	23	397	28	,16
2	—	22	,11	2	34	35	,84	24	414	46	,08
3	—	33	,17	3	51	53	,76	25	432	00	—
4	—	44	,23	4	69	7	,68	26	449	17	,92
5	—	55	,29	5	86	25	,6	27	466	35	,84
6	1	2	,35	6	103	43	,52	28	483	53	,76
7	1	13	,41	7	120	61	,44	29	501	7	,68
8	1	24	,47	8	138	15	,36	30	518	25	,6
9	1	35	,53	9	155	33	,28	31	535	43	,52
10	1	46	,59	10	172	51	,2	32	552	61	,44
20	3	29	,18	11	190	5	,12	33	570	15	,36
30	5	11	,77	12	207	23	,04	34	587	33	,28
40	6	58	,36	13	224	40	,96	35	604	51	,2
50	8	40	,95	14	241	58	,88	36	622	5	,12
60	10	23	,54	15	259	12	,8	37	639	23	,04
70	12	6	,13	16	276	30	,72	38	656	40	,96
80	13	52	,72	17	293	48	,64	39	673	58	,88
90	15	35	,31	18	311	2	,56	40	691	12	,8
				19	328	20	,48	41	708	30	,72
				20	345	38	,4	42	725	48	,64
				21	362	56	,32	43	743	2	,56
				22	380	10	,24	44	760	20	,48

118 *A Set of New Decimal Tables, &c.*

N ^o .	<i>l. c.</i>	<i>Qc</i>	<i>Pt.</i>	N ^o .	<i>l. c.</i>	<i>Qc</i>	<i>Pt.</i>	N ^o .	<i>l. c.</i>	<i>Qc</i>	<i>Pt.</i>
45	777	38	,4	64	1105	58	,88	83	1434	15	,36
46	794	56	,32	65	1123	12	,8	84	1451	33	,28
47	812	10	,24	66	1140	30	,72	85	1468	51	,2
48	829	28	,16	67	1157	48	,64	86	1486	5	,12
49	846	46	,08	68	1175	2	,56	87	1503	23	,04
50	864	0	,—	69	1192	20	,48	88	1520	40	,96
51	881	17	,92	70	1209	38	,4	89	1537	58	,88
52	898	35	,84	71	1226	56	,32	90	1555	12	,8
53	915	53	,76	72	1244	10	,24	91	1572	30	,72
54	933	7	,68	73	1261	28	,16	92	1589	48	,64
55	950	25	,6	74	1278	46	,08	93	1607	2	,56
56	967	43	,52	75	1296	0	,—	94	1624	20	,48
57	984	61	,44	76	1313	17	,92	95	1641	38	,4
58	1002	15	,36	77	1330	35	,84	96	1658	56	,32
59	1019	33	,28	78	1347	53	,76	97	1676	10	,24
60	1036	51	,2	79	1365	7	,68	98	1693	28	,16
61	1054	5	,12	80	1382	25	,6	99	1710	46	,08
62	1071	23	,04	81	1399	43	,52				
63	1088	40	,96	82	1416	61	,44				

Table I.

Table II. { Land Measure, one
Acre the Integer.

N ^o .	<i>Pq</i>	<i>Yq</i>	<i>Pt.</i>	<i>Rods</i>	0	1	2	3	<i>Pq</i>	<i>Yq</i>	<i>Pt.</i>
1	—	—	,48	N ^o .	0	25	50	75	0	0	,0
2	—	—	,96		1	26	51	76	1	18	,15
3	—	1	,44		2	27	52	77	3	6	,05
4	—	1	,92		3	28	53	78	4	24	,2
5	—	2	,4		4	29	54	79	6	12	,1
6	—	2	,88		5	30	55	80	8	0	,—
7	—	3	,36		6	31	56	81	9	18	,15
8	—	3	,84		7	32	57	82	11	6	,05
9	—	4	,32		8	33	58	83	12	24	,2
10	—	4	,84		9	34	59	84	14	12	,1
20	—	9	,68		10	35	60	85	16	0	,—
30	—	14	,52		11	36	61	86	17	18	,15
40	—	19	,36		12	37	62	87	19	6	,05

N^o.

N ^o .	Pq	Yq.	Pt.	Rods	0	1	2	3	Pq	Yq	Pt.
50	—	24	,2	N ^o .	13	38	63	88	20	24	,2
60	—	29	,04		14	39	64	89	22	12	,1
70	1	3	,88		15	40	65	90	24	0	,—
80	1	8	,72		16	41	66	91	25	18	,15
90	1	13	,56		17	42	67	92	27	6	,05
					18	43	68	93	28	24	,2
					19	44	69	94	30	12	,1
					20	45	70	95	32	0	,—
					21	46	71	96	33	18	,15
					22	47	72	97	35	6	,05
					23	48	73	98	36	24	,2
					24	49	74	99	38	12	,1

Table I. | Table II. { Time. One Year the Integer.

N ^o .	D.	H.	Pt.	N ^o .	MW.	D.	H.	Pt.	N ^o .	M	W.	D.	H.	Pt.
1	—	—	,87	1	—	—	3	15,6	19	2	1	6	8,4	
2	—	1	,74	2	—	1	0	7,2	20	2	2	3	0, —	
3	—	2	,62	3	—	1	3	22,8	21	2	2	6	15,6	
4	—	3	,5	4	—	2	0	14,4	22	2	3	3	7,2	
5	—	4	,38	5	—	2	4	6, —	23	2	3	6	22,8	
6	—	5	,25	6	—	3	0	21,6	24	3	0	3	14,4	
7	—	6	,12	7	—	3	4	13,2	25	3	1	0	6, —	
8	—	7	, —	8	1	0	1	4,8	26	3	1	3	21,6	
9	—	7	,88	9	1	0	4	20,4	27	3	2	0	13,2	
10	—	8	,76	10	1	1	1	12, —	28	3	2	4	4,8	
20	—	17	,52	11	1	1	5	3,6	29	3	3	0	20,4	
30	1	2	,28	12	1	2	1	19,2	30	3	3	4	12, —	
40	1	11	,04	13	1	2	5	10,8	31	4	0	1	3,6	
50	1	19	,8	14	1	3	2	2,4	32	4	0	4	19,2	
60	2	4	,56	15	1	3	6	18, —	33	4	1	1	10,8	
70	2	13	,32	16	2	0	2	9,6	34	4	1	5	2,4	
80	2	22	,08	17	2	0	6	1,2	35	4	2	1	18,6	
90	3	6	,84	18	2	1	3	16,8	36	4	2	5	9,6	

N^o.

120 *A Set of New Decimal Tables, &c.*

N ^o .	M.	W.	D.	H.	Pt.	N ^o .	M.	W.	D.	H.	Pt.
37	4	3	2	1	,2	69	8	3	6	20	,4
38	4	3	5	16	,8	70	9	0	3	12	,—
39	5	0	2	8	,4	71	9	1	0	3	,6
40	5	0	6	0	,—	72	9	1	3	19	,2
41	5	1	2	15	,6	73	9	2	0	10	,8
42	5	1	6	7	,2	74	9	2	4	2	,4
43	5	2	2	22	,8	75	9	3	0	18	,—
44	5	2	6	14	,4	76	9	3	4	9	,6
45	5	3	3	6	,—	77	10	0	1	1	,2
46	5	3	6	21	,6	78	10	0	4	16	,8
47	6	0	3	13	,2	79	10	1	1	8	,4
48	6	1	0	4	,8	80	10	1	5	0	,—
49	6	1	3	20	,4	81	10	2	1	15	,6
50	6	2	0	12	,—	82	10	2	5	7	,2
51	6	2	4	3	,6	83	10	3	2	22	,8
52	6	3	0	19	,2	84	10	3	5	14	,4
53	6	3	4	10	,8	85	11	0	2	6	,—
54	7	0	1	2	,4	86	11	0	6	21	,6
55	7	0	4	18	,—	87	11	1	2	13	,2
56	7	1	1	9	,6	88	11	1	6	4	,8
57	7	1	5	1	,2	89	11	2	2	20	,4
58	7	2	1	16	,8	90	11	2	6	12	,—
59	7	2	5	8	,4	91	11	3	3	3	,6
60	7	3	2	0	,—	92	11	3	6	19	,2
61	7	3	5	15	,6	93	12	0	3	10	,8
62	8	0	2	7	,2	94	12	1	0	2	,4
63	8	0	6	22	,8	95	12	1	3	18	,—
64	8	1	2	14	,4	96	12	2	0	9	,6
65	8	1	6	6	,—	97	12	2	4	1	,2
66	8	2	2	21	,6	98	12	3	0	16	,8
67	8	2	6	13	,2	99	12	3	4	8	,4
68	8	3	3	4	,8						

N^o.

Table I. | Table II. { Time. One Month the Integer.

N ^o .	H.	Pt.	W.	0	1	2	3	D.	H.	Pt.
1	—	,06	N ^o .	0	25	50	75	0	0	—
2	—	,13		1	26	51	76	0	6	,72
3	—	,2		2	27	52	77	0	13	,44
4	—	,26		3	28	53	78	0	20	,16
5	—	,33		4	29	54	79	1	2	,88
6	—	,4		5	30	55	80	1	9	,6
7	—	,46		6	31	56	81	1	16	,32
8	—	,53		7	32	57	82	1	23	,04
9	—	,6		8	33	58	83	2	5	,76
10	—	,67		9	34	59	84	2	12	,48
20	1	,34		10	35	60	85	2	19	,2
30	2	,01		11	36	61	86	3	1	,92
40	2	,68		12	37	62	87	3	8	,64
50	3	,35		13	38	63	88	3	15	,36
60	4	,02		14	39	64	89	3	22	,08
70	4	,69		15	40	65	90	4	4	,8
80	5	,36		16	41	66	91	4	11	,52
90	6	,03		17	42	67	92	4	18	,24
				18	43	68	93	5	0	,96
				19	44	69	94	5	7	,68
				20	45	70	95	5	14	,4
				21	46	71	96	5	21	,12
				22	47	72	97	6	3	,84
				23	48	73	98	6	10	,56
				24	49	74	99	6	17	,28

Table I. | Table II. { Time. One Day the Integer.

N ^o .	M'	Pt.	N ^o .	H.	M'	Pt.	N ^o .	H.	M'	Pt.
1	—	,14	1	0	14	,4	6	1	26	,4
2	—	,28	2	0	28	,8	7	1	40	,8
3	—	,43	3	0	43	,2	8	1	55	,2
4	—	,57	4	0	57	,6	9	2	9	,6
5	—	,72	5	1	12	—	10	2	24	—

R

N^o.

22 *A Set of New Decimal Tables, &c.*

N ^o .	M.	Pt.	N ^o .	H.	M.	Pt.	N ^o .	H.	M.	Pt.
6	—	,86	11	2	38	,4	51	12	14	,4
7	1	,—	12	2	52	,8	52	12	28	,8
8	1	,15	13	3	7	,2	53	12	43	,2
9	1	,29	14	3	21	,6	54	12	57	,6
10	1	,44	15	3	36	,—	55	13	12	,—
20	2	,88	16	3	50	,4	56	13	26	,4
30	4	,32	17	4	4	,8	57	13	40	,8
40	5	,76	18	4	19	,2	58	13	55	,2
50	7	,2	19	4	33	,6	59	14	9	,6
60	8	,64	20	4	48	,—	60	14	24	,—
70	10	,08	21	5	2	,4	61	14	38	,4
80	11	,52	22	5	16	,8	62	14	52	,8
90	12	,96	23	5	31	,2	63	15	7	,2
			24	5	45	,6	64	15	21	,6
			25	6	0	,—	65	15	36	,—
			26	6	14	,4	66	15	50	,4
			27	6	28	,8	67	16	4	,8
			28	6	43	,2	68	16	19	,2
			29	6	57	,6	69	16	33	,6
			30	7	12	,—	70	16	48	,—
			31	7	26	,4	71	17	2	,4
			32	7	40	,8	72	17	16	,8
			33	7	55	,2	73	17	31	,2
			34	8	9	,6	74	17	45	,6
			35	8	24	,—	75	18	0	,—
			36	8	38	,4	76	18	14	,4
			37	8	52	,8	77	18	28	,8
			38	9	7	,2	78	18	43	,2
			39	9	21	,6	79	18	57	,6
			40	9	36	,—	80	19	12	,—
			41	9	50	,4	81	19	26	,4
			42	10	4	,8	82	19	40	,8
			43	10	19	,2	83	19	55	,2
			44	10	33	,6	84	20	9	,6
			45	10	48	,—	85	20	24	,—
			46	11	2	,4	86	20	38	,4
			47	11	16	,8	87	20	52	,8
			48	11	31	,2	88	21	7	,2
			49	11	45	,6	89	21	21	,6
			50	12	0	,—	90	21	36	,—

N ^o .	H.	M.	Pt.	N ^o .	H.	M.	Pt.	N ^o .	H.	M.	Pt.
91	21	50	,4	94	22	33	,6	97	23	16	,8
92	22	4	,8	95	22	48	,—	98	23	31	,2
93	22	19	,2	96	23	2	,4	99	23	45	,6

Table I.

Table II.

*Time or Motion.
One Hour, or De-
gree the Integer.*

N ^o .	H.	M.	Pt.	N ^o .	H.	M.	Pt.	N ^o .	H.	M.	Pt.
1	—	—	,36	1	0	—	36	29	17	—	24
2	—	—	,72	2	1	—	12	30	18	—	,—
3	1	—	,08	3	1	—	48	31	18	—	36
4	1	—	,44	4	2	—	24	32	19	—	12
5	1	—	,8	5	3	—	,—	33	19	—	48
6	2	—	,16	6	3	—	36	34	20	—	24
7	2	—	,52	7	4	—	12	35	21	—	,—
8	2	—	,88	8	4	—	48	36	21	—	36
9	3	—	,24	9	5	—	24	37	22	—	12
10	3	—	,6	10	6	—	,—	38	22	—	48
20	7	—	,2	11	6	—	36	39	23	—	24
30	10	—	,8	12	7	—	12	40	24	—	,—
40	14	—	,4	13	7	—	48	41	24	—	36
50	18	—	,—	14	8	—	24	42	25	—	12
60	21	—	,6	15	9	—	,—	43	25	—	48
70	25	—	,2	16	9	—	36	44	26	—	24
80	28	—	,8	17	10	—	12	45	27	—	,—
90	32	—	,4	18	10	—	48	46	27	—	36
	8	—	,72	19	11	—	24	47	28	—	12
	8	—	,88	20	12	—	,—	48	28	—	48
	9	—	,24	21	12	—	36	49	29	—	24
	9	—	,44	22	13	—	12	50	30	—	,—
	9	—	,8	23	13	—	48	51	30	—	36
	10	—	,16	24	14	—	24	52	31	—	12
	10	—	,52	25	15	—	,—	53	31	—	48
	10	—	,88	26	15	—	36	54	32	—	24
	11	—	,24	27	16	—	12	55	33	—	,—
	11	—	,6	28	16	—	48	56	33	—	36

N ^o .	'	''	N ^o .	'	''	N ^o .	'	''
57	34	12	72	43	12	87	52	12
58	34	48	73	43	48	88	52	48
59	35	24	74	44	24	89	53	24
60	36	—	75	45	—	90	54	—
61	36	36	76	45	36	91	54	36
62	37	12	77	46	12	92	55	12
63	37	48	78	46	48	93	55	48
64	38	24	79	47	24	94	56	24
65	39	—	80	48	—	95	57	—
66	39	36	81	48	36	96	57	36
67	40	12	82	49	12	97	58	12
68	40	48	83	49	48	98	58	48
69	41	24	84	50	24	99	59	24
70	42	—	85	51	—			
71	42	36	86	51	36			

Table I.

Table II. { Motion. *One Sign*
the Integer.

N ^o .	'	''	N ^o .	gr ^o .	'	N ^o .	gr ^o .	'
1	—	10	1	0	18	19	5	42
2	—	21	2	0	36	20	6	—
3	—	32	3	0	54	21	6	18
4	—	43	4	1	12	22	6	36
5	—	54	5	1	30	23	6	54
6	1	4	6	1	48	24	7	12
7	1	15	7	2	6	25	7	30
8	1	26	8	2	24	26	7	48
9	1	37	9	2	42	27	8	6
10	1	48	10	3	—	28	8	24
20	3	36	11	3	18	29	8	42
30	5	24	12	3	36	30	9	—
40	7	12	13	3	54	31	9	18
50	9	—	14	4	12	32	9	36
60	10	48	15	4	30	33	9	54
70	12	36	16	4	48	34	10	12
80	14	24	17	5	6	35	10	30
90	16	12	18	5	24	36	10	48

N ^o .	gr ^o .	'	N ^o .	gr ^o .	'	N ^o .	gr ^o .	'
37	11	6	58	17	24	79	23	42
38	11	24	59	17	42	80	24	—
39	11	42	60	18	—	81	24	18
40	12	—	61	18	18	82	24	36
41	12	18	62	18	36	83	24	54
42	12	36	63	18	54	84	25	12
43	12	54	64	19	12	85	25	30
44	13	12	65	19	30	86	25	48
45	13	30	66	19	48	87	26	6
46	13	48	67	20	6	88	26	24
47	14	6	68	20	24	89	26	42
48	14	24	69	20	42	90	27	—
49	14	42	70	21	—	91	27	18
50	15	—	71	21	18	92	27	36
51	15	18	72	21	36	93	27	54
52	15	36	73	21	54	94	28	12
53	15	54	74	22	12	95	28	30
54	16	12	75	22	30	96	28	48
55	16	30	76	22	48	97	29	6
56	16	48	77	23	6	98	29	24
57	17	6	78	23	24	99	29	42

Now as these Rules of Proportion have some certain Rules
 for giving to Numbers in the same Proportion, and the same
 being generally known and understood, those given Rules
 are often omitted of these Parts and Denominations, which
 therefore are to be reduced to the same, in order for the Quotient
 on to be written in the simplest Manner, and with the greatest
 ease and expedition; which ought to be the Aim of every
 Writer.

But as the Manner of Reducing the Parts of Coins,
 Measures, Weights, &c. hath been fully taught already, I
 shall only here express the Question vaguely, but late and

P. A. H.

Question, What is the Value of such and such a Sum?
 What will such and such a Sum cost at such a Rate?

C H A P. VI.

The Use of DECIMALS in all the Rules of Proportion, viz. The Rule of Three direct and Inverse ; and the double Rule of Five Numbers.

DIRECT Proportion is, when of four Numbers, the first is in the same Proportion bigger or less than the second, as the third is bigger or less than the fourth Number. Thus as 4 is to 12 so is 6 to 18 ; consequently the two middle Numbers multiplied together, are equal to the Product of the two extreme ones ; as $12 \times 6 = 18 \times 4 = 72$. Hence is deduced the general Rule for working all Questions in the Rule of Three direct, viz.

Multiply the second Number by the third, and divide by the first ; and the Quotient will be the fourth Number sought, or Answer.

Now as these Rules of Proportion have some certain Numbers given to find others in the same Proportion, and their Subject being generally Trade and Merchandise, those given Numbers often consist of *diverse Parts and Denominations*, which therefore are to be reduced to *Decimals*, in order for the Question to be wrought in the simplest Manner, and with the greatest ease and expedition ; which ought to be the Aim of every Artist.

But as the Manner of Reducing the Parts of Coins, Measures, Weights, &c. hath been fully taught already ; I shall only here express the Question vulgarly, but state and work it decimally.

Question 1. If $7\frac{3}{4}$ Yards of Cloth cost 2*l.* 12*s.* 9*d.* What will $140\frac{1}{2}$ Yards cost at that Rate ?

Thus

The Use of Decimals

127

Thus stated { Vulgarly $7\frac{1}{4} : 2-12-9 :: 140\frac{1}{2}$
 { Decimally $7,75 : 2,6375 :: 140,5$

$$\begin{array}{r}
 140,5 \\
 \hline
 131875 \\
 105500 \\
 \hline
 26375 \qquad L. s. d. \\
 7,75) 370,56875 (47,8153 = 47:16:3\frac{1}{4} \\
 \underline{3100} \\
 \cdot 6056 \\
 \underline{5425} \\
 \cdot 6318 \\
 \underline{6200} \\
 \cdot 1187 \\
 \underline{775} \\
 4125 \\
 \underline{3875} \\
 2500 \\
 \underline{2325} \\
 175
 \end{array}$$

Question 2. At the Rate of 5*l.* 8*s.* 4*d.* per *C. Weight*,
 What will 19 *C.* 2 *qr.* 14 *lb.* cost?

Stated thus { C. l. s. d. C. Q. lb.
 { Vulgarly $1 : 5-8-4 :: 19:2:14$
 { Decimally $1 : 5,416 :: 19,625$

$$\begin{array}{r}
 5,416 \\
 \hline
 9) 117750 \\
 \underline{130833} \\
 19625 \\
 \underline{78500} \\
 98125
 \end{array}$$

The Answer, $106\text{ l. } 6\text{ s. } 0\frac{1}{2} = \underline{\underline{106,302083}}$

Question

Question 3. Suppose four *Hogheads*, three *Firkins*, and five *Gallons* of *Beer* cost 6 *l.* 14 *s.* 8 *d.* How much is that *per Hoghead*, and *per Gallon*?

Hds. Fr. G. l. s. d.

Stated thus { Vulgarly 4—3—5 : 6—14—8 :: 1
Decimally 4,5825 : 6,73 :: 1

O P E R A T I O N.

$$\begin{array}{r}
 4,5825 \overline{) 6,73333} \quad (\\
 \underline{45} \quad \quad \quad 673 \\
 21386 \quad \quad \quad 54 \\
 \underline{18352} \quad \quad \quad 54 \\
 30340 \quad \quad \quad 27 \\
 \underline{27528} \quad \quad \quad 27 \\
 28120 \quad \quad \quad 27 \\
 \underline{27528} \\
 5920 \\
 \underline{4588} \\
 1332
 \end{array}$$

l. l. s. d.

The Answer { 1,4661 = 1 : 9 : 3 $\frac{1}{4}$ *per Hoghead*.
0,0271 = 0 : 0 : 6 $\frac{1}{2}$ *per Gallon*.

Question 4. The *mean Motion* of the *Sun* being known to be 59' 8" each Day, 'tis required to know in what time He performs one intire Revolution through the whole Circle of the *Ecliptic*, or 360 Degrees.

o ' " D. o ' "

This Question { Vulgarly 0—59—8 : 1 :: 360—00—00
is thus stated. { Decimally 5983 : 1 :: 360

.985

O P E

OPERATION.

$$\begin{array}{r}
 .98\frac{1}{2}) 360 (\\
 \underline{98} \\
 262 \\
 \underline{184} \\
 780 \\
 \underline{780} \\
 0
 \end{array}
 \quad
 \begin{array}{r}
 360 \\
 \underline{360} \\
 0
 \end{array}
 \quad
 \begin{array}{r}
 365,2762 = 365 : 6 : 37 : 43 \\
 \underline{2661} \\
 5790 \\
 \underline{5322} \\
 4680 \\
 \underline{4435} \\
 2450 \\
 \underline{1774} \\
 6760 \\
 \underline{6209} \\
 5510 \\
 \underline{5322} \\
 1880 \\
 \underline{1774} \\
 106
 \end{array}$$

These are the various Cases which may happen in the Rule of Three Direct; by which any one may observe the Advantage of Decimals, and the absolute Necessity of understanding the Management of circulating or repeating Decimals.

The Rule of Three Inverse.

Inverse Proportion is, when of four Numbers, the third bears the same Ratio or Analogy to the first, as the second does to the fourth.

Whence the Rule is; to multiply the first and second of the given Numbers, and divide that Product by the third; the Quotient will be the fourth Number, or Answer.

To know when the Terms of a proposed Question are in this reciprocal or inverse Proportion, observe this Rule; viz. When the third { Bigger } than the first, { Less, } The Number is { Lesser } and requires { More, } Terms

Terms are in the *inverse Ratio*; and are to be worked by the *Rule* above; as in the following *Instances*.

Question 1. If when Wheat is sold for 6s. 6d. per Bushel, the Penny White Loaf ought to weigh eight Ounces Troy; What must it weigh when it is at 4s. per Bushel?

Thus stated $\left\{ \begin{array}{l} \text{Vulgarly } 6 \text{ s. } 6 \text{ d.} : 8 \text{ oz.} :: 4 \text{ s.} \\ \text{Decimally } 6,5 : 8 \text{ lb.} :: 4 \text{ s.} \end{array} \right.$

9) 3,90 lb. lb. oz.
4) 4,8 (1,083 = 1 : 1. or 13 Oun.
4
33
32
13
12 } *ad infinitum.*
1

Question 2. Two Equal *Parallelograms* *A*, *B* given, the Length of *A* is 8 Feet 8 Inches, and its Breadth 4 Feet and 3 Inches; the Breadth of *B* is 2 Feet 10 Inches, Quere its Length?

Thus stated $\left\{ \begin{array}{l} \text{Vulgarly} \\ \text{Decimally} \end{array} \right. \begin{array}{l} F. \text{ In. } 4-3 : 8-8 :: 2-10 \\ 4,25 : 8,6 :: 2,83 \\ 8,6 \end{array}$

$4,25 \quad \boxed{8,8} \quad A \quad \underline{9) 2550}$

$\begin{array}{r} 28\cancel{3} \\ 3400 \end{array}$

$\begin{array}{r} 2,8\cancel{3} \\ 28 \end{array}) \overline{\begin{array}{r} 36,8\cancel{3} \\ 368 \end{array}} \left(\begin{array}{l} 13 \text{ Inches the Answer.} \end{array} \right.$

$\begin{array}{r} 2,55 \\ 255 \end{array}) \overline{\begin{array}{r} 3315 \\ 255 \end{array}}$

$\begin{array}{r} 765 \\ 765 \end{array}$

...

Question

The Use of Decimals

131

Question 3. A Piece of Land 4 Rod broad and 40 long, being a Statute-Acre; tis required to know what Length, with 10 Rod and 2 Yards Breadth, will make an Acre?

Stated thus $\left\{ \begin{array}{l} \text{Vulgarly} \\ \text{Decimally} \end{array} \right. \begin{array}{l} R. \quad Y. \\ 4 : 40 :: 10 - 2 \\ 4 : 40 :: 10,38 \\ 4 \end{array}$

10,38) 160,0

10 16

10,26) 158,4 (15,438 = 15:2:1:2

1026

5580

5130

4900

4104

396

The Double Rule of Three ; or Rule of Five Numbers.

In this Rule of Proportion there are *Five Numbers* given to find a *Sixth* in Proportion ; which is either *Direct* or *Inverse*, according to the Nature of the Question.

Questions in this Rule are performed at two Operations, that is, by a *double stating* the Question, most generally.

Question 1. What is the *Interest* of 364 *l.* 5 *s.* for seven Months, three Weeks, at the Rate of 4 *l.* 10 *s.* *per Cent. per Annum*?

Thus $\left\{ \begin{array}{l} \text{Vulgarly} \\ \text{stated} \end{array} \right. \begin{array}{l} l. \quad M. \quad l. \quad s. \quad l. \quad s. \quad M. \quad W. \\ 100 : 12 : 4 - 10 :: 364 - 5 : 7 - 3 \\ \text{Decimally} \quad 100 : 12 : 4,5 \quad :: 364,25 : 7,75 \end{array}$

First Operation, $\begin{array}{l} l. \quad l. \quad l. \\ 100 : 4,5 :: 364,25 \\ 4,5 \end{array}$

Hence the Interest of 364,25 *l.* } 182125
for one Year, is 16,39125 *l.* } 145700
100) 1639,125 (16,39125

S 2

Then

Then, $M^o. \quad l. \quad M^o.$
Second Operation $12 : 16,39125 : 7,75$

$$\begin{array}{r} 57,7 \\ \hline 114739 \\ 11474 \\ \hline 819 \end{array}$$

$12) 127,032 (10,586 = 10 : 11 : 8 \frac{1}{2}$
The Answer.

Question 2. Suppose it were required to know what *Principal* would gain $15l. 14s. 8d.$ in nine Months and three Days, at the Rate of $4l. 10s. \text{ per Cent, per Annum, Quere.}$ the Answer?

$l. \quad s. \quad M^o. \quad l. \quad l. \quad s. \quad d. \quad M^o \quad D.$
Thus $\left\{ \begin{array}{l} \text{Vulgarly} \\ \text{stated} \end{array} \right. \left\{ \begin{array}{l} 4 : 10 - 12 - 100 - 15 : 14 : 8 - 9 : 3 \\ \text{Decimally} 4,5 \quad - 12 - 100 - 15,78 \quad - 9,107 \end{array} \right.$

The First Operation *Direct.*

$$\begin{array}{r} 4,5 : 100 :: 15,78 \\ \hline 100 \\ \hline 4,5) 1578,3 (349,62 \\ \hline 135 \\ \hline 223 \\ 180 \\ \hline 433 \\ 405 \\ \hline 283 \\ 270 \\ \hline 133 \\ 90 \\ \hline 43 \end{array}$$

The Principal from whence $15,78l.$ comes in 12 Months is $349,62l.$

Ad infinitum.

The Second Operation therefore must be wrought *inversely.*

Thus,

Thus, $12 : 349,62 :: 9,107$

$$\begin{array}{r}
 69828 \\
 349629 \quad l. \quad l. \quad s. \quad d. \\
 9,107) 4198,5 \quad (460,6956 = 460 : 13 : 11 \\
 \underline{36428} \\
 55275 \\
 54642 \\
 \underline{63355} \\
 54642 \\
 \underline{87135} \\
 81963 \\
 \underline{5172}
 \end{array}$$

But any Question in this Rule may be answered at *one Operation*, by the following *Rule* :

Make the $\left. \begin{array}{l} 1^{st} \\ 2^{d} \\ 3^{d} \end{array} \right\} \left\{ \begin{array}{l} \text{Number of the} \\ \text{Five, the same} \\ \text{kind with the} \end{array} \right. \left. \begin{array}{l} 4^{th} \\ 5^{th} \\ 6^{th} \end{array} \right\} \left\{ \begin{array}{l} \text{Number given.} \\ \text{the Num. sought.} \end{array} \right.$

Then, Multiply the *Three* Numbers to the Right-hand together, and the *two first* to the Left-hand ; and divide the *first Product* by this *last*, and the *Quotient* will be the sixth Number, or *Answer*, if the Proportion be *Dire&*.

Question 3. If a 1000 Men can dig a Trench 500 Feet Long in 24 Hours, what Length of such a Trench can 9800 Men dig in 10 Hours ?

Thus

Thus stated, $\begin{array}{c} \text{Men} \quad \text{Hours} \quad \text{Feet} \quad \text{Men} \quad \text{Hours} \\ 1000. \quad 24 : 500 :: 9800. \quad 10 \\ \hline 24 \quad \hline 24000 \end{array}$

$24 \overline{) 24000} 98000 \overline{) 98000} 500$

48...

100

96

40

24

160

144

16

} In Infinitum.

The Answer is
Feet Feet In.
 $2041,6 = 2041 : 8$
The Length required.

If any Part of the Question be in reciprocal or inverse Proportion; place the Three first Numbers as in the last Question; and of the other two, place *That* the fourth, which is of the same kind as the second; and consequently the other must be the fifth Number.

Question 4. If 1000 Men can dig a Trench in 24 Hours 500 Feet Long, How many Men will dig 2041,6 Feet in 10 Hours?

Stated thus, $\begin{array}{c} \text{Men} \quad \text{H.} \quad \text{Feet.} \quad \text{H.} \quad \text{Feet.} \\ 1000 : 24 : 500 :: 10 : 2041,6 \\ \hline 10 \quad \hline 1000 \end{array}$

Rule. Multiply the 1st, 2d, and 5th Numbers, and the 3d, and 4th; then divide the first Product by the last, the Quotient is the Answer; viz. 9800 Men. }

5000

2041,666,6

24

8106666,6

4083333,3

$5 \overline{) 8106666,6} 1621333,3$

45

40

40

.. 00

But for the greater readiness and ease of the ingenious Arithmetician, I shall transcribe that famous general Theorem in

in Mr. Ward's *Young Mathematician's Guide*, which shews at once how to answer any Question of *Five Numbers* at one Operation, without regard to the Proportion of the Terms; be that *Direct* or *Indirect* as it will.

The *Theorem* is this, $TgP = Gpt$. In this *Theorem* you observe three *Capital Letters*, viz. *T, P, G*, and the same three Letters in *small Characters*, *g, p, t*. The three *Capitals* signify the *Three first conditional Terms* of the Question,

Thus, $\begin{cases} P, \text{ Is the Principal Cause of Gain, Loss, Action, \&c.} \\ T, \text{ Is the Time, Space, Distance, \&c.} \\ G, \text{ Is the Gain, Loss, Action, \&c.} \end{cases}$

Of the *Three small Letters*, (which correspond to, and signify the same with the *Capitals*) two always move the Question, the other shews the Answer; which, as the Letters are three, is *threefold*; and answered by the same *Theorem* disposed in these three proper *Terms*.

Viz. If $\begin{cases} P \\ t \\ g \end{cases}$ be sought, the *Theorem* is $\begin{cases} \frac{TgP}{tG} = P. \\ \frac{TgP}{Gp} = t. \\ \frac{GtP}{TP} = g. \end{cases}$

Or thus, $TgP \div Gt = p$. $TgP \div Gp = t$. $Gpt \div TP = g$.

If any *Arithmetician* should complain he does not understand such *Algebraic Forms and Characters*, all that I have to answer is, That 'tis a very *necessary Part* of his *Business and Profession*, and *highly concerns him to learn it*.

C H A P. VII.

*A New Decimal Practice; or a short Way
of computing all kind of Merchandise by
DECIMALS.*

TH^O there is scarce any Part of Arithmetick in which *Decimals* are of greater (or indeed so great) Service, as *Practice*; yet this of all others has been the least improv'd by it; hardly an Author can be met with on this Head; and those who have undertaken it, have presented us with but imperfect Sketches, and left the Matter unfinished. I hope what follows in this Chapter will give Satisfaction in this Point.

The *Tables of aliquot Parts* commonly used, are these

s.	d.	Parts.
10	— 0	— 2
6	— 8	— 3
5	— 0	— 4
4	— 0	— 5
3	— 4	— 6
2	— 6	— 8
2	— 0	— 10
1	— 8	— 12

The *even* or *aliquot Parts* of a Pound Sterling. By which dividing, gives an Answer in Pounds.

d.	Parts.
6	— 2
4	— 3
3	— 4
2	— 6
1½	— 8
1	— 12

The *even* or *aliquot Parts* of a Shilling Sterling. By which dividing, gives an Answer in Shillings.

But the Table following is far more general, expeditious and useful; and has not yet been applied to *Decimal Practice*.

A General Table for Decimal Practice.

<i>Price.</i>	<i>Divisors.</i>	<i>Price.</i>	<i>Divisors.</i>	<i>Price.</i>	<i>Divisors.</i>
<i>d. q.</i>		<i>d. q.</i>		<i>s.</i>	
0 : 1	3,4,80	6 : 1	80, X2, +12	1	20
0 : 2	6,80	6 : 2	40, +12	2	10
0 : 3	4,80	6 : 3	40, +8	3	10, +2
1 : 0	4,60	7 : 0	40, +6	4	10, X2
1 : 1	4,6,8	7 : 1	40, +6,4	5	4
1 : 2	2,80	7 : 2	40, +4	6	10, X3
1 : 3	3,40, —8	7 : 3	40, +4,6	7	10, X3, +2
2 : 0	3,40	8 : 0	30.	8	10, X4
2 : 1	3,40, +8	8 : 1	80, X3, —4	9	10, X4, +2
2 : 2	3,40, +4	8 : 2	60, X2, +8	10	2
2 : 3	80, —12	8 : 3	80, X3, —12	11	2, +10
3 : 0	80.	9 : 0	40, +2	12	10, X6
3 : 1	80, +12	9 : 1	80, X3, +12	13	10, X6, +2
3 : 2	80, +6	9 : 2	80, X3, +6	14	10, X7
3 : 3	80, +4	9 : 3	80, X3, +4	15	2, +2
4 : 0	60.	10 : 0	3,8	16	10, X8
4 : 1	3,40, X2, +8	10 : 1	30, +4,8	17	10, X8, +2
4 : 2	60, +8	10 : 2	20, —8	18	10, X9
4 : 3	60, +8,2	10 : 3	40, +2,2,6	19	1, —20
5 : 0	6,8	11 : 0	20, —12		
5 : 1	40, —8	11 : 1	40, X2, —8		
5 : 2	40, —12	11 : 2	40, X2, —12		
5 : 3	80, X2, —12	11 : 3	30, +2, —16		
6 : 0	40.				

An Explanation of the preceeding Table.

The *first Column* shews the *Price* of the *Commodity*, either in *Pence* and *Farthings*, or in *Shillings*, for one of a sort ; as one *Pound*, *Yard*, *Piece*, &c.

Against the *Piece*, you observe in the second *Column*, several Numbers, of which those which *stand first*, and have nothing prefixt to them, are *Divisors*; by which any given *Quantity* or Number of *Yards*, *Ells*, *Pounds*, are to be *divided*.

If any Number follow these with any *Character prefix* to them as $3,40, \times 2, + 8$. $80, \times 3, - 12$, &c. They are to be *understood*, and *read*, as in the following Examples.

$$3,4, - 8$$

$$3,4, + 8$$

$$3,40, \times 2, + 8 \}$$

$$60, + 8, 2 \}$$

$$3, + 2, - 16 \}$$

$$1, - 20 \}$$

From a 4th of a 3d, take one 8th of that 4th

To a 4th of a 3d add an 8th of that 4th.

To a 40th of a 3d, multiplied by 2, add an 8th of that 40th.

To a 60th add an 8th of that 60th, and half of that 8th.

To a 3d add an half of that 3d, then subtract a 16th of that 2d.

From the given Number take one 20th Part.

These being well understood, 'twill not be difficult to use the Table on all occasions with ease; especially after perusing the Examples ensuing, which are chosen for the more difficult Parts thereof.

Note, When the *Price* consists of *Shillings* only, the Number may be multiplied by the *Decimal*, that is, *half the Number of Shillings*, and the Answer will be the same.

Ex. 1. At 1*q.* per Yard, What cost 144 Yards?

$$\text{One 3d} = 48$$

$$\text{One 4th of that} = 12$$

$$\text{One 80th of that} = 0,15 = 3 \text{ s. Answer.}$$

Ex. 2. At 3*q.* per Yard, What cost 172,5 Yards?

$$\text{One 4th} = 43,125$$

$$\text{One 80th of that} = 0,53907 = 10 \text{ s. } 9 \text{ d. } \frac{1}{4}.$$

Ex. 3. At 1*d.* 1*q.* What cost 1792,25

$$\text{One 4th} = 448,0625$$

$$\text{A 6th of that} = 74,67708$$

$$\text{An 8th of that} = 9,33463 = 9 \text{ l. } 6 \text{ s. } 8 \frac{1}{2} \text{ d.}$$

Ex. 4. At 1*d.* 3*q.* What cost 9742,6

$$\text{One 3d} = 3247,5$$

$$\text{A 40th of that} = 81,18888$$

$$\text{From whi h take an 8th} = 10,1436 \text{ l. s. d.}$$

$$\text{Remains the Answer} = 71,04527 = 71 : 00 : 10 \frac{1}{4}$$

Ex.

Ex. 5. At 2 d. 1 q. What cost 3698,24

One 3d = 1232,08

A 40th of that = 30,80208

To which add one 8th = 3,85025 l. s. d.

The Sum the Answer = 34,6523 = 34 : 13 : 0½

Ex. 6. At 2 d. 3 q. What cost 4130,21 ?

One 80th = 51,627

From which take a 12th = 4,302 l. s. d.

Remains the Answer = 47,325 = 47 : 6 : 6

Ex. 7. At 4 d. 1 q. What cost 1932,49 ?

One 3d = 644,16

A 40th of that = 16,104

Ditto = 16,104

Of which add an 8th = 2,013 l. s. d.

The Sum is the Answer = 34,221 = 34 : 4 : 5

Ex. 8. At 4 d. 3 q. What cost 948,48 ?

One 60th = 15,807

One 8th of that = 1,975

One half of that = 0,987 l. s. d.

The Sum is the Answer = 18,771 = 18 : 15 : 5

Ex. 9. At 5 d. 3 q. What cost 1012,2 ?

One 80th = 12,6

One 80th, × 2 = 25,33

Subtract a 12th of an 80th = 1,03 l. s. d.

Remains the Answer = 24,27 = 24 : 5 : 6½

Ex. 10. At 8*d.* 1*q.* What cost 2640?

$$\begin{array}{r} \text{One 80th} = 33 \\ \underline{3} \end{array}$$

$$\text{That multiplied by 3} = 99$$

$$\text{Subtract a 4th of an 80th} = 8,25 \quad \text{l. s. d.}$$

$$\text{Remains the Answer} = 90,75 = 90 : 15 : 0$$

Ex. 11. At 9*d.* 1*q.* What cost 961,92?

$$\begin{array}{r} \text{One 80th} = 12,024 \\ \underline{3} \end{array}$$

$$\text{An 80th} \times 3 = 36,072$$

$$\text{To which add 12th of an 80th} = 1,002 \quad \text{l. s. d.}$$

$$\text{The Sum is the Answer} = 37,074 = 37 : 1 : 5\frac{1}{4}$$

Ex. 12. At 10*d.* 3*q.* What cost 1600

$$\text{One 40th} = 40$$

$$\text{Add an half} = 20$$

$$\text{Add half that} = 10$$

$$\text{Add a sixth of that} = 1,6 \quad \text{l. s. d.}$$

$$\text{The Sum is the Answer} = 71,6 = 71 : 13 : 4$$

Ex. 13. At 11*d.* 3*q.* What cost 908,18?

$$\text{One 30th} = 30,208$$

$$\text{To which add an half} = 15,103$$

$$\text{From the Sum} = 45,309$$

$$\text{Subtract a 16th} = 0,943 \quad \text{l. s. d.}$$

$$\text{Remains the Answer} = 44,368 = 44 : 7 : 3\frac{1}{2}$$

Ex. 14. At 7*s.* What cost 365,25?

$$\text{One 10th} = 36,525$$

$$\underline{3}$$

$$\text{One 10th} \times 3 = 109,575$$

$$\text{To which add half that 10th} = 18,2625 \quad \text{l. s. d.}$$

$$\text{The Sum is the Answer} = 127,8375 = 127 : 16 : 9$$

Ex.

Ex. 15. At 19 s. What cost 257,6?
 Subtract a 20th = $\frac{12,883}{100}$ l. s. d.
 Remains the Answer = $\frac{244,783}{100} = 244 : 15 : 8$

Ex. 16. At 13 s. 9 $\frac{1}{4}$ d. What cost 96x,92?

$\begin{array}{r} 96x,92 \\ 10th = 96,192 \\ \hline 6 \\ 10th \times 6 = 577,153 \\ \frac{1}{2} \text{ of } 10th = 48,096 \\ \hline \text{The Sum} = 625,249 \end{array}$	For 13 <i>Shil.</i>	$\begin{array}{r} 80th = 12,024 \\ \hline 3 \\ 80th \times 3 = 36,072 \\ 12 \text{ of } 80 = 1,002 \\ \hline \text{Add } \left\{ \begin{array}{l} 37,074 \text{ for } 0:9\frac{1}{4} \\ 625,249 \text{ for } 13:00 \end{array} \right. \end{array}$
---	------------------------	---

For the 9 $\frac{1}{4}$ d. s. d.
 The Answer = $\frac{662,323}{100}$ for 13:9 $\frac{1}{4}$

In this Example (and any other) the Answer for the *Shillings* is found with least trouble, and Figures, by *Multiplying the given Number* by the *Decimal of the Shillings*,

Thus, $\left\{ \begin{array}{l} 96x,92 \text{ The given Number.} \\ \quad ,65 \text{ The Decimal of a Pound for 13 s.} \\ \hline 480960 \\ 5771531 \\ \hline 625,249 \end{array} \right.$ The same as before.

If the *Price* or *Value* consist of *Pounds, Shillings, Pence, &c.* the most *ready* and *practical* way, is to turn the *whole* into *Decimals*, then multiply the given Number (turn'd into *Decimals* also, if express'd in *diverse Parts*;) and the *Product* will be the *Answer*.

Ex. 17. At 5 l. 16 s. 8 d. What cost 270? l. s. d.

$$\begin{array}{r} 5,83 = 5 : 16 : 8 \\ \hline 9) 810 \\ \hline 900 \\ 2160 \\ 1350 \\ \hline \text{The Answer} = 1575,00 \end{array}$$

Ex.

142 The Use of Decimals in Fellowship.

Ex. 18. At 1 : 17 : 3 $\frac{1}{4}$ What cost 14 : 1 : 14 : 10?

Then, $\left\{ \begin{array}{l} 1438058 \\ 526568,1 \end{array} \right.$ The Multiplier inverted.

143805

115044

8628

719

86

3

The Answer L. 26,8285 = 26 l. 16 s. 6 $\frac{1}{4}$ d.

These Examples are sufficient to the ingenious Practical Student of *Decimal Arithmetick*; who with those Instructions will easily (*proprio Marte*) apply this noble Art to all Cases of Common Trade and Merchandise.

CHAP. VIII.

The Use of DECIMALS in the Rules of Fellowship, Tare and Trett, Barter, Gain and Loss, Exchange, Alligation, Rule of False Position, Extraction of Roots,

8 Single Fellowship; or That without Time.

THE Rules of Fellowship are proper to Merchants and those who Trade in Company, or Partnership; where they have a common Joint-Stock to traffick withal; for to every one of the Company is distributed his due share of Gain or Loss acquired by Trading, in proportion to his Stock laid out, by this following

Rule

The Use of Decimals in Fellowship. 143

Rule.

As the total Sum of the Stocks, is to the total Gain or Loss; so is each Man's particular Stock, to his particular Gain or Loss.

Example. Suppose Four Merchants, A, B, C, D, make a Joint Stock of 421 l. 8 s. 6 d.

$$\begin{array}{lcl} \text{Thus, } \left\{ \begin{array}{l} A \text{ puts in } 154 : 13 : 4 = 154,8666 \\ B \text{ — } 110 : 18 : 6 = 110,9250 \\ C \text{ — } 95 : 00 : 8 = 95,0333 \\ D \text{ — } 60 : 16 : 0 = 60,8 \end{array} \right. \end{array}$$

$$\text{The whole Stock} = 421 : 08 : 6 = 421,425$$

The Trade and Gain 88 l. 17 s. 10 d. = 88,8916 l. Tis required to find each Man's Part or Share of that Gain.

$$\begin{array}{lcl} \text{Then as } 421,425 : 88,8916 \left\{ \begin{array}{l} :: 154,8666 : 32,6192 \\ :: 110,9250 : 23,394 \\ :: 95,0333 : 20,0425 \\ :: 60,8 : 12,8227 \end{array} \right. \end{array}$$

$$\text{The Sum of the several Shares} = 88,8784$$

Which being the same with, or equal to, the total Gains always proves the Truth of the Work.

But all Cases in the Rules of Fellowship are soonest and easiest answer'd by finding the Proportional Part of the Gain or Loss due to one Pound; and then by that to multiply each Man's particular Part of the Stock; for the several Products in such Cases, are the several Answers; viz. each Man's Part of the Gain or Loss.

Thus, As 421,425 : 88,8916 :: 1 : ,2109 the common Multiplier.

$$A's \text{ Part of Stock} = 154,8666 \quad B's \text{ Part of Stock} = 110,925$$

$$\text{Multiply by } ,2109 \quad \text{Multiply by } ,2109$$

$$= 32,6192 \quad = 23,394$$

$$= 20,0425 \quad = 12,8227$$

$$= 30,9333 \quad = 22,1850$$

$$A's \text{ Part of Gain} = 32,6192 \quad B's \text{ P. of Gain} = 23,394 \quad \&c.$$

C's

144 The Use of Decimals in Fellowship.

<i>l.</i> C's Part of Stock = 95,03 Multiply by ,2109 <hr style="width: 50%; margin: 0 auto;"/> 85528 950333 19008666 <hr style="width: 50%; margin: 0 auto;"/> C's Part of Gain = 20,04253	<i>l.</i> D's Part of Stock = 60,8 Multiply by ,2109 <hr style="width: 50%; margin: 0 auto;"/> 5472 6080 1216 <hr style="width: 50%; margin: 0 auto;"/> D's Part of Gain = 12,82272
---	---

Here every Man's Share is the same as before.

Double Fellowship, or That with Time.

Fellowship with Time considers the *Share* of the *Gain* or *Loss* with regard to the *Money*, and the *Time* it was employed, and proportionates it to both by the following

Rule.

Multiply each Man's Stock by the Time it was employed; then say, As the sum of those Products, is to the whole Gain or Loss; so is every one of the Products, to its proportional Part of the Gain or Loss.

Example. Three Merchants A, B, and C, enter into Partnership, thus;

<i>l.</i>	
A puts in 65,5	for 8 Months, 2 Weeks, and 3 Days.
B — 78,6	— 12 Months, 3 Weeks, and 1 Day.
C — 84	— 6 Months, and 6 Days.

They traffick and gain 140,016 *l.* 'Tis required to find each Man's Share proportional to his Stock, and Time 'twas in.

	<i>l.</i>	Months.		Products.
First {	A's Stock	65,5 × 8,607	=	563,7585
	B's Stock	78,6 × 12,3357	=	970,4084
	C's Stock	84 × 6,214	=	521,976
The Sum of the Products =				2056,1429

Then

The Use of Decimals in Tare and Trett. 145

l.

Then, As $2056,1429 : 140,018 \left\{ \begin{array}{l} :: 563,7585 : 38,3918 = A \\ :: 970,4084 : 66,0846 = B \\ :: 521,976 : 35,5465 = C \end{array} \right.$

The whole Gain very near = 140,0229 l.

Questions in this Rule also are much better answer'd by finding the Proportional Part to one Pound, for a common Multiplier, as before.

l.

Thus, as $2056,1429 : 140,018 :: 1 : ,0681$ Common Multiplier.

The Operation for A , = $563,7585$. For B = $970,4084$

The Multiplier inverted $\frac{1860,0}{}$

<u>338255</u>	<u>582244</u>
45100	77632
563	970
<u>38,3918</u>	<u>66,0806</u>

For C = $521,9760$
 $\frac{1860,0}{}$

313185
 41758
522

35,5465

{ Their several Parts of the Gain, as before.

Thus appears the excellent Use of Decimals in the Rules of Fellowship.

Tare and Trett.

Tare is the Weight of the Hogshead, Chest, Bag, Cask, &c. which contain the Goods bought or sold.

Trett is an Allowance of 4 lb. in 100, or 104 lb. for Goods wherein is Loss by refuse, &c.

Cloff is an Allowance of 2 Pound upon every Draught which exceedeth 300 Gross Weight.

146 The Use of Decimals in Tare and Trett.

Subtile is the *Weight* when the *Tare* is deducted, but not the *Trett*.

Neat Weight is the *Remainder* when *Tare*, *Trett*, and *Cloff*, if all are allowed, are taken away.

For resolving Questions in this Rule there are several *Methods*; but those by *Decimals* are much the shortest and best, and are as follow.

Question 1. What is the *Neat Weight* of 9 C. 2 qr. 7 lb. *Tare* at 14 lb. *per Cent.* to be deducted.

First, This may be answered by the *Golden Rule* in *Decimals*.

lb. C.
If 1 C. allow 14, What will 9,5625 allow for Tare?

$$\begin{array}{r} 14 \\ \hline 382500 \\ 95625 \end{array}$$

Total Tare lb. = 133,875 = 1,1953 C.

Then, $\left\{ \begin{array}{l} \text{The Gross Weight} = 9,5625 \\ \text{The Whole Tare} = 1,1953 \text{ to be subtracted.} \\ \text{The Difference is} = 8,3672 = 8 \text{ C. } 1 \text{ qr. } 13 \text{ lb.} \\ \text{the Neat Weight.} \end{array} \right.$

A *Second Way*, is to Multiply the *Gross Weight* by the *Decimal* of C. Weight, equal to the *Tare* allow'd.

$$\begin{array}{r} C. \\ \text{The Gross Weight} = 9,5625 \\ \text{The Decimal of 14 lb.} = ,125 \\ \hline 478125 \\ 191250 \\ \hline 95625 \end{array}$$

The Tare (as before) = 1,1953125 to be subtracted.

A *Third Way*, is to multiply the *Gross Weight* by the *Decimal* of the *Neat Part* of a *Hundred Weight*.

Thus,

The Use of Decimals in Tare and Trett. 147

	C.	C.	
Thus, from	1,000	Then	9,5625
Subtract	,125		578,0
			Multiplier inverted.
The Neat of C.	,875		76500
			6693
			478

The Neat Weight total = 8,3671 as above.

A Fourth Way, is to work by Aliquot Parts as in Practice. Thus 14 being the 8th Part of 112; if you take an 8th of 9,5625, that will be the Tare of the Whole.

C.
Thus, $\left\{ \begin{array}{l} 9,5625 \text{ the Gross Weight.} \\ \frac{1}{8} = 1,1953 \text{ the Tare, as before.} \end{array} \right.$

For the more expeditious finding the Tare by Aliquot Parts, I have inserted the following Table of Tare and proper Divisors.

Tare.	Divisors.	Tare.	Divisors.	Tare.	Divisors.
lb.					
1	2,7,8	8	2,7	15	2,8,X2,+7
2	7,8	9	2,7,+8	16	7
3	7,8,+2	10	2,7,+4	17	2,7,X2,+8
4	4,7	11	2,7,+4,2	18	7,+8
5	4,7,+4	12	2,7,+2	19	7,+4,-20
6	4,7,+2	13	2,7,+2,4	20	7,+4
7	2,8	14	8	21	8,+2

The Construction and Use of this Table of Tare is the same with the Table of Prices or Values in Practice, which see there taught.

Having shewn how to find the Tare, the next Business is to find the Trett, or the Neat Weight when the Trett is deducted from the Subtile;

Thus Multiply $\left\{ \begin{array}{l} ,0384 \text{ the Product is the Trett.} \\ \text{the Subtile by } ,9616 \text{ the Product is the Neat Weight.} \end{array} \right.$

148. *The Use of Decimals in Tare and Trett.*

Question 2. In 72 C. 3 qr. 12 lb. Gross, Tare at 12 lb. per C. Trett 4 lb. per 104. How many C. neat?

$$\begin{array}{r}
 \text{C.} \\
 \text{The Gross} = 72,8571 \\
 \text{Multiply by the Decimal of 12 lb. inverted} = 1701,0 \\
 \hline
 72857 \\
 5100 \\
 72 \\
 \hline
 \text{The Tare (subtile)} = 7,8029 \\
 \text{The Subtile} = 65,0542 \\
 \text{Multiplier inverted} = 4830,0 \\
 \hline
 19516 \\
 5204 \\
 260 \\
 \hline
 \text{The Trett to be subtile} = 2,4980 \\
 \text{The Neat Wr.} = 62,5562
 \end{array}$$

A Shorter Way, thus;

$$\begin{array}{r}
 \text{C.} \\
 \text{The Gross} = 72,8571 \\
 \text{Mult. by the Neat Dec. of 112 lb. inverted} = 8298,0 \\
 \hline
 582856 \\
 65571 \\
 1457 \\
 582 \\
 \hline
 \text{The Subtile} = 65,0466 \\
 \text{The Multiplier inverted} = 6169,0 \\
 \hline
 585419 \\
 39027 \\
 650 \\
 390 \\
 \hline
 \text{The Neat Weight} = 62,5486
 \end{array}$$

These are the *best Methods* for finding *Tare* and *Trett*; and that I have here given for finding the *Trett* is *new* to me, not having seen it in any Author I have met with.

Barter.

Barter.

By the *Rule of Barter*, Merchants and Traders exchange Goods of *different Values, Kinds, and Quantities*, so as to sustain no *Loss or Disadvantage* by such a *Barter or Change*.

Question 1. Two Merchants, *A* and *B* barter; *A* would exchange 5 C. 3 qr. 14 lb. of *Pepper*, worth 3 l. 10 s. per C. with *B* for *Cotton* worth 10 d. per lb. How much *Cotton* must *B* give *A* for his *Pepper*?

Proceed thus by *Decimals* to find the Value of the *Pepper*.

$$\begin{array}{r}
 \text{C.} \quad \text{l.} \quad \text{C.} \\
 \text{Say, As } 1 : 3,5 :: 5,875 \\
 \underline{3,5} \\
 29375 \\
 17625 \quad \text{l. s. d.}
 \end{array}$$

The Value of the *Pepper* = 20,5625 = 20 : 11 : 3

Then to find the Quantity of *Cotton* equal to the Value of the *Pepper*;

$$\begin{array}{r}
 \text{l.} \quad \text{C.} \quad \text{l.} \\
 \text{Say, As } ,0416 : ,00892 :: 20,5625 \\
 \underline{29800,0} \\
 1645 \\
 185 \\
 4 \\
 ,0416 \quad ,1834 \\
 \underline{041} \quad 1834 \quad \text{l.} \\
 ,0375 \quad 16506 \quad (4,4016 \\
 \underline{1500} \\
 1506 \\
 1500 \\
 \dots 600 \\
 375 \\
 2250 \\
 2250 \\
 \dots
 \end{array}$$

Thus *B* must give *A*
 C. C. qr. lb.
 4,4016 = 4 : 1 : 17½
 of *Cotton* for his *Pepper*.

Question

2350 The Use of Decimals in Barter.

Question 2. A has 52 Dozen of Hats, worth in ready Money 2 s. 6 d. but barterers at 2 s. 9 d. per Hat. B has Cotton at 10 d. per Pound, ready Money. Quere at what Rate per Pound B must barter his Cotton, and how much he must give for the Hats?

First; say, As 25 : 25 :: 38 : 38 = One Penny.
So that B's Cotton is to be advanced a Penny a Pound in Barter.

Secondly, to find the Value of the (624) Hats in barter;

Hat l. Hats l.
Say, As 1 : 1375 :: 624 : 85,8

624
5500
2750
8250

l. s.
85,8000 = 85 : 16 the Price of all the Hats.

Thirdly to know what Cotton at 11 d. per Pound can be had for that Money; l. C. l. C.

Say, As 10458 : 100892 :: 85,8 : 16,6982

85,8
7136
4460
7136

10458) 765336
0458 765336

104125) 6888024 (16,6982 C.
4125

27630
24750

28802
24750

740524
37125

33990
33000

9900
8250

1650

These

Hence it appears that B must give in Exchange
C. C. Q. lb.
16,6982 = 16 : 2 : 24
of Cotton at 11 d. per lb.
for 52 Dozen of Hats at
2 s. 9 d. per Hat.

The Use of Decimals in Gain and Loss. 151

These two Questions well understood, are sufficient for all other Cases in this Rule.

Gain and Loss.

By this Rule Men of Trade and Business know what they get by Retailing Goods; and in case of Damage, what they lose, by selling it at any given Rate; and whether they gain or lose, to know at what Rate per Cent.

Question 1. If I buy Cambricks at 5 s. 6 d. per Yard, and sell them at 8 s. 9 d. What is the Gain per Cent.?

$$\begin{array}{r} \text{Say } \left\{ \begin{array}{l} \text{Vulg. } 5-6 : 8-9 :: 100 : 159-1-9\frac{1}{4} \\ \text{Dec. } .275 : .4375 :: 100 : 159,09 \end{array} \right. \end{array}$$

$$\begin{array}{r} \text{Or, thus, } \left\{ \begin{array}{l} \text{Vulg. } 5-6 : 3-3 :: 100 : 59-1-9\frac{1}{4} \\ \text{Dec. } .275 : .1625 :: 100 : 59,09 \end{array} \right. \left. \begin{array}{l} \text{The Gain} \\ \text{per Cent.} \end{array} \right\} \end{array}$$

By the very same Manner of Working you find the Loss.

Question 2. If I buy Cambrick at 5 s. 6 d. per Yard, How must I sell it to Gain 59 l. 1 s. 9½ d. per Cent.?

The Converse of the last Method solves this Question.

$$\begin{array}{r} \text{Thus } \left\{ \begin{array}{l} \text{Vulg. } 100 : 159-1-9\frac{1}{4} :: 5-6 : 8-9 \\ \text{Dec. } 100 : 159,09 :: .275 : .4375 \end{array} \right\} \left. \begin{array}{l} \text{The Price} \\ \text{per Yard.} \end{array} \right\} \end{array}$$

Question 3. If I buy a C. Weight of Tobacco for 4 l. 13 s. 4 d. and sell it at 11 d. per Pound; What do I Gain or Lose, and at what per Cent.?

First find what a C. Weight will amount to at 11 d. per Pound.

$$\begin{array}{r} \text{Thus, } \left\{ \begin{array}{l} \text{Vulg. } \begin{array}{ccc} \text{lb.} & \text{d.} & \text{lb. l. s. d.} \\ 1 & : & 11 \\ & & : 112 : 5-2-8 \end{array} \\ \text{C.} & \text{l.} & \text{C. lb.} \\ \text{Dec. } .00892 : .04583 :: 1 : 5,13 \end{array} \right\} \text{Answ.} \end{array}$$

$$\begin{array}{r} \text{Then } \left\{ \begin{array}{l} \text{Vulg. } 4-13-4 : 5-2-8 :: 100 : 110 \\ \text{Dec. } 4,6 : 5,13 :: 100 : 110 \end{array} \right\} \left. \begin{array}{l} \text{viz. } 10 \text{ l.} \\ \text{per Cent.} \end{array} \right\} \text{Then}$$

152 The Use of Decimals in Exchange.

$\begin{array}{r} \text{Then from } 5-2-8 \\ \text{Take } 4-13-4 \\ \hline \end{array}$

$\begin{array}{r} \text{And from } 110 \\ \text{take } 100 \\ \hline \end{array}$

Remains 0—9—4 the Gain, at the Rate of 10 l. per C.

The *Converse* of this needs no Example.

Question 4. If I buy $5\frac{1}{2}$ Loads of Wheat for 45 l. 16 s. 8 d. For how much must I sell it *per* Quarter, to gain 6 l. 10 s. by the Bargain?

$\begin{array}{r} \text{First } \left\{ \begin{array}{l} \text{To the given Price} = 45-16-8 \\ \text{Add the designed Gains} = 6-10-0 \\ \hline \end{array} \right. \\ \text{The Sum is} = 52-6-8 \end{array}$

for which the said Wheat must be sold. $5\frac{1}{2}$ Load = 27,5 Quarters.

$\begin{array}{r} \text{Therefore say } 27,5 : 52,8 :: 1 : 1,903 = 1-18-0\frac{1}{2} \\ \text{the Answer.} \end{array}$

These being the *principal Cases* of this Rule, are sufficient if well understood; and the Operations at large are omitted for the Exercise of the Ingenious.

Exchange.

Both the *Name* and *Eufiness* of *Exchange* is analogous to that of *Barter*; only that relates to *Goods* and *Commodities*; whereas this is concern'd in *Foreign Coins*, *Weights*, and *Measures*.

Exchange then consists in finding the *true Sum* or *Value* of one *Country Coin*, &c. equivalent to any given *Sum* or *Value* of that of any other *Country*.

The *Par* of *Exchange* is the *fixt* and *Standard Value* of *Foreign Coins*, &c. express'd in *Sterling Money* of our own; and is that in the *Tables*. 'Tis so called because in *Exchange*, *Par pro Pari*, i. e. One equal Value for another, is given.

The *Course* of *Exchange* is the *current Price* of *Exchange*, always unsettled, being sometimes *above* and sometimes *below* the *Par*; according to the various *Circumstances* and *Accidents* of *Trade* and *Nations*.

The

Decimal Tables of Foreign Coins. 153

The *Course of Exchange* is published in the Weekly Papers and Pamphlets, which compar'd with the *Par* in the *Tables*, it appears whether it be above or below it at any Time.

EXAMPLE.

<i>Course of Exchange.</i>		<i>The Par.</i>		<i>Difference.</i>	
	s. d.	s. d.	s. d.		
June 1733. at	Amsterdam 35 : 0	Shil. d.	{ —33 : 4 }	{ —1 : 8 }	above.
	Rotterdam 35 : 1				
	Hamburgh 34 : 1				
	Antwerp 35 : 4			{ —0 : 12½ }	below.
	Madrid 41 : 2½				
	Leghorn 50 : 3¾				
	Genoa 53 : 1½				
	Venice 48 : ½	Pence.	{ —52 : ½ }	{ —0 : 3½ }	

It is to be observed, that when the *Course of Exchange* is above the *Par*, tis a general Indication that our Trade is prosperous, and the Nation on the *Gainful Side*; as on the contrary, if tis below the *Par*, the Trade is bad, and the Nation looser.

The *Par of Exchange* in Coins, Measures, Weights, &c. between Us and Foreigners, are express'd in the following Tables, and which I have reduced to *Decimals* for more convenient and ready resolving of Questions in this Rule.

Low Country Coins.

	l.	s.	d.	l.
A Stiver	—	—	0 : 0 : 1½	0,005
A Flemish Shilling (= 6 Stivers)	—	—	0 : 0 : 7½	0,03
A Gilder (= 20 Stivers)	—	—	0 : 2 : 0	0,1
A Flem. Pound (= 33 s. 4 d. Flemish)	—	—	1 : 0 : 0	1,0
An Emblem Doller	—	—	0 : 2 : 3¾	0,115
A Campen Doller	—	—	0 : 7 : 7½	0,13
A Zeland Doller	—	—	0 : 3 : 0	0,15
A Lyons Doller	—	—	0 : 4 : 0	0,2
A Specie Doller	—	—	0 : 5 : 0	0,25
A Duccatoon	—	—	0 : 6 : 3½	0,315

X

Ger-

154 *Decimal Tables of Foreign Coins.*

German Coins.

		<i>l.</i>	<i>s.</i>	<i>d.</i>	<i>l.</i>	
<i>A Rix Doller of the Empire</i>	—	0	4	3 $\frac{3}{4}$		0,2156
<i>A Gilder of Noremberg</i>	—	0	7	1		0,354

French Coins.

<i>A Denier</i>	—	—	0	0	0 $\frac{3}{4}$		0,00031
<i>A Soulz (= 12 Deniers)</i>	—	—	0	0	0 $\frac{3}{4}$		0,00374
<i>A Livre (= 20 Soulz)</i>	—	—	0	1	6		0,075
<i>A Crown (= 3 Livres)</i>	—	—	0	4	6		0,225

Spanish Coins.

<i>Malvadies 13,7 make</i>	—	—	0	0	0 $\frac{1}{4}$		0,00104
<i>A Rial (= 372 Malvadies)</i>	—	—	0	0	6 $\frac{3}{4}$		0,02812
<i>A Piece of 8 (Rials) Pillar</i>	—	—	0	4	6 $\frac{3}{4}$		0,22812
<i>A Piece of 8 Mexico</i>	—	—	0	4	6		0,225
<i>A Piece of 8 Peru</i>	—	—	0	4	5		0,22083
<i>A Piece of 8 Seville</i>	—	—	0	4	6		0,225

Portugal Coins.

<i>Rees, 12,4 of which make</i>	—	—	0	0	1		0,00416
<i>Mill Ree (= 1000 Rees)</i>	—	—	0	6	8 $\frac{1}{2}$		0,33541
<i>A Testoon</i>	—	—	0	1	3		0,0625

Italian Coins.

<i>The Livre at Leghorn</i>	—	—	0	0	9		0,0375
<i>Crown currant at Florence</i>	—	—	0	5	3		0,2625
<i>Ducat de Banco, at Venice</i>	—	—	0	4	4		0,216
<i>The Courrant Du at</i>	—	—	0	3	4		0,16
<i>Ducat at Naples</i>	—	—	0	5	0		0,25
<i>A St. Mark</i>	—	—	0	2	10		0,1416
<i>A Palermo Florin</i>	—	—	0	2	6		0,125

Deci-

Decimal Tables of Foreign Long Measures.

London	Foot	1,000	Lyons	—	Ell	3,976
Paris	—	1,068	Bologn	—		2,056
Amsterdam	—	0,942	Amsterdam	—		2,269
Brill	—	1,103	Antwerp	—		2,273
Antwerp	—	0,946	Leyden	—	—	2,260
Dort	—	1,184	Frankford	—		1,826
Leyden	—	1,033	Hamburgh	—		1,905
Lorrain	—	0,958	Leipsick	—	—	2,260
Mechlin	—	0,919	Lubeck	—	—	1,908
Middleburgh	—	0,991	Noremburgh	—		2,227
Strasburgh	—	0,920	Bavaria	—		0,954
Bremen	—	0,964	Vienna	—		1,054
Cologne	—	0,954	Bononia	—		2,147
Frankford ad Moen	—	0,948	Dantzick	—		1,903
Spanilh	—	1,001	Florence	—	Brace	1,913
Toledo	—	0,900	Spanilh	—	Palm	0,751
Roman	—	0,967	Spanilh	—	Vare	3,001
Bononia	—	1,204	Lisbon	—		2,750
Mantua	—	1,569	Gibraltar	—		2,760
Venice	—	1,162	Toledo	—		2,685
Dantzick	—	0,944	Naples	—	Canna	6,880
Copenhagen	—	0,965	Genoa	—	Palm	0,830
Prague	—	1,026	Milan	—	Calamus	6,544
Riga	—	1,831	Parma	—	Cubit	1,866
Turin	—	1,062	China	—		1,016
Greek	—	1,007	Cairo	—	—	1,824
			Turkisha	—	Pike	2,200
			Persian	—	Arash	3,197

	<i>lb.</i>		<i>lb.</i>
London, The Pound } Averdupois }	1,00	London, The Pound } Averdupois }	1,00
Paris —	0,93	Middleburgh —	0,98
Lyons — —	1,09	Strasburgh —	0,93
Bologn —	0,89	Bremen —	0,94
Amsterdam —	0,93	Cologn —	0,97
Antwerp —	0,98	Frankford —	0,93
Leyden —	0,96	Hamborough —	0,95
Lorain —	0,98	Leipsick —	1,15
Mechlin —	0,98	Noremburgh —	0,94
		X 2	Copen

156 *Decimal Tables of the Course of Exchange.*

		l.			l.
London, the Pound	}	1,00	London, the Pound	{	1,00
Averdupois			Averdupois		
Copenhagen	—	0 94	Genoa	—	1,42
Vienna	—	0,83	Mantua	—	1,43
Castile	—	0,99	Milan	—	1,40
Lisbon	—	1,06	Parma	—	1,43
Gibraltar	—	1,03	Venice	—	1,53
Toledo	—	1,00	Dantzick	—	1,19
Rome	—	1,23	Prague	—	1,06
Bononia	—	1,27	Cairo	—	1,61
Florence	—	1,23	Constantinople	—	0,86
Naples	—	1,43			

Having presented the Reader with large Tables of the *Par of Exchange*; I shall next exhibit a *Table of the Course of Exchange*, in *Pence*, and *Shillings and Pence*, (into which *Foreign Coins* are reduced) in *Decimal Parts* of a *Pound Sterling*.

Decimal Tables of the Course of Exchange.

P.	l.	P.	l.	P.	l.	P.	l.
1	,001	16	,06	37	,15416	58	,2416
2	,002	17	,07083	38	,1583	59	,24583
3	,0031	18	,075	39	,1625	60	,25
4	,00416	19	,07916	40	,16	61	,25416
5	,00516	20	,083	41	,17083	62	,2583
6	,00616	21	,0875	42	,175	63	,2625
7	,00716	22	,0916	43	,17916	64	,26
8	,0083	23	,09583	44	,183	65	,27083
9	,0093	24	,1	45	,1875	66	,275
10	,0103	25	,10416	46	,1916	67	,27916
11	,0113	26	,1083	47	,19583	68	,283
12	,0123	27	,1125	48	,2	69	,2875
13	,0133	28	,116	49	,20416	70	,2916
14	,0143	29	,12083	50	,2083	71	,29583
15	,0153	30	,125	51	,2125	72	,3
		31	,12916	52	,216	73	,30416
		32	,13	53	,22083	74	,3083
		33	,1375	54	,225	75	,3125
		34	,1416	55	,22916	76	,316
		35	,14583	56	,23	77	,32083
		36	,15	57	,2375	78	,325

Decimal Tables of the Course of Exchange. 157

P.	l.	P.	l.	P.	l.	P.	l.
79	,32918	85	,35418	91	,37918	97	,40418
80	,33	86	,3583	92	,383	98	,4083
81	,3375	87	,3625	93	,3875	99	,4125
82	,3418	88	,36	94	,3918	100	,418
83	,34583	89	,37083	95	,39583		
84	,35	90	,375	96	,4		

S. P.	Flem. P.	Eng. P.	S. P.	Flem. P.	Eng. P.
32.0	1,6	,625	8	1,73	,576923
1	1,60418	,623376	9	1,7375	,575539
2	1,6083	,621761	10	1,7418	,574162
3	1,6125	,620155	11	1,74583	,572792
4	1,616	,618556	35.0	1,75	,571428
5	1,62083	,616966	1	1,75418	,570071
6	1,625	,615384	2	1,7583	,568720
7	1,62918	,613810	3	1,7625	,567375
8	1,63	,612244	4	1,76	,566037
9	1,6375	,610687	5	1,77083	,564705
10	1,6418	,609137	6	1,775	,563380
11	1,64583	,607594	7	1,77918	,562060
33.0	1,65	,606060	8	1,783	,560747
1	1,65418	,604534	9	1,7875	,559440
2	1,6583	,603015	10	1,7918	,558139
3	1,6625	,601503	11	1,79583	,556844
4	1,66	,6	36.0	1,8	,55
5	1,67083	,598503	1	1,80418	,554272
6	1,675	,597084	2	1,8083	,552995
7	1,67918	,595533	3	1,8125	,551724
8	1,683	,594059	4	1,816	,550458
9	1,6875	,592592	5	1,82083	,549199
10	1,6918	,591133	6	1,825	,547945
11	1,69583	,589680	7	1,82918	,546697
34.0	1,7	,588235	8	1,83	,54
1	1,70418	,586797	9	1,8375	,544217
2	1,7083	,585365	10	1,8418	,542986
3	1,7125	,583941	11	1,84583	,541760
4	1,716	,582524	37.0	1,85	,540
5	1,72083	,581113	1	1,85418	,539325
6	1,725	,579710	2	1,8583	,538116
7	1,72918	,578313	3	1,8625	,536912

158 The Use of Decimals in Exchange.

S. P.	Flem. P.	Eng. P.	S. P.	Flem. P.	Eng. P.
4	1,86	,535714	9	1,8875	,529801
5	1,8708½	,534521	10	1,8916	,528634
6	1,875	,53	11	1,8958½	,527472
7	1,87916	,532150	38.0	1,9	,526315
8	1,883	,530973			

From these Sets of Tables of the *Par* and *Course* of *Exchange*, (which are more compleat than any I have yet seem in the *Common Books* of *Arithmetick*;) the ingenious Accomptant will readily cast up any *Bill of Exchange*; or convert the *Coins, Weights* and *Measures* of any other Country into the *same* of our own. And by comparing the *Course* with the *Par*, may see whether our *Nation Gains* or *Loses* by trading to any *Foreign Parts*, and in what Proportion.

Question 1. Suppose at *Venice* I would exchange 175 *l.* 12 *s.* 6 *d.* for their *Ducats de Banco* at 4 *s.* 4 *d.* per *Piece*, How many must I have?

l. s. d. l. s. d. l.
First, 175—12—6 = 175,625; and 4—4 = ,216.

Then ,216) 175,625
 21 175625
 — —
 ,195) 158,0625
 1365
 —
 1156
 975
 —
 1813
 1755
 —
 . 575

Ducats }
 (759,3 }
 fere. }

The Answer,

Question 2. The *Course* of *Exchange* at *Madrid* being now 41½ *d.* per *Piece* of 8 *Mexico*, what Number of those *Pieces* may I have in exchange for 533,766 *l.*?

Per Table the First, of the *Course* of *Exchanges* 41½ *d.* = ,1744½ *l.*

Then ,1744½) 533,766 (=3060 *Pieces*, the Answer.

Quest-

The Use of Decimals in Exchange. 159

Question 3. A Bill of Exchange was accepted at *London* for the Payment of 933,93*l.* for the same value delivered at *Lisbon* in *Milrees*; Exchange at 5*s.* 4*d.* per Piece. How many of those *Milrees* was paid at *Lisbon*?

First 64*d.* (= 5*s.* 4*d.*) = ,26*l.*

Then ,26) 933,93 (= 3502,25 *Milrees*, the Answer.

Now had these 3502,25 *Milrees* been exchanged at *Par* (viz. 6*s.* 8½*d.*) they would have amounted to 1174,6895*l.* which is above 240*l.* more; and consequently there was so much loss.

Question 4. In 1421 Pieces of 8 *Peru*, How many *Eng-lish* Pounds Sterling, Exchange at *Par*?

Multiply — 1421
By the Par = ,22083

9) 4263

4738

11368

28420

2842

Answer *l.* 313,80416

Another Way.

l.

{	For	1 =	0,22083
		20 =	4,41666
		400 =	88,33333
		1000 =	220,83333
		1421 =	313,80416

Question 5. When the Exchange from *Antwerp* to *Lon-don* is at 1*l.* 4*s.* 7*d.* (= 34*s.* 7*d.*) *Flemish*; How many Pounds *Eng-lish* at *London* will ballance 236 *l.* *Flemish* at *Antwerp*?

Proceed thus, {
 Multiply the Tabular Num-
ber for *Eng-lish* Pounds } 57831, &c.
 By the given Number — 236
 The Product is the Answer *l.* 136,4819, &c.

Question 6. What Number of *Flemish* Pounds will be equivalent to 400 *l.* Sterling, Exchange at 1*l.* 13*s.* 6*d.* (= 33*s.* 6*d.*)?

Multiply the Pound *Flemish* 1,675
By the Number of Pounds Sterling 400

The Product is the Answer = 670,000 Pounds Sterl.

Question 7. A Dutch Man sells 2550 *Flemish* Ells of *Hol-land* to an *Eng-lish* Man, a *Spaniard*, a *Venetian*, an *Italian*, and

160 The Use of Decimals in Alligation

and a Portuguese; who are to have each a like Quantity; Quere how much in their own Country Measure?

First 2050 *Flemish Ells* are equal to 1230 *Ells English*, equal to 4612,5 Feet, which divided by 5 quotes 922,5 Feet each.

Then by the	(307,5	Yards for the Briton.)	
Table of Measure	793,89	Feet for the Venetian.		
922,5 Feet are e-	{ 1228,36	Palms for the Spaniard.	}	Ans.
qual to —	482,2	Braces for the Italian.		
	(335,43	Vares for the Portuguese.)	

Question 8. What Number of Pounds *Averdupois* at *Vienna* will *Equiponderate* 270 Pounds *Averdupois Weight* at *London*?

Divide 270 by ,83. the Quotient 325,3 l. is the Answer.

But if 'twas required to know what Number of Pounds *Averdupois Weight* at *London* would equal any given Number at any other place, then you must multiply by the *Tabular Number*.

Alligation, or Rule of Composition.

Alligation (so called of the Latin Word *Alligo*, to bind or tie together; because the vulgar Way is to tie or connect together the Numbers concern'd in the Work,) is a Rule for Compounding or Mixing several Ingredients of different Sorts together, in any Manner or Proportion. And is divided into *Alligation*, *Medial* and *Alternate*.

Alligation Medial is that by which the Mean Rate or Price of any Mixture is found when the particular Quantities, and their Prices, are given; and it is perform'd by this

Rule,

Multiply each Quantity by its Price; then say, As the Sum of all the Quantities, is to the Sum of the said Products, so is any Part of the Mixture, to the Mean Price of that Part.

Question 1. A Tobacconist would mix 20 lb. of Tobacco at 9 d. the Pound with 60 lb. at 14 d. per lb. with 40 lb. at 18 d. per lb. and with 12 $\frac{1}{2}$ lb. at 2 s. per lb. Quere what a Pound of such a Mixture is worth?

First

The Use of Decimals in Alligation. 161

	lb.	Rate.		Products.
First,	20	X, 0375	produceth	— 0,75
	60	X, 0583	produceth	— 3,5
	40	X, 075	produceth	— 3,0
	12,75	X, 1	produceth	— 1,275

The Sum 132,75 of the Quan. The Sum 8,525 the Prod.

lb. l. lb. l.

Then say, As 132,75 : 8,525 :: 1 : 0642

132,75) 8,5250 (,0642 = 1 s. 3 $\frac{1}{4}$ d. per lb. Answer.

79650

• 56000

53100

• 29000

26550

2450

Question 2. A Goldsmith hath Gold 12 oz. worth 4*l.* per oz. 8 $\frac{1}{2}$ oz. at 4*l.* 5*s.*; 3 oz. at 4*l.* 6*s.* 8*d.*; and 9 oz. at 4*l.* 13*s.* 4*d.* Suppose these all melted down together, *Quere* what an Ounce of that *Mixture* would be worth?

	oz.	l.		l.
First,	12	X 4	produceth	48
	83	X 4,25	produceth	35,275
	3	X 4,3	produceth	13
	9	X 4,6	produceth	42

The Sum 32,3

The Sum 138,275 of the Prod.

Then say, As 32,3 oz. : 138,275 *l.* :: 1 oz. : 4,2809 *l.*

32,3) 138,275 (4,2809 = 4*l.* 5*s.* 7 $\frac{1}{4}$ d.

1292

• 907

646

2615

2584

• 3100

2907

• 193

Y

Answer 4*l.* 5*s.* 7 $\frac{1}{4}$ d.

Alli.

162 The Use of Decimals in Alligation.

Alligation Alternate is that by which the *particular Quantities* of every Ingredient in any *Mixture* are found; when the *particular Rates* of every one of the Ingredients; and the *Mean Rates* are given.

This is (as it were) the *Converse* of the former, and admits of *three Cases*.

Case 1. The *particular Rates* and the *Mean Rate* being given, to find the *Quantity* of each Ingredient for the *Mixture* proposed.

Question 1. A *Vintner* would make a *Mixture* of *Malaga* at 7 s. 6 d. per Gallon; with *Canary* at 6 s. 9 d. per Gallon; *Sherry* at 5 s. per Gallon, and *White Wine* at 4 s. 3 d. per Gallon: What *Quantity* of each Sort must he take, that the *Whole Measure* may be sold for 5 s. 10 d. per Gallon?

Note, In all Questions of this Nature, where two or four Things are mixt together, when one half of the Prices are Greater, and the other half lesser than the *Mean Rate*, you must set a greater and lesser Price above, and the same below the *mean Price*; then take the *Difference* between the *mean Rate* and the *particular Rates*, and place them alternately, and they will be the *Quantities* required.

	<i>Rates.</i>	<i>Differences.</i>
	7,5 <i>Malaga</i>	1,583 Gal. of <i>Malaga</i> .
	4,25 <i>White</i>	1,6 Gal. of <i>White</i> .
Mean Rate = 5,83	5 <i>Sherry</i>	0,916 Gal. of <i>Sherry</i> .
	6,75 <i>Canary</i>	0,83 Gal. of <i>Canary</i> .

The Sum of those Differences is = 5,0 Gallons the whole *Mixture*.

Note, The *Differences* are not only the *Quantities*, which answer the Question, but any other Numbers, in the same *Proportion* as they are, will answer the Question as well.

For	—	—	1,583	1,6	0,916	0,83				
All multiplied by	—	—	—	—	—	3				
Produce the Proportionals	—	—	4,75	5	2,75	2,5				
These multiplied by	—	—	—	—	—	4				
Produce these whole Numbers in the same Ratio, and so on <i>In infinitum</i> .	<table> <tr> <td>19.</td> <td>20.</td> <td>11.</td> <td>10.</td> </tr> </table>						19.	20.	11.	10.
19.	20.	11.	10.							

In *Case one* of the *Given Rates* (when more than two) be { Greater } and all the rest { Lesser } than the *mean Rate*. Then

The Use of Decimals in Alligation. 163

Then the *mean Rate*, *particular Rates*, and *Differences* must stand as in the following Examples.

P. R. Differences.			P. R. Differ.		
Mean Rate 20	18	4 + 2	Mean Rate 20	14	4
	22	2		18	4
	24	2		24	6 + 2

R. Differences.			R. Differences.		
M. Rate 4	2	1 + 5 + 13	Or 40	10	8
	5	2		25	8
	9	2		36	8
	17	2		48	30 + 15 + 4

The Method is the same for any other given Rates, or Prices.

Case 2. When the *Particular Rates*, the *Mean Rate*, and the *Quantity* of one Ingredient is given; to find the *Quantity of all the rest* of the Ingredients.

This is call'd *Alligation Partial*, because a *Part* of the Mix'd Ingredients only are known.

In this Case you must set down the *mean Rate*, the *particular Rates*, and their *Differences* just as before; then say,
Rule.

As the *Difference* opposite to the known *Quantity*, is to the known given *Quantity*; so is any other *Difference*, to the *Quantity* of its opposite *Name*.

Question 2. How much *Malaga* at 7 s. 6 d.; *Sherry* at 5 s.; *White Wine* at 4 s. 3 d. the Gallon, must be mixt with eighteen Gallons of *Canary* at 9 s. 9 d. per Gallon, that the Whole may be sold for 5 s. 10 d. per Gallon?

Rates.					
Mean Rate 5,83	7,5	Malaga	0,83	Differences.	
	5	Sherry	1,6		
	6,75	Canary	1,583		
	4,25	White	0,916		

Then, As 1,583 : 18 :: $\left\{ \begin{array}{l} 0,83 \text{ to the Gallons of Malaga.} \\ 1,6 \text{ to the Gallons of Sherry.} \\ 0,916 \text{ to the Gallons of W. Wine.} \end{array} \right.$

I leave the Work to exercise the Learner.

164 The Use of Decimals in Alligation.

Case 3. The particular Rates, the mean Rate, and the Sum of all the Quantities of the Ingredients given; hence to find the particular Quantities of the Mixture.

This is call'd *Alligation Total*; because the whole Quantity of the Mixture is given.

It is thus perform'd; set down the mean Rate, the particular Rates; and find their Differences as before. Then say,

Rule. As the Sum of all the Differences, is to the Sum of all the Quantities; so is each particular Difference, to its particular Quantity.

Question 3. Suppose it required to mix Malaga at 7 s. 6 d. with Canary at 6 s. 9 d. Sherry at 5 s. and White Wine at 4 s. 3 d. per Gallon; and the whole Quantity to be $84\frac{1}{2}$ Gallons, and to be sold at 5 s. 10 d. per Gallon; Quere the Quantity of each Sort for the Mixture?

Mean Rate 5,83	{	7,5	Malaga	{	0,83	{	Differences.
		5	Sherry		1,6		
		6,75	Canary		1,583		
		4,25	White		0,916		
				<hr/>		5 = The Sum.	

Then, As 5 : $84,625 :: \left\{ \begin{array}{l} 0,83 \\ 1,6 \\ 1,583 \\ 0,916 \end{array} \right\}$ to the Quantity in Gallons of $\left\{ \begin{array}{l} \text{Malaga.} \\ \text{Sherry.} \\ \text{Canary.} \\ \text{White.} \end{array} \right.$

Note, The Work of these, and such like Proportions, may be very much shortened, and easierly perform'd by a common Multiplier as in Fellowship.

Now because *Alligation alternate* answers not Questions compleatly, that is, does not give all the Answers such Questions are capable of; and so perhaps not always those which best suit the occasion; I shall shew (from Mr. Ward) how this Imperfection of common Arithmetick is supplied by Algebra, and all the possible Answers to any Questions may be clearly and easily discover'd.

Question 4. A Tobacconist hath three Sorts of Tobacco, viz. one of 2 s. 8 d. per Pound; another of 20 d. per Pound; a third Sort of 16 d. per Pound; of these he would make a Mixture to contain 56 Pound that may be sold for 22 d. per Pound; How much of each Sort may he take?

Let

The Use of Decimals in Alligation. 165

Let $\begin{cases} a = \text{the Quantity of that worth } 2 \text{ s. } 8 \text{ d.} \\ e = \text{that of } 20 \text{ d. per Pound.} \\ y = \text{that of } 16 \text{ d. per Pound.} \end{cases} = 32 \text{ d.}$

Then 1. $a + e + y = 56$ Hence 'tis evident
And 2. $32a + 20e + 16y = 1232$ from the 7th Step
1-a 3. $e + y = 56 - a$ that the Quantity sig-
2-32a 4. $20e + 16y = 1232 - 32a$ nified by a must be
3 X 16 5. $16e + 16y = 896 - 16a$ less than 21, and (by
4-5 6. $4e = 336 - 16a$ the 8th) Step greater
6 ÷ 4 7. $e = 84 - 4a$ than $9\frac{1}{4}$. That is a
3-7 8. $y = 3a - 28$ may be any Number
between 21 and $9\frac{1}{4}$.

If there be *more* than *three* Quantities concerned in the Question, the Work will be more large; because the *Limits* of all the Quantities above *two*, must be found.

Question 5. Suppose it were required to mix *four* Sorts of *Wine* together; viz. one worth 7 s. 4 d. per Gallon; a second worth 4 s. 7 d. a third worth 3 s. 8 d. and a fourth worth 2 s. 9 d. per Gallon. How much of each Sort must be taken to make a Mixture of 63 Gallons, to be sold for 5 s. 6 d. per Gallon, without Loss?

$\begin{matrix} & & & \text{s.} & \text{d.} & \text{d.} \\ & & & & & \\ \text{First let } \left\{ \begin{array}{l} a = \text{that Quantity worth } 7 \text{ } 4 = 88 \\ e = \text{that worth } \text{---} 4 \text{ } 7 = 55 \\ y = \text{that worth } \text{---} 3 \text{ } 8 = 44 \\ u = \text{that worth } \text{---} 2 \text{ } 9 = 33 \\ \text{the mean Rate } \text{---} 5 \text{ } 6 = 66 \end{array} \right. \end{matrix}$

Then 1. $a + e + y + u = 63$
And 2. $88a + 55e + 44y + 33u = 4158$.
1-a 3. $e + y + u = 63 - a$
2-88a 4. $55e + 44y + 33u = 4158 - 88a$
3 X 33 5. $33e + 33y + 33u = 2079 - 33a$
4-5 6. $22e + 11y = 2079 - 55a$
6 ÷ 11 7. $2e + y = 189 - 5a$
3 X 55 8. $55e + 55y + 55u = 3465 - 55a$
8-4 9. $11y + 22u = 33a - 93$
9-11 10. $y + 2u = 3a - 63$
Suppose 11. $a = 22$. Then $5a = 110$, and $3a = 66$
Per 7th 12. $2e + y = 189 - 5a = 79$

266 The Use of Decimals in Position.

$$\begin{array}{l|l}
 13-2e & 13 \quad y = 79 - 2e \\
 \text{Per 3d.} & 14 \quad e + y + u = 63 - a = 41 \\
 14-e & 15 \quad y + u = 41 - e \\
 15-13 & 16 \quad u = e - 38
 \end{array}$$

From the seventh and tenth Steps it appears, that the Quantity denoted by a , must be less than $37\frac{1}{2}$, and greater than 21 Gallons; whence 16 answer flow from the Limits of a only. Then if a be put $= 22$, by the thirteenth and sixteenth Steps it appears $e = 39$, $y = 1$, and $u = 1$. And thus proceeding with each single value of a , above 120 Answers may be found to this Question in whole Numbers; in Fractions, infinite.

Position, or Rule of False.

This Rule of *Position*, or rather *Supposition*, is so call'd, because we *suppose* or make a *Position* of some *uncertain* Numbers, in order that by reasoning from them we may gain the *true Number* sought; and because those *Positions* are altogether at *random* or *adventure*, the *Rule* is also call'd *False*.

The Use of this *Rule*, before the common Knowledge of *Algebra*, was much more considerable than since; because that Art supplies *Theorems* for resolving all kind of Questions in this Rule in a better and more curious a manner than here; Yea some of the *best Pieces* of *Arithmetick* have intirely discarded it, and others post-pone it, as *obsolete* and of little use, since *Algebra*.

Questions in this are mostly perform'd by one or two *Suppositions*; if by one, the Rule is said to be of *Single Position*; if two *Suppositions* are necessary, 'tis called *Double Position*.

Single Position.

Question 1. Three Merchants A, B, C trade in Company, and gain 100 $l.$ of which A had a certain Part, B had twice as much, and C had thrice as much as B ; How much had each Man?

Suppose A had 4 $l.$ then B must have 8 $l.$ and C would have 24 $l.$ which together make 36 $l.$ but shou'd have been an 100 $l.$

There-

Therefore Reason by Proportion *l.*

$$\text{Thus, As } 36 : 100 :: \begin{cases} 4 : 11, \dot{x} = A \\ 8 : 22, \dot{x} = B \\ 24 : 66, \dot{x} = C \end{cases}$$

Their several Parts added make — 100 for Proof.

Question 2. A Schoolmaster being asked how many Scholars he had; answer'd, if I had as many, and $\frac{1}{2}$ as many, and $\frac{1}{4}$ as many, I should have 99. How many had he?

Suppose he had 40; Then $40 + 40 + 20 + 10 = 110$. but it should have been but 99. Therefore say

As $110 : 40 :: 99 : 36$ Scholars, the Answer.

Question 3. Three Men *A, B, C* buy a Ship for 310*l.* 15*s.* of which *A* paid an unknown Sum; *B* paid $2\frac{1}{2}$ as much; and *C* $3\frac{1}{3}$ as much: How much did each Man pay?

Suppose *A* paid 48*l.* then *B* paid $48 \times 2,5 = 120$ *l.* and *C* must pay $48 \times 3,3 = 160$ *l.* But $48 + 120 + 160 = 328$ instead of 310,75*l.*

Say therefore, As $328 : 48 :: 310,75 : 45,4756$, &c.

Then <i>A</i> paid	—	—	45,4756
<i>B</i> paid ($45,4756 \times 2,5 =$)			113,689
<i>C</i> paid ($45,4756 \times 3,3 =$)			151,5853
Proof is the Sum	—	—	310,75

Double Position.

In the *Double Rule*, two Suppositions are used, because here the Numbers cannot be parted to find the Answer by Proportion as before.

Therefore when we make two Suppositions, and miss in both, observe the Nature of the *Errours*, whether they be *Greater* or *Lesser* than the *Number proposed*; and accordingly mark them with the *Signs More* or *Less*, viz. +, —; and place them precisely against their proper Suppositions; then observe the general

Rule,

168 The Use of Decimals in Position.

Rule,

Viz. As the Difference of the Errors if alike, (or their Sum if unlike) is to the Difference of the Suppositions; so is either of the Errors, to a fourth Number.

The fourth Number add to, or subtract from, the Supposition opposite to it; and you have the Number sought.

Question 1. Admit three Merchants build a Ship which cost 1360 Pounds. A pays a certain Part unknown; B paid $2\frac{1}{2}$ as much, wanting 15,5 l. and C paid as much as both A and B, and 75,25 l. over; How much did each Man pay?

First, Suppose A paid 200 l. then B must have paid 484,5 l. and C paid 759,75 l. But those three Sums, viz. $200 + 484,5 + 759,75 = 1444,25$ l. which is more than 1360 by 84,25 l. Wherefore the first Error is $- + 84,25$ l.

Secondly, Suppose A paid 180 l. then B paid 434,5 l. and C paid 689,75 l. But $180 + 434,5 + 689,75 = 1304,25$ l. which is too little by 55,75 l. therefore the Suppositions and their Errors will stand thus,

The First Supposition 200, + 84,25 Error.

The Second Supposition 180, — 55,75 Error.

The Difference of Supposi. = 20 140 = Sum of Errors.

Then by the General Rule, say,

As, { 140 : 20 :: 55,75 : 7,964 Ec. }
Or, { 140 : 20 :: 84,25 : 12,035 Ec. }

Then, { $200 - 12,035 =$ } $187,964 = A's$ Part.

Then B must have paid — 454,410 = B's Part.

And C must have paid — 717,624 = C's Part.

The Sum of which is = 1360 for Proof.

Note, When the Errors are equal and have unlike Signs; half the Sum of the Suppositions, is the Number sought.

Ex-

Extraction of Roots.

The *extream Use of Decimals* in all kinds of *Extractions* is sufficiently known to all versed in *Arithmetical Knowledge*; and its *absolute Necessity* in *some Parts* of *Arithmetick*, and its *Excellency* beyond even *Logarithms* themselves in others, is also as well known.

I would know what is the *Square Root* of 161,29?

Thus 161,29 (12,7 = the Answer.

$$\begin{array}{r}
 \text{I} \\
 22) \overline{61} \\
 \underline{44} \\
 247) \overline{1729} \\
 \underline{1729} \\
 \dots
 \end{array}$$

What is the *Square Root* of 3477?

In such Cases as this, you must add *twice* as many *Cyphers* to the given Number, as you design to have *Decimal Places* in the *Root* of the above Number to three Places of *Decimals*.

Then 3477,000000 (58,881 the Root requir'd.

$$\begin{array}{r}
 25 \\
 (108) \overline{977} \\
 \underline{864} \\
 1168) \overline{10300} \\
 \underline{9344} \\
 11768) \overline{95600} \\
 \underline{94144} \\
 117761) \overline{145600} \\
 \underline{117761} \\
 27839
 \end{array}$$

170 The Use of Decimals in Extractions.

Required the *Square Root* of 2, to 6 Places of Decimals.

$$\begin{array}{r}
 \begin{array}{c}
 \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \\
 2,000000000000 \text{ (1,414213 the Root required.} \\
 \hline
 1 \\
 24 \overline{) 100} \\
 \underline{96} \\
 281 \overline{) \cdot \cdot 400} \\
 \underline{281} \\
 2824 \overline{) 11900} \\
 \underline{11296} \\
 2828 \overline{) \cdot \cdot 604} \\
 \underline{565} \\
 \cdot 38 \\
 \underline{28} \\
 10 \\
 8 \\
 \underline{} \\
 (2)
 \end{array}
 \end{array}$$

Having here got 3 Places of the 6, I work by the *contracted way of Division*, and gain the other 3 as truly as if wrought at *large*.

What is the *Square Root* of ,4489?

$$\begin{array}{r}
 \cdot \cdot \cdot \\
 ,4489 \text{ (,67 the Root requir'd.} \\
 \hline
 36 \\
 127 \overline{) \cdot 889} \\
 \underline{889} \\
 \dots
 \end{array}$$

What is the *Square Root* of ,00576?

$$\begin{array}{r}
 \cdot \cdot \cdot \cdot \cdot \\
 ,00576 \text{ (,024 the Root sought.} \\
 \hline
 004 \\
 ,044 \overline{) \cdot \cdot 176} \\
 \underline{176} \\
 \dots
 \end{array}$$

What

The Use of Decimals in Extractions. 171

What is the Square Root of, 00005625 ?

,00005625 (,0075 = the Root sought.

$$\begin{array}{r}
 \dots 49 \\
 \hline
 145) \cdot 725 \\
 \quad \cdot 725 \\
 \hline
 \dots
 \end{array}$$

To Extract the Roots of Single Repetends.

What is the Square Root of ,x or Unity infinitely repeated ?

$$\begin{array}{r}
 \cdot xiiiiiii \text{ \&c.} \\
 9 \quad (,3333 \text{ \&c. the Square Root.} \\
 \hline
 63) \cdot 211 \\
 \quad 189 \\
 \hline
 663) \cdot 2211 \\
 \quad 1989 \\
 \hline
 6663) \cdot 22211 \\
 \quad 19989 \\
 \hline
 \quad 222211
 \end{array}
 \left. \begin{array}{l} \\ \\ \\ \end{array} \right\} \begin{array}{l} \text{So downward} \\ \\ \text{Ad Infinitum.} \end{array}$$

What is the Square Root of ,44,4 \&c. ?

,44,4444 \&c. (6,66 \&c. the Root.

$$\begin{array}{r}
 36 \\
 \hline
 126) \cdot 844 \\
 \quad 756 \\
 \hline
 1326) \cdot 8844 \\
 \quad 7956 \\
 \hline
 \quad 88844
 \end{array}
 \left. \begin{array}{l} \\ \\ \end{array} \right\} \begin{array}{l} \text{So downward} \\ \\ \text{Ad Infinitum.} \end{array}$$

If the Root does not repeat in the first Figure, 'twill be very uncertain when it will repeat.

Note, Only these two Digits 1 and 4 (of all the Nine) when infinitely repeated ; have their Roots, pure single Repetends.

[To Extract the Square Root of Compound Repetends.

What is the *Square Root* of 196,84?

Thus, 196,85 (14,030568658 the Root.

$$\begin{array}{r}
 \text{I} \\
 \hline
 24) \cdot 96 \\
 \underline{96} \\
 2803) \cdot \cdot 8568 \\
 \underline{8409} \\
 280605) \cdot 1595685 \\
 \underline{1403025} \\
 280611) 192660 \\
 \underline{168366} \\
 \cdot 24294 \\
 \underline{22448} \\
 \cdot 1846 \\
 \underline{1683} \\
 \cdot 163 \\
 \underline{140} \\
 \cdot 23 \\
 \underline{22} \\
 (I)
 \end{array}$$

In this Manner the *skilful Artist* may proceed and gain the *Root* of any *Repetend* to what Number of Places he pleaseth. I omit the *Extraction* of the *Cube Root* here; because I shall have occasion to shew the Method and *Rationale* of that, and of the *Square Root*. both, when I come to shew the Use of Decimals in Algebra.

CHAPTER.

C H A P. IX.

The Use of DECIMALS in the Business of Interest both Single and Compound ; Of Annuities, Pensions, &c. their Value in Present Worth, and in Arrears ; Of Rebate or Discount ; Of Free-hold or Real Estates.

I N T E R E S T is a *small Sum of Money* paid for the Use of any *greater Sum*, according to any *Rate* agreed on ; as 5 *l.* per 100 *l.* &c. for a Year ; and it is either *Simple* or *Compound*.

Simple Interest is that which arises only from the *Principal* or *Sum of Money* lent ; and both *Interest* and *Principal* are always the same as at first.

Compound Interest is that which ariseth from the *Principal* and its *Simple Interest* (when *due* and *forborn*) reckoned together as a *New Sum*, so that both *Principal* and *Interest* here are always increasing.

Annuities, Pensions, Salaries, &c. are *Rents, Profits, and Payments* made *Yearly, or Half Yearly, &c.* and they are said to be in *Arrears*, when they are due and unpaid for any *Number of Payments*.

Rebate or Discount is an *Abatement* of *Part* of a *Sum of Money* due sometime hence, in Consideration of *prompt* or present Payment of the Remainder ; and this is done at any *Rate of Interest*.

In exemplifying the *wonderful Use of Decimals* in the *Affair of Interest, &c.* I need only shew the Reader the Solutions of those *admirable Theorems* in Numbers at large, which Mr. Ward (in his *Mathematician's Guide*) has with great *Invention* contriv'd from the following *Data*, and Method of Reasoning from thence.

In

In Simple Interest.

He puts $\left\{ \begin{array}{l} P = \text{Any Principal or Sum put to Interest.} \\ R = \text{The Ratio of the Rate per Cent. per Annum.} \\ T = \text{The Time of the Principal at Interest.} \\ A = \text{The Amount of the Principal and its Interest.} \end{array} \right.$

In Annuities, &c. at Simple Interest.

He puts $\left\{ \begin{array}{l} U = \text{The Annuity, Pension, or Yearly Rent, &c.} \\ T = \text{Time of Forbearance, or being Unpaid.} \\ R = \text{The Ratio of Interest, as before.} \\ A = \text{The Amount of the Annuity and its Interest.} \\ \frac{2}{R} - 1 = x \text{ By way of Substitution, in Arrears.} \\ \frac{2}{R} - \frac{2P}{U} + 1 = x \text{ By Substitution also, in pres. worth.} \\ P = \text{The Present Worth of Annuities, &c.} \end{array} \right.$

In *Compound Interest* these *Characters* signify the same Things as here; only *T* here, is there wrote in a small Letter (*t*) and denotes the *Power* of *R*; or is the *Index* of the *Power* to which *R* is to be involved; and *R* = Amount of 1 *l.* and its Interest one Year. From the above *Data*, he makes the following Proportions.

Simple Interest.

The *Ratio* of the *Rate* of *Interest* signified by *R* is thus found (for 'tis only the Interest of 1 *l.* for 1 Year.)

	<i>l.</i>	<i>In.</i>	<i>l.</i>	<i>Ratio.</i>	
As	{	100 : 4	:: 1 :	,04	= <i>R</i> at 4 per Cent.
		100 : 4,5	:: 1 :	,045	= <i>R</i> at 4 $\frac{1}{2}$ per Cent.
		100 : 5	:: 1 :	,05	= <i>R</i> at 5 per Cent.
		100 : 5,5	:: 1 :	,055	= <i>R</i> at 5 $\frac{1}{2}$ per Cent.
		100 : 6	:: 1 :	,06	= <i>R</i> at 6 per Cent.

And $\left\{ \begin{array}{l} 1. \quad 2. \quad 3. \quad 4. \quad 5. \quad 6. \quad \&c. = \text{Years.} \\ R. \quad 2R. \quad 3R. \quad 4R. \quad 5R \quad 6R. \quad = \text{Interest.} \end{array} \right.$

Hence 'tis evident the *Simple Interest* of 1 *l.* is a *Series* of *Terms* in *Arithmetical Progression* increasing.

Wherein $\left\{ \begin{array}{l} R = \text{The first Term, and also the Common Differ.} \\ T = \text{The Number of all the Terms.} \\ TR = \text{The last Term of the Series.} \end{array} \right.$

Then

1. Int.

Then, As $1 : TR :: P : TRP = \text{Interest of } P$. But the *Principal* and its *Interest* added is equal to the *Amount*. Whence this *General Theorem*,

$$TRP = P = A.$$

Annuities, Simple Interest.

Here $U = \text{the Yearly Rent}$, and $R = \text{the Ratio of Interest}$. Then $2U = \text{the Rent}$, and $RU = \text{the Interest for the second Year}$; and thus the following *Progressions* for five Years.

$$\text{Thus } \left\{ \begin{array}{llllll} 1. & 2. & 3. & 4. & 5. & \&c. \text{ The Years.} \\ U. & 2U. & 3U. & 4U. & 5U. & \&c. \text{ The Rent.} \\ RU. & 2RU. & 3RU. & 4RU. & 5RU. & \&c. \text{ The Interest.} \end{array} \right.$$

Hence 'tis plain, that $RU + 2RU + 3RU + 4RU + 5U = A$. The *Sum* of all the *Rents* and their *Interests* being *forborn* five Years. From whence it follows

That $RU + 2RU + 3RU + 4RU = A - TU$. For here $T = 5$.

Divide all by U . Then $R + 2R + 3R + 4R = \frac{A - TU}{U}$

Then by *Substitution*, put $R + 2R + 3R + 4R = Z$.

Then $1 + 2 + 3 + 4 = \frac{Z}{R}$ Now the *first* and *last* Terms of the *Progression* are $1 + 4 = 5 = T$. Therefore $\frac{T - 1}{2} \times T = \text{Sum of all the Terms}$.

Now $\frac{TT - T}{2} = \frac{Z}{R}$ Hence $\frac{TTR - TR}{2} = Z$. Consequently $\frac{TTR - TR}{2} = \frac{A - TU}{U}$ The *General Theorem* for *Annuities in Arrears*.

But because $P = \text{the Present Worth}$, is not in the *last General Theorem*, That will answer no Questions relating thereto; Wherefore a *New one* must be contriv'd: Now because A denotes the same Thing, viz. the *Amount*, in both the *last General Theorems*; and because any two Quantities, equal to one and the same thing, are equal to one another. And $PTR + P = A$ in the *first General Theorem*; And $\frac{TTRU - TRU + 2TU}{2} = A$ in the *latter General Theor.*

There-

176 The Use of Decimals in Interest,

Therefore $PTR + P = \frac{PTRU - TRU + 2TU}{2}$ is the *General Theorem* for Questions about the *present Worth* or *Purchasing of Annuities*.

Compound Interest.

The *Proportion* for finding R the *Ratio* of the *Rate* of *Compound Interest*, (which is only the *Amount* of 1 *l.* and its *Interest* for one *Year*,) is This

1.

As $\begin{cases} 100 : 105 :: 1 : 1,05 = R \text{ at } 5 \text{ per Cent.} \\ 100 : 106 :: 1 : 1,06 = R \text{ at } 6 \text{ per Cent, \&c.} \end{cases}$

But as one *Pound*, is to the *Amount* of one *Pound*, at one *Year's End*; so is that *Amount*, to the *Amount* of one *Pound* at two *Year's End*; and so on continually.

That is, $1 : R :: R : RR :: RR : RRR :: R^3 : R^4 :: R^4 : R^5 :: \&c.$

Then $\begin{cases} 1. & 2. & 3. & 4. & 5. & = \text{Years.} \\ R. & R^2. & R^3. & R^4. & R^5. & = \text{The Amount of 1 l. at} \\ & & & & & \text{any Rate.} \end{cases}$

Hence 'tis evident the *Amount* proceeds in a *Geometrical Proportion*, wherein the *Time* ($= t$), or *Number of Years*, is always equal to, or the same with, the *Index* of the *Power* of the *last* and *highest Term* of the *Series*; viz. R^t , or R^5 .

But, as one *Pound* : is to the *Amount* of one *Pound* for any given *Time* :: so is any proposed *Principal* or *Sum* : to its *Amount* for the same *Time*.

That is, As $1 : R^t :: P : PR^t$.

But $PR^t = A$ The *General Theorem*.

Annuities. Compound Interest.

Let $\begin{cases} R = \text{One Pound and its Interest for one Year, as before.} \\ U = \text{The first Year's Rent without Interest.} \end{cases}$

Then $RU =$ The *Amount* of the *first Year's Rent*, and its *Interest*.

And hence is form'd the following *Progression* of *Amounts* in continued *Geometrical Proportion*.

Thus $\begin{cases} 1. & 2. & 3. & 4. & 5. & \&c. & \text{The Years.} \\ U & +UR & +UR^2 & +UR^3 & +UR^4 & , \&c. & \text{The Amounts.} \end{cases}$
Hence

Hence $U + UR + UR^2 + UR^3 + UR^4 = A$ The Amount of any Yearly Rent or Annuity forborn five Years.

Now the last Term in the above Series is $UR^4 = UR^{t-1}$.

Therefore $A - UR^{t-1} =$ The Sum of all the Antecedents.

And $A - U =$ The Sum of all the Consequents in the Series.

So that it will be, $U : RU :: A - UR^{t-1} : A - U$.

Therefore $AU - UU = RUA - UUR^t$. Divide all by U .

Then $A - U = RA - UR^t$. The General Theorem.

For the present Worth, we must proceed as in Simple Interest in this Case, to gain an Equation or general Theorem, wherein shall be P .

The Theorem for Interest is $PR^t = A$.

And in the last Theorem $\frac{UR^t - U}{R - 1} = A$.

Consequently, $PR^t = \frac{UR^t - U}{R - 1}$ The General Theorem.

Free-hold Estates, Compound Interest.

Free-hold or Real Estates are supposed to be purchased for ever. And the Computation of the Value of such Estates is grounded on a Series of Geometrical Proportionals decreasing ad Infinitum.

Let P, U, R , denote the same as before ; then the Series will be $\frac{U}{R} \cdot \frac{U}{R^2} \cdot \frac{U}{R^3} \cdot \frac{U}{R^4} \cdot \frac{U}{R^5} (= P)$ &c. till the last Term be $= 0$. Then will $P - 0 =$ Sum of all the Antecedents ; and $P - \frac{U}{R} =$ Sum of all the Consequents.

Therefore, as $\frac{U}{R} : \frac{U}{R^2} :: P : P - \frac{U}{R}$ which gives $PR - U = P$. The general Theorem.

Theorems Resolving all Questions concerning Simple Interest.

Given P, R, T ; To find A ?

Theorem 1. $TRP + P = A$.

Given T, R, A ; To find P ?

Theorem 2. $\frac{A}{TR + 1} = P$.

A a

Given

178 The Use of Decimals in Interest.

Given A, P, T ; To find R ?

$$\text{Theorem 3. } \left\{ \frac{A-P}{TP} = R. \right.$$

Given P, R, A ; To find T ?

$$\text{Theorem 4. } \left\{ \frac{A-P}{RP} = T. \right.$$

Question 1. What will 256 l. 10 s. Amount to in three Years, one Quarter, two Months, and eighteen Days, at 6 l. per Cent. per Ann.

$$\text{Here is given } \left\{ \begin{array}{l} P = 256,5 \\ R = 0,06 \\ T = 3,46598 \end{array} \right\} \text{ To find } A, \text{ per The. 1.}$$

$$\begin{array}{rcl} \text{Multiply} & - & 3,46598 = T \\ \text{By} & - & 0,06 = R \end{array}$$

$$\begin{array}{rcl} \text{Product} & - & 0,2079588 = TR \\ \text{Mult. by Inversion} & - & 5,652 = P. \end{array}$$

$$\begin{array}{r} 4159176 \\ 1039794 \\ 124775 \\ 10397 \end{array}$$

$$\begin{array}{rcl} \text{The Product} & - & 53,34142 = TRP \\ \text{Add} & - & +256,5 = P \end{array}$$

$$\text{Answer} - \text{ l. } 309,84142 = A = 309 \text{ l. } 16 \text{ s. } 10 \text{ d.}$$

Question 2. What Principal or Sum of Money put to Interest, will raise a Stock of (or be worth) 405 l. 6 s. in five Years, and eight Months, at the Rate of 5 l. per Cent. per Ann.?

$$\text{Here is given } \left\{ \begin{array}{l} A = 405,30016 \\ R = 0,05 \\ T = 5,613698 \end{array} \right\} \text{ To find } P, \text{ per The. 2.}$$

Multi-

$$\begin{array}{rcl}
 \text{Multiply} & 5,613698 & = T \\
 \text{By} & \text{—} & 0,05 = R \\
 \hline
 \text{Product} & 0,2806849 & = TR \\
 \text{Add Unity} & 1, & \\
 \hline
 TR + 1 = & 1,2806849 & 405,30016 (316,47144 l. = P.
 \end{array}$$

Hence the *Principal* is 316,47144 l. which, in common Coin, is 316l. 9s. 5d. for the Answer.

$$\begin{array}{r}
 38420547 \\
 \hline
 2109469 \\
 1280684 \\
 \hline
 828785 \\
 768410 \\
 \hline
 60375 \\
 51227 \\
 \hline
 9148 \\
 8964 \\
 \hline
 184 \\
 128 \\
 \hline
 56 \\
 51 \\
 \hline
 5 \\
 5 \\
 \hline
 0
 \end{array}$$

Note, By this Theorem 'tis you find what *present Money*, or *prompt Payment*, will satisfy a *Debt* due any Time hereafter, *Abating* or *Discounting* at any Rate per Cent.

Question 3. At what Rate of Interest per Cent. will 36 l. amount to 36 l. 18s. 11d. $\frac{1}{4}$ d. in six Months, three Weeks, and three Days?

Here is given $\left\{ \begin{array}{l} P = 36 \\ T = 526 \\ A = 36,9468 \end{array} \right\}$ To find *R*, per The. 3.

180 *The Use of Decimals in Interest.*

$$\begin{array}{r} \text{Multiply } 528 = T \\ \text{By } \quad \quad 36 = P \\ \hline 3156 \\ 1578 \\ \hline \end{array}$$

$$\begin{array}{r} \text{From } 36,9468 = A \\ \text{Subst. } 36, \quad = P \\ 18,936), 94680(,05 = R \\ \hline 94680 \\ \hline \end{array}$$

$$\text{Prod. } 18,936 = TP$$

Then, As 1 : 0,05 :: 100 : 5 l. Answer 5 per Cent.

Question 4. In what Time will 200 l. 1 s. 8 d. amount to 250 l. at 4 l. 10 s. per Cent. Interest?

$$\text{Here is given } \left\{ \begin{array}{l} P = 200,083 \\ R = 0,045 \\ A = 250, \end{array} \right\} \text{To find } T, \text{ per Theo. 4.}$$

$$\begin{array}{r} \text{Mult. } 200,083 = P \\ \text{By } \quad \quad 0,045 = R \\ \hline 1000416 \\ 8003333 \\ \hline \end{array}$$

$$\begin{array}{r} \text{From } 250,000 = A \\ \text{Subst. } 200,083 = P \\ \hline 9,00375) \end{array}$$

$$\text{Prod. } 9,00375 = PR$$

The Time then is

Years

$$5,54399 = 5 \text{ Years,}$$

$$7 \text{ Months, and } 3$$

$$\text{Days.}$$

$$\begin{array}{r} 49,91866(5,54399 = T \\ 4501875 \\ \hline 489791 \\ 450187 \\ \hline 39604 \\ 36012 \\ \hline 3592 \\ 2701 \\ \hline 891 \\ 810 \\ \hline 81 \\ 81 \\ \hline \end{array}$$

Theorems resolving all Questions concerning Annuities, Pensions, &c. in Arrears, Simple Interest.

Given U, T, R ; to find A ?

$$\text{Theorem 1. } \left\{ \frac{TTU - TU}{2} R + TU = A. \right.$$

Given

Given A, T, R ; To find U ?

$$\text{Theorem 2. } \left\{ \frac{2A}{TTR - TR + 2T} = U. \right.$$

Given A, T, U ; To find R ?

$$\text{Theorem 3. } \left\{ \frac{2A - 2TU}{TTU - TU} = R. \right.$$

Given U, R, A ; To find T ?

$$\text{Theorem 4. } \left\{ \sqrt{\frac{2A}{RU} + \frac{x}{4}} - \frac{1}{2}x = T \right.$$

Question 1. If 250*l.* Yearly Rent (or Annuity, &c.) be forborn or unpaid 7 Years; what will it amount to in that Time, at the Rate of 6*l.* per Cent. per Annum?

Here is given $\left\{ \begin{array}{l} U = 250 \\ T = 7 \\ R = 0,06 \end{array} \right\}$ To find A , per Theorem 1.

$$\text{Multiply } \quad \quad \quad 250 = U$$

$$\text{By } \quad \quad \quad 7 = T$$

$$\text{Product } \quad \quad \quad 1750 = TU.$$

$$\text{By } \quad \quad \quad 7 = T.$$

$$\quad \quad \quad 12250 = TTU.$$

$$\text{Subtract } \quad \quad \quad 1750 = TU.$$

$$\text{Remains } \quad \quad \quad 10500 = TTU - TU.$$

$$\text{Halve } \quad \quad \quad 5250 = TTU - TU \div 2$$

$$\text{Multiply by } \quad \quad \quad 0,06 = R$$

$$\quad \quad \quad 315,00$$

$$\text{Add } \quad \quad \quad 1750, \quad = TU$$

$$\text{Sum } \quad \quad \quad 2065 \text{ l. } = A \text{ The Answer.}$$

If the Payment of the aforefaid Annuity had been made half Yearly, then would $U = 125 = \frac{250}{2}$, and $T = 14 =$ Number of Payments; and $R = 0,03 = \frac{0,06}{2}$, and working as per Theorem; A will be found = 2091 *l.* 5 *s.* which is

182 The Use of Decimals in Interest.

is more than the Yearly Payment by 26l. 5s. Hence the oftner the Payment, the more advantageous.

Question 2: What Annuity, or Yearly Payment, being unpaid $8\frac{1}{2}$ Years, will raise a Stock of 572 l. 12 s. 8 d. at 5 per Cent, per Annum.

Here is given $\left\{ \begin{array}{l} A = 572,63 \\ T = 8,5 \\ R = 0,05 \end{array} \right\}$ To find U , per Theor. 2.

Multiply — 8,5 = T
By — ,05 = R

Again by — ,425 = TR
— 8,5 = T

Product — 3,6125 = TTR
Subtract — 0,4250 = TR

Remains — 3,1875 = $TTR - TR$
Add $8,5 \times 2 = 17,0000 = 2T$

$TTR - TR + 2T = 20,1875$ $1145,2866$ &c. = $2A$ (56,73148

1009375
1358916
1211250
147666
141312

The Annuity there-
fore is 56,73148 = 56 l.
14 s. 7 $\frac{1}{2}$ d. The An-
swer.

6354
6056
298
201
97
80
17
16
1

Quest.

The Use of Decimals in Interest. 183

Question 3. At what Rate of Interest, per Cent. per Ann. will 40 l. 13 s. 4 d. Yearly Rent, amount to 450 l. 13 s. 4 d. in 9 Years?

Here is given $\left\{ \begin{array}{l} A = 450,8 \\ T = 9 \\ U = 40,8 \end{array} \right\}$ To find R , per Theorem 3.

$$\begin{array}{rcl}
 \text{Multiply} & - & 40,8 = U \\
 \text{By} & - & 9 = T \\
 \hline
 \text{Product} & - & 366 = TU \\
 \text{Multiply again by} & 9 & = T \\
 \hline
 \text{From that Prod.} & 3294 & = TTU \\
 \text{Subtract} & - & 366 = TU \\
 \hline
 \text{Remains} & - & 2928 = TTU - TU \text{ The Divisor.} \\
 \text{Then from} & 901,3 & = 2A \\
 \text{Subtract} & - & 732 = 2TU \\
 \hline
 \text{Remains} & - & 169,3 = 2A - 2TU \text{ The Dividend.} \\
 \text{Then } 2928) 169,33 & & (0,05784 = R \\
 & 14640 & \\
 & \hline
 & \cdot 2293 & \\
 & 2049 & \\
 & \hline
 & \cdot 244 & \\
 & 233 & \\
 & \hline
 & \cdot 11 & \\
 & 11 & \\
 & \hline
 & .. &
 \end{array}$$

Therefore as 1 : 0,05784 :: 100 : 5,784 = 5 : 15 : 8 $\frac{1}{4}$
the Rate per Cent. required.

Question 4. In what Time will 250 l. Yearly Rent, raise a Stock of 2065 l. allowing 6 per Cent. &c. for the Forbearance of the Payments as they become due?

Here is given $\left\{ \begin{array}{l} U = 250 \\ A = 2065 \\ R = 0,06 \end{array} \right\}$ To find T , per Theorem 4.

First

184 *The Use of Decimals in Interest.*

$$\begin{array}{rcll}
 \text{First} & - & 4130 & = 2A \\
 \text{Then multiply} & - & 250 & = U \\
 \text{By} & - & 0,06 & = R \\
 \text{Product is} & - & 15 & = UR \\
 \text{Then} & - & 33,3 & = \frac{2}{R} \\
 \text{And } 33,3 - 1 & = & 32,3 & = \frac{2}{R} - 1 = x. \\
 \text{And } 32,3 \div 2 & = & 16,16 & = \frac{1}{2}x \\
 \text{The Square of it is } 261,36x & = & \frac{1}{4}xx = \frac{xx}{4} \\
 \text{To which add} & 275,3 & = & \frac{2A}{RU} \\
 \text{The Sum is} & 536,694 & = & \frac{2A}{RU} + \frac{xx}{4} \\
 \text{The Square Root of which is} & \left. \begin{array}{l} \\ \end{array} \right\} 23,16 & = & \sqrt{\frac{2A}{RU} + \frac{xx}{4}} \\
 \text{From which take} & 16,16 & = & \frac{1}{2}x \\
 \text{There remains} & 7 & = & T \text{ The Time required,} \\
 & \text{viz. 7 Years.} & &
 \end{array}$$

The *Divisions* and *Extractions* at large I have omitted for the *Learner's Exercise*; but I have represented all the Numbers in One; which Mr. *Ward's* Method could not do as being *deficient* in the *Doctrine of Calculating Numbers*, as may be observ'd in his Work of this and other Questions of Interest,

N. B. In all Questions about *Yearly*, or *stated Rents* and *Payments*, the *Interest* is reckoned for *every Payment* after it becomes *due*, thro' the whole Time of *Forbearance*.

Theorems resolving all Questions concerning the Present Worth of Annuities, Pensions, &c. at Simple Interest.

Given U, R, T ; To find P ?

$$\text{Theorem 1. } \left\{ \frac{TTR - TR + 2T}{2TR + 2} U = P \right.$$

Given

The Use of Decimals in Interest. 185

Given P, R, T ; To find U ?

$$\text{Theorem 2. } \left\{ \frac{TR+1}{TTR-TR+2T} 2P = U \right.$$

Given P, U, T ; To find R ?

$$\text{Theorem 3. } \left\{ \frac{2P-2TU}{TTU-TU-2PT} = R \right.$$

Given U, P, R ; To find T ?

$$\text{Theorem 4. } \sqrt{\frac{2P}{RU} + \frac{xx}{4}} \pm \frac{1}{2}x = T$$

Question 1. What is 75^l. Yearly Rent, to continue 9 Years, worth in ready Money, at 6 per Cent. &c.

Here is given $\left\{ \begin{array}{l} U = 75 \\ R = 0,06 \\ T = 9 \end{array} \right\}$ To find P , per Theorem 1.

Multiply — $0,06 = R$

By — $9 = T$

The Product $54 = TR$

Again by $9 = T$

From which $4,86 = TTR$

Take — $54 = TR$

Remains $4,32 = TTR - TR$

To which add $18,00 = 2T$

Dividend = $22,32 = TTR - TR + 2T$

Divisor = $3,08 = 2TR + 2$

B b

Then The Dividend = 22,32 = TTR - TR + 2T

186 *The Use of Decimals in Interest.*

$$\begin{array}{r}
 \text{Then } 3,08) 22,32 \text{ (} 7,24675 \\
 \underline{2156} \qquad \qquad \qquad 75 = U \\
 \cdot 760 \qquad \qquad \qquad 3623375 \\
 \underline{616} \qquad \qquad \qquad 5072725 \\
 1440 \qquad \left\{ \begin{array}{l} 543,50625 = P \\ 1232 \qquad = 543 \text{ l. } 10 \text{ s. } 1 \frac{1}{2} \text{ d.} \end{array} \right. \\
 \underline{1232} \qquad \text{Answer.} \\
 \cdot 2080 \\
 \underline{1848} \\
 \cdot 2320 \\
 \underline{2156} \\
 \cdot 1640 \\
 \underline{1540} \\
 \cdot 100
 \end{array}$$

Question 2. What *Annuity*, to continue 21 Years, will 192 l. 1 s. 5 $\frac{1}{2}$ d. purchase, at 5 per Cent.?

Here is given $\left\{ \begin{array}{l} P = 192,0731 \\ T = 21 \\ R = 0,05 \end{array} \right\}$ To find U , per *The. 2.*

$$\begin{array}{rcl}
 \text{Multiply} & - & 21 = T \\
 \text{By} & - & ,05 = R \\
 \hline
 \text{That Product} & & 1,05 = TR \\
 \text{Again by} & - & 21 = T \\
 \hline
 & & 105 = \\
 & & 210
 \end{array}$$

$$\begin{array}{rcl}
 \text{The Product is} & 22,05 = TTR \\
 \text{From which take} & 1,05 = TR \\
 \hline
 \end{array}$$

$$\begin{array}{rcl}
 \text{There remains} & 21,00 = TTR - TR \\
 \text{To which add} & 42,00 = 2T \\
 \hline
 \end{array}$$

$$\begin{array}{rcl}
 \text{The Divisor} & = & 63,00 = TTR - TR + 2T \\
 \text{Then to} & - & 1,05 = TR \\
 \text{Add Unity} & & 1,00 = 1 \\
 \hline
 \end{array}$$

$$\text{The Dividend} = 2,05 = TR + 1$$

Then

The Use of Decimals in Interest. 187

Then 63) 205 (03254

189

160

126

340

315

250

252

But — 384,1462 = 2P
Multiply by 45230,8 invert

115244

7683

1920

153

Product — 12,5 = U

The Annuity then is 12,5 l. = 12 l. 10 s. the Answer.

Note, This is a very frequent and useful Question; and ought to be work'd with great Exactness; and therefore if a Person be not very ready at, nor rightly understands the Manner of Contracted Multiplication and Division, 'twill be best to work the common Way. Which also is to be observed in all Questions of Moment.

Question 3. At what Rate of Simple Interest, will 250,3 l. or 250 l. 6 s. 8 d. purchase an Annuity of 30 l. 10 s. per Annum, to continue 10 Years?

Here is given $\left\{ \begin{array}{l} P = 250,3 \\ U = 30,5 \\ T = 10 \end{array} \right\}$ To find R, per Theorem 3.

B b 2

Multiply

$$\begin{array}{rcl}
 \text{Multiply} & - & 305 = U \\
 \text{By} & - & 10 = T \\
 \hline
 \text{The Product is} & - & 305 = TU \\
 \text{Which again mult. by} & 10 = T & \left. \begin{array}{l} \text{Differ. } 109,8 = 2P \\ - 2TU \text{ the Divi-} \\ \text{dend.} \end{array} \right\} \\
 \text{Produceth} & - & 3050 = TrU \\
 \text{From which take} & - & 305 = TU \\
 \hline
 \text{There remains} & & 2745 = TTU - TU. \\
 \text{Then} & - & 5006,8 = 2PT \\
 \hline
 \text{The Difference} & & 2261,8 = TTU - TU - 2PT \\
 \text{the Divisor.} & &
 \end{array}$$

$$\begin{array}{rcl}
 \text{Then} & 2261,8 \overline{) 109,8} & \left. \begin{array}{l} 2261 \quad 109 \\ 2035,5 \end{array} \right\} \text{See Division of Repetends.} \\
 \hline
 & 2035,5 &
 \end{array}$$

$$\begin{array}{rcl}
 & 8840 & (0,04343 = R \\
 & 8142 & \\
 \hline
 & 698 & \\
 & 610 & \\
 \hline
 & 88 & \\
 & 81 & \\
 \hline
 & 7 & \\
 & 6 &
 \end{array}$$

Then say, As 1 : 0,04343 :: 100 : 4,343 = 4 l. 6 s. 10 $\frac{1}{4}$ d. the Rate *per Cent.* sought.

Question 4. In what Time will 7 l. *per Annum* pay a Debt of 120 l. 8 s. at 6 l. *per Cent.* Or, For how long a Time may an *Annuity* of 7 l. *per Annum* be purchas'd or enjoy'd for 120 l. 8 s. at the aforesaid Rate?

$$\begin{array}{rcl}
 \text{Here is given} & \left\{ \begin{array}{l} U = 7 \\ R = 0,06 \\ P = 120,4 \end{array} \right. & \left. \begin{array}{l} \\ \\ \end{array} \right\} \text{To find } T, \text{ per Theorem 4.}
 \end{array}$$

First

First	—	$240,8 = 2P$	$\left\{ \begin{array}{l} 1,03 = \frac{1}{2}x \\ 103 = \frac{1}{2}xx \\ 9)309 = \\ \hline 344 \\ 10333 \\ \hline 1,067 = \frac{1}{4}xx \end{array} \right.$
And	—	$34,4 = \frac{2P}{U}$	
From which take	$33,2 = \frac{2}{R}$		
To the Remaind.	$1,06 = \frac{2}{R} - \frac{2P}{U}$		
Add Unity	—	$1 = + 1$	
The Sum is	$2,06 = \frac{2}{R} - \frac{2P}{U} + 1 = x$	by Substitut.	
Then	—	$1,03 = \frac{1}{2}x$	
And	—	$1,067 = \frac{1}{4}xx$	
Again	—	$0,42 = RU$	
And	—	$573,3 = \frac{2P}{RU}$	
Then	—	$574,4 = \frac{2P}{RU} + \frac{xx}{4}$	
Sq. Root of that	$23,96 = \sqrt{\frac{2P}{RU} + \frac{xx}{4}}$		
To which add	$1,03 = \frac{1}{2}x$		
The Sum is	$24,9 = 25 = T$	the Time sought.	

Having thus, in a most perspicuous Manner, shew'd the great and invaluable Service of Decimals in working Questions of Simple Interest, &c. I shall proceed to treat of the same things, in the like Manner, in Compound Interest.

Theorems resolving all Questions of Compound Interest.

Given P, R, t ; To find A ?

Theorem 1. $PR^t = A$?

Given A, R, t ; To find P ?

Theorem 2. $\left\{ \frac{A}{R^t} = P \right.$

Given $\left\{ \begin{array}{l} P, A, R, \\ P, A, t \end{array} \right\}$ to find $\left\{ \begin{array}{l} t? \\ R? \end{array} \right.$

Theorem 3. $\left\{ \frac{A}{P} = R^t \right.$

Rates.

190 The Use of Decimals in Interest.

Rates. Logarithms. Rates. Logarithms.

1,10 = 0,041392	1,055 = 0,023252
1,08 = 0,033423	1,045 = 0,019116
1,06 = 0,025305	1,04 = 0,017033
1,05 = 0,021189	1,03 = 0,012837

Question 1. What will 256 l. 10 s. amount to in 7 Years, at 6 l. per Cent. &c. Compound Interest?

Here is given $\left\{ \begin{array}{l} P = 256,5 \\ R = 1,06 \\ t = 7 \end{array} \right\}$ To find A , per Theorem 1.

Multiply the Logarithm of the Rate $1,06 = 0,025305$
By the Index of its Power (viz. $t =$) 7

The Product the Logarit. of $R^t = 1,50363 = 0,177135$
Multiply that by P — $= 256,5 = 2,409087$

The Product is the Amount $A = 385,6811 = 2,586222$
That is, 385 l. 13 s. 7 $\frac{1}{2}$ d. the Answer required.

Question 2. What Principal, or Sum of Money, will raise a Stock of 20,6 in 5 Years, at 5 l. per Cent. per Annum Compound Interest?

Here is given $\left\{ \begin{array}{l} A = 20,5 \\ t = 5 \\ R = 1,05 \end{array} \right\}$ To find P , per Theorem 2.

Multiply the Logarithm of the Rate $1,05 = 0,021189$
By the Index of its Power $t =$ — 5

The Product is the Logar. of $R^t = 1,27628 = 0,105945$
By which divide the Amount $A = 20,6 = 1,313867$
l.

The Quotient is the Principal $P = 16,1407 = 1,207922$
That is 16 l. 7 s. the Sum required.

Question 3. In what Time will 37 l. 15 s. amount to 76,65 l. (or 76 l. 13 s.) at 4 l. 10 s. per Cent?

Here is given $\left\{ \begin{array}{l} A = 76,65 \\ P = 37,75 \\ R = 1,045 \end{array} \right\}$ To find t , per Theorem 3.

Divide

Divide the Amount — $A = 76,65 = 1,884512$
 By the Principal (or Sum) — $P = 37,75 = 1,576916$
 The Quotient is — $\frac{A}{P} = R^t = 2,03046 = 0,307596$

Then 2,03046 divided by ($R =$) 1,045; and that Quotient again by 1,045; and thus continually dividing the Quotients by 1,045, 'till nothing remains, the *Number* of such *Divisions* will be equal to ($t =$) the *Time* sought. But this is sooner, and easier done by much, by *Logarithms*.

Thus, *Divide* the *Logarithm* of 2,03046 ($= R^t$) by the *Logarithm* of 1,045 ($= R$) and the Quotient is the *Time*.

0,0191116) 0,307596 (16,091 = t the *Time* sought.

19116

116436

114696

1740

1720

20

19

1

{ *Viz.* 16 Years, 1 Month,
and 5 Days, the *Ans.*

Question 4. At what *Rate* of *Compound Interest*, will 51 *l.* 15 *s.* amount to 70 *l.* 18 *s.* in 5 *Years*?

Here is given $\left\{ \begin{array}{l} P = 51,75 \\ A = 70,9 \\ t = 5 \end{array} \right\}$ To find R , per *Theorem 3.*

Divide the Amount — $A = 70,9 = 1,850646$
 By the Principal or Sum — $P = 51,75 = 1,713910$

The Quotient will be $R^t = R^5 = 1,370048 = 0,136736$

The Surfolid Root of which is $R = 1,065 = 0,027347$
l. l. l. l. l. s.

Then say, As 1 : 1,065 :: 100 : 106,5 = 106 — 10 = R the *Rate per Cent. per Annum* sought.

Note; R^5 being equal to 1,370048, of Consequence $R = \sqrt[5]{1,370048}$, which may be extracted by an *Algebraick converging Series*; the Manner of doing it, see in *Chap. 11.* of the *Use of Decimals in Algebra.*

Theo-

192 *The Use of Decimals in Interest.*

Theorems resolving all Questions relating to Annuities, &c. in Arrear, calculated at Compound Interest.

Given U, R, t ; To find A ?

Theorem 1. $\left\{ \frac{UR^t - U}{R - 1} = A \right.$

Given A, R, t ; To find U ?

Theorem 2. $\left\{ \frac{RA - A}{R^t - 1} = U \right.$

Given U, A, R ; To find t ?

Theorem 3. $\left\{ \frac{RA + U - A}{U} = R^t \right.$

Given A, U, t ; To find R ?

Theorem 4. $\left\{ \frac{A}{U} R - R^t = \frac{A - U}{U} \right.$

Question 1. If 30*l.* Yearly Rent be forborn or unpaid 9 Years; What will it amount to at the Rate of 6*l.* per Cent. &c. Compound Interest?

Here is given $\left\{ \begin{array}{l} U = 30 \\ t = 9 \\ R = 1,06 \end{array} \right\}$ To find A , per *Theorem 1.*

In the first Place, let $R = \quad \quad \quad 1,06 = 0,025305$
Be involved to the 9 Power (*viz.* R^t) $\quad \quad \quad 9$

That will be $\quad \quad \quad R^9 = 1,689451 = 0,227745$
Multiply by $\quad \quad \quad U = 30 = 1,477121$

The Product is $\quad \quad \quad UR^t = 50,683530 = 1,704866$
From that Subtract $\quad \quad \quad U = 30$

The Remainder is the Divid. $= 20,68353 = UR^t - U.$

Divide therefore $UR^t - U = 20,68353 = 1,315626$
By $\quad \quad \quad R - 1 = \quad \quad \quad 0,06 = 8,778151$

The Quotient is $\quad \quad \quad A = 1.344,7267 = 2,537475$
That is the Amount $= 344*l.* 14*s.* 6 $\frac{1}{4}$ *d.* the Answer.$

Que stion

Question 2. What Annuity 3 l. 10 s. per Cent. Compound Interest, will raise a Stock of 344 l. 5 s. being forborn 8 Years?

Here is given $\left\{ \begin{array}{l} A = 344,25 \\ R = 1,035 \\ t = 8 \end{array} \right\}$ To find U , per Theor. 2.

Multiply the Amount — $A = 344,25 = 2,536874$
By the Rate — $R = 1,035 = 0,014940$

From that Product — $RA = 356,29875 = 2,551814$
Subtract the Amount — $A = 344,25$

The Remainder is $RA - A = 12,04875$, the Divid.

Then involve — $R = 1,035 = 0,014940$
To the 8th Power — viz. $R^8 = 1,316803 = 0,119520$

That Power will be $R^8 = 1,316803 = 0,119520$
The same less Unity is $R^8 - 1 = 0,316803$, the Divisor.

Therefore divide $RA - A = 12,04875 = 1,080908$
By — $R^8 - 1 = 0,316803 = 9,500785$

The Quotient is — $U = 38,0297 = 1,580123$

The Annuity therefore which was sought, is found to be 38,0297 l. = 38 l. 0 s. 7 d. per Annum, Answer.

Question 3. In what Time will 38 l. 0 s. 7 d. raise a Stock of 344 l. 5 s. at 3 l. 10 s. per Cent. per Annum, Compound Interest?

Here is given $\left\{ \begin{array}{l} U = 38,0297 \\ A = 344,25 \\ R = 1,035 \end{array} \right\}$ To find t , per Theor. 3.

Ce

First

194 *The Use of Decimals in Interest.*

First multiply the Amount $A = 344,25 = 2,536874$
 By the given Rate $R = 1,035 = 0,014940$

To that Product $RA = 356,29875 = 2,551814$
 Add the Annuity $U = 38,0297$

From the Sum $RA + U = 394,32845$
 Take the Amount $A = 344,25$

The Remainder is the $RA + U - A = 50,07845 = 1,699651$
 Divi. which $U = 38,0297 = 1,580123$
 divide by

The Quotient will be $Rt = 1,316803 = 0,119528$

Then Divide $1,316803$ continually by the Rate $1,035$ until nothing remains, and the Number of those Divisions will be $8 = t =$ Time required.

But much better by Logarithms thus; Divide the Logarithm of the Power by the Logarithm of the Rate, the Quotient is $= t$ the Time sought.

Thus $0,014941 \div 0,119528 (8 = t \text{ the Time, viz. } 8 \text{ Years; Answer.})$
 $0,119528$

Question 4. At what Rate per Cent. Compound Interest, will $30l.$ Yearly Rent, being forborne or unpaid 9 Years, amount to $344l. 14s. 6\frac{1}{4}d.$

Here is given $\left\{ \begin{array}{l} U = 30 \\ A = 344,7267 \\ t = 9 \end{array} \right\}$ To find R , per The. 4.

First divide the Amount $A = 344,7267 = 2,537475$
 By the Annuity $U = 30 = 1,477121$

The Quotient is $\frac{A}{U} = 11,4909 = 1,060354$

Again the Amount $A - U = 314,7267 = 2,497933$
 less the Annuity is

Which divide by $U = 30 = 1,477121$

The Quotient is $\frac{A - U}{U} = 10,49014 = 1,020812$

Now $Rt = R9$. Therefore the Theorem affords this Equation, viz. $11,4909R - R^9 = 10,49014$

This

The Use of Decimals in Interest. . 195

This Equation is easily resolv'd by a *Converging Series* (which see in the *Use of Decimals in Algebra*.)

Note, This Question may be very easily and expeditiously answer'd, by the Rule of *False Position*; thus

Make two *Suppositions* of the Rate, which may include between them the Rate you seek.

Then find what the *Amounts* of the given *Annuity* would be at the two *supposed Rates* of Interest, per *Theorem 1*.

Lastly; Observe the *Errors* of those Amount from the Amount here given, then by those *Suppositions* and their *Errors*, find the true Rate, (*viz.* 1,06) as is there taught.

Theorems resolving all Questions concerning the Present Worth of Annuities, Pensions, or Leases in Reversion, at Compound Interest.

Given U, R, t ; To find P ?

$$\text{Theorem 1. } \left\{ \frac{U - \frac{U}{R^t}}{R - 1} = P. \right.$$

Given P, R, t ; To find U ?

$$\text{Theorem 2. } \left\{ \frac{PR^t \times R - PR^t}{R^t - 1} = U. \right.$$

Given U, R, P ; To find t ?

$$\text{Theorem 3. } \left\{ \frac{U}{P + U - PR} = R^t. \right.$$

Given U, P, t ; To find R ?

$$\text{Theorem 4. } \left\{ \frac{U}{P} = \frac{U}{P} R^t + R^t - R^{t+1}. \right.$$

Question 1. What is 30 l. Yearly Rent, Worth in ready Money, for its Continuance 7 Years, allowing 6 l. per Cent. Compound Interest to the Purchaser?

$$\text{Here is given } \left\{ \begin{array}{l} U = 30 \\ R = 1,06 \\ t = 7 \end{array} \right\} \text{ To find } P, \text{ per Theorem 1.}$$

C c 2

First

196. *The Use of Decimals in Interest.*

First, involve $R = 1,06 = 0,025305$
 To the 7th Power, (viz. R^7) 7
 That will be $R^7 = 1,50061 = 0,177135$
 Then divide $U = 30 = 1,477121$
 By R^7 , there will remain $\frac{U}{R^7} = 19,9520 = 1,299986$
 Then from the Annuity $U = 30$
 Subtract $\frac{U}{R^7} = 19,952$
 Remains the Dividend $U - \frac{U}{R^7} = 10,048 = 1,002079$
 Which divide by the Rate less $\frac{U - \frac{U}{R^7}}{R - 1} = 0,06 = 8,778151$
 Unity $= R - 1$
 The Quotient is the present Worth $= P = 167,4716 = 2,223928$
 The Present Worth, In ready Money is 167*l.* 9*s.* 5*d.* the Answer.

N. B. Suppose this were an *Annuity in Reversion*, or not to be entered on till after 7 Years are past, and thence to continue 7 Years; and you would know the *present Worth*; find by the second Theorem of *Compound Interest*, what *ready Money* will amount to 167*l.* 9*s.* 5*d.* in 7 Years, at the *same Rate* of Interest; and that will be its *present Worth*; and so for any other *Annuity in Reversion*.

Question 2. What *Annuity*, to continue 7 Years, may be purchased for 120*l.* 5*s.* at 6 per Cent. *Compound Interest*?

Here is given $\left\{ \begin{array}{l} P = 120,25 \\ R = 1,06 \\ t = 7 \end{array} \right\}$ To find U , per Theor. 2.

Involve the Rate $R = 1,06 = 0,025305$
 To the Index of its Power (viz. $= t$) 7

The Power of R will be $R^7 = 1,50361 = 0,177135$
 Which mult. by the present Worth $P = 120,25 = 2,08084$

The Product is $PR^7 = 180,8087 = 2,257219$
 Multiply that by the Rate $R = 1,06 = 0,025305$

That Product is $PR^7 \times R = 191,65722 = 2,282524$
 From which subtract $PR^7 = 180,8087$

There Remains the Dividend $10,84852 = PR^7 \times R$
 Divide $- PR^7$

Divide therefore $PR^t \times R - PR^t = 10,84852 = 1,035369$

By the Power of R less 1 $= R^t - 1 = 0,50361 = 9,702113$

The Quotient is the Annuity $U = 21,54057 = 1,333256$

The Annuity sought therefore is $21,54057l. = 21l.$

$10s. 9\frac{1}{2}d.$

Question 3. For what Time will $167l. 9s. 5d.$ purchase an Annuity of $30l.$ per Annum, at 6 per Cent. Compound Interest?

Here is given $\left\{ \begin{array}{l} P = 167,4716 \\ U = 30 \\ R = 1,06 \end{array} \right\}$ To find t , per Theor. 3.

To the present Worth $P = 167,4716$

Add the Annuity $U = 30$

The Sum is $P + U = 197,4716$

Then mult. the present Worth $P = 167,4716 = 2,223928$

By the Rate $R = 1,06 = 0,025305$

The Product is $PR = 177,5199 = 2,249233$

Which subst. from Sum of $P + U = 197,4716$

The Remainder is $P + U - PR = 19,9517$ the Divisor.

Then Divide the Annuity $U = 30 = 1,477121$

By the Divisor $P + U - PR = 19,9517 = 1,299986$

The Quotient is $R^t = 1,50361 = 0,177135$

Lastly, Divide the Logarithm of R^t , by the Logarithm of the Rate; the Quotient will be the Time $= t$ sought.

Thus, $0,025305) 0,177135$ ($7 = t$ the Time sought,)
 $0,177135$ viz. 7 Years. Answer, }
 }

Question 4. Suppose I purchase an Annuity of $21l. 10s. 9\frac{1}{2}d.$ to continue 7 Years, for $120l. 5s.$ ready Money; at what Rate per Cent. Compound Interest, was the Purchase made?

Here is given $\left\{ \begin{array}{l} P = 120,25 \\ U = 21,54057 \\ t = 7 \end{array} \right\}$ To find R , per Theor. 4.

First,

198 *The Use of Decimals in Interest.*

First, divide the Annuity — $U = 21,54057 = 1,333256$
 By the present Worth — $P = 120,25 = 2,080084$

The Quotient is — $\frac{U}{P} = 0,17915 = 9,253172$

Then multiply it into the given Power of the Rate, to which add the Power, &c. as per Theorem; and you have this Equation, viz. $0,17915R^7 + R^7 - R^8 = 0,17915$; whence a Person ready at *Algebraick Extractions*, may soon discover $R = 1,06$. Then say, as 1 l. : 0,06 l. :: 100 l. : 6 l. the Rate per Cent. requir'd.

Note; This (and all Questions of this Nature) may be answered by the Rule of Position; in the same manner as was directed in the fourth Question of *Annuities in Arrears*.

Theorems resolving all Questions relating to the Purchasing of Free-hold or Real Estates, at Compound Interest.

Given P, R ; To find U ?

Theorem 1. $P R - P = U$.

Given U, R ; To find P ?

Theorem 2. $\frac{U}{R - 1} = P$.

Given P, U ; To find R ?

Theorem 3. $\frac{P + U}{P} = R$.

Question 1. Suppose a *Free-hold* Estate of 25 l. per Annum were to be sold; What is the Worth, allowing 5 l. 10 s. per Cent. &c. Compound Interest to the Buyer?

Here is given $\left\{ \begin{array}{l} U = 25 \\ R = 1,055 \end{array} \right\}$ To find P , per Theor. 2.

Divide the Annual Rent — $U = 25 = 1,397940$
 By the Rate less Unity $R - 1 = 0,055 = 8,740362$

The Quotient is the Worth $P = 454,5 = 2,657578$

The Value of that Estate therefore is 454 l. 10 s. 10 $\frac{3}{4}$ d.

Q. E. I.

Question

Question 2. Suppose a Person would lay out 418 l. 13 s. 4 d. on a *Free-hold Estate*, and so as to be allowed 6 l. per Cent. for his Money, *Compound Interest*; What must be the *Annual Rent* of such an Estate?

Here is given $\left\{ \begin{array}{l} P = 418,6 \\ R = 1,06 \end{array} \right\}$ To find U , per *Theor. 1.*

Multiply the present Worth — $P = 418,6 = 2,619789$
By the Rate — $R = 1,06 = 0,025305$

The Product is — $PR = 441,8 = 2,645094$
From which subtract the Worth $P = 418,6$

There remains the Annual Rent } $U = 25 l.$ per Annum.
The Answer

Question 3. Suppose one give 416 l. 13 s. 4 d. for a *Free-hold Estate* of 25 l. per Annum; What Rate per Cent. *Compound Interest*, has the Purchaser for his Money?

Here is given $\left\{ \begin{array}{l} P = 416,6 \\ U = 25 \end{array} \right\}$ To find R , per *Theor. 3.*

To the present Worth — $P = 416,6$

Add the Annual Rent — $U = 25,0$

Divide their Sum — $P + U = 441,6 = 2,645094$

By the present Worth — $P = 416,6 = 2,619789$

The Quotient is the Rate sought $R = 1,06 = 0,025305$

Then say, As 1 l. : ,06 l. :: 100 l. : 6 l. per Cent. the Answer.

Rebate or Discount.

What this is I have already defin'd in the Beginning of this Chapter; The *Interest*, and *Discount*, of the same Parcel of Money, is very different, tho' vulgarly understood (and accordingly is reckoned) the same thing.

In order therefore to have a right Notion of *Discount*, and how it differs from *Interest*; we must consider, that *Interest* is the Increase of any *Principal*, or Sum of Money, according to any Rate, or Proportion, agreed on; and in computing it, we have Regard only to the bare *Principal*; But what is properly call'd *Discount*, is the Difference between a Sum of Money due any Time hence, and such another Sum as, being put to *Interest*, would, with its Increase by *Interest*, become equal to the said Sum hereafter due.

Thus,

200 *The Use of Decimals in Interest.*

Thus, for *Example*, If I have 105 *l.* due to me 12 Months hence; the *Discount* for *prompt Payment* thereof at 5 *l. per Cent. Simple Interest* must be 5 *l.* and the *present Money*, or *Worth* of that 105 *l.* is 100 *l.*; Because if I put 100 *l.* out at the *aforesaid Rate*, it would in *that Time* be equal or amount to 105 *l.* Wherefore the *Interest* of 105 *l.* discounted (as is the common way) I should receive but 99 *l.* 15 *s.*; the *Interest* of the 105 *l.* being 5 *s.* more than the true *Discount*; and consequently the reckoning *Interest* for *Discount* is very disadvantageous to those who make such *Discounts*.

The Proportion for *Rebate* or *Discount* then is,

As 100 *l.* and the *Rate* : is to the *Rate* :: so is any other Sum : to its true *Discount* for the same *Time*.

The *Theorem* for finding at once both the *Discount* and *present Worth* of any Sum of *Money*, due any time hereafter, is the second *Theorem* of *Simple* and *Compound Interest*, as I there observ'd.

By the second *Theorem* of *Simple Interest*, it was found that the *present Worth* of 405 *l.* 6 *s.* 0 *d.* due 5 Years and 8 Months hence at 5 *per Cent.*

	<i>l.</i>	<i>s.</i>	<i>d.</i>
Must be	—	—	—
Which subtracted from the <i>Debt</i> ,	—	—	—
leaves the true <i>Discount</i>	—	—	—
But the <i>Interest</i> of that Sum is	—	—	—
for that Time	—	—	—
Which exceeds the true <i>Discount</i> by	—	—	—

Above a fourth Part loss to the *Discounter* of *Interest* for such a Sum.

Note. This *Theorem* of Mr. *Ward's*, is far more easy, concise, and elegant, than any other extant, for finding the *present Worth*, or *Discount* for *prompt Payment* of any *Debt*.

A Table of Days for any given Time.

Days	January	February	March	April	May	June	July	August	September	October	November	December
1	1	32	60	91	121	152	182	213	244	274	305	335
2	2	33	61	92	122	153	183	214	245	275	306	336
3	3	34	62	93	123	154	184	215	246	276	307	337
4	4	35	63	94	124	155	185	216	247	277	308	338
5	5	36	64	95	125	156	186	217	248	278	309	339
6	6	37	65	96	126	157	187	218	249	279	310	340
7	7	38	66	97	127	158	188	219	250	280	311	341
8	8	39	67	98	128	159	189	220	251	281	312	342
9	9	40	68	99	129	160	190	221	252	282	313	343
10	10	41	69	100	130	161	191	222	253	283	314	344
11	11	42	70	101	131	162	192	223	254	284	315	345
12	12	43	71	102	132	163	193	224	255	285	316	346
13	13	44	72	103	133	164	194	225	256	286	317	347
14	14	45	73	104	134	165	195	226	257	287	318	348
15	15	46	74	105	135	166	196	227	258	288	319	349
16	16	47	75	106	136	167	197	228	259	289	320	350
17	17	48	76	107	137	168	198	229	260	290	321	351
18	18	49	77	108	138	169	199	230	261	291	322	352
19	19	50	78	109	139	170	200	231	262	292	323	353
20	20	51	79	110	140	171	201	232	263	293	324	354
21	21	52	80	111	141	172	202	233	264	294	325	355
22	22	53	81	112	142	173	203	234	265	295	326	356
23	23	54	82	113	143	174	204	235	266	296	327	357
24	24	55	83	114	144	175	205	236	267	297	328	358
25	25	56	84	115	145	176	206	237	268	298	329	359
26	26	57	85	116	146	177	207	238	269	299	330	360
27	27	58	86	117	147	178	208	239	270	300	331	361
28	28	59	87	118	148	179	209	240	271	301	332	362
29	29		88	119	149	180	210	241	272	302	333	363
30	30		89	120	150	181	211	242	273	303	334	364
31	31		90		151		212	243		304		365

The Use of the Table.

First; To know the *Number of Days* from the *Beginning* of the *Year*, to any given *Day* of any *Month*.

D d

This

202 *The Use of the preceeding Table.*

This is obtain'd by Inspection only ; Thus from *January* the 1st, to *September* the 7th, is 250 Days ; To *November* the 27th are 331, &c.

Secondly, To know what is the *Number of Days* from any given *Day* of any *Month*, to the *End* of the *Year*.

Suppose *September* the seventh, then from — 365

Subtract the Number answering to *Sept. 7* — 250

There remains the Number of Days sought, viz. 115 Days.

Thirdly, To find the *Number of Days* between the given *Day* of any one *Month*, and any given *Day* of any other *Month*, in the same *Year*.

For Instance, To know how many *Days* there are between *April* the 17th, and *October* 23.

Thus, From the Number answering to *October* 23 — 296

Subtract that answering to *April* 17 — 107

The *Remainder* is the Number of Days sought — 189

Fourthly, To find the *Number of Days*, from any given *Day* of any *Month* in one *Year*, to any given *Day* of any *Month* in the next *Year*.

How many Days is it from *September* the 7th, 1733, to *April* the 19th, 1734?

From the Days of a Whole Year — 365

Subtract the Number to *September* 7 — 250

Remains the Number to the *End* of the *Year* — 115

To which *add* the Number to *April* 19 — 109

The *Sum* is the *Number of Days* required — 224

And thus is the *Number of Days* readily found for any *Interval* of *Time* given, in the same *Year* compleatly ; or which is part of one, or part of another *Year*.

How very *necessary* and *useful* a *Table* this is in all *Parts* of *Arithmetical Science* relating to *Time* is sufficiently evident to the *Skilful* therein ; but because it is more *particularly* so in the whole *Affair* of *Interest*, I have therefore prefixed it to the other *Tables*.

Having then the *Number of Days*, 'tis easy to find what *Decimal Part* of the *Year*, they make ; and having found that, you have the *T, t*, in the foregoing *Theorems* representing any *Part* of a *Year*. An

An Example in *Simple and Compound Interest*, will make the whole Matter easy and conspicuous.

Example 1. What will 65 *l.* amount to, being lent from *March* the 7th to *November* the 3d, at 5 *l.* per Cent. per *Annum Simple Interest*?

From *March* the 7th to *November* the 3d are 241 Days; those make 8 *Months*, 2 *Weeks*, and 3 *Days*, = 660273 *Decimals* of a Year. Then by *Theorem 1.*

Multiply the *Time* — — $T = 0,660273$

By the *Ratio* of the *Rate* — — $R = 0,05$

And that *Product* — — $TR = 0,03301365$

Multiply by the *Principal* — — $P = 65$

The *Product* is — — $TRP = 2,14588725$

To which add the *Principal* $P = 65$

The *Sum* is the *Amount* sought = 67,1458 &c. *l.*

Example 2. What is the *Amount* thereof at *Compound Interest*, the *Rate* and *Time* being the same?

The *Logarithm* of the *Rate* $R = 1,05 = 0,0211893$

Multiply by the *Time* — — $t = ,6603$

The *Product* is the *Logar.* of $R^t = R^{0,6603} = 0,0139912$

To which add the *Log.* of the *Prin.* $P = 63 = 1,8129133$

The *Sum* is the *Log.* of *Amount* $A = 67,1281 = 1,8269045$

And thus the *Theorems* serve to answer *Questions*, when the *Time* is only *part* of a Year, as well as when *complete* Years.

Proem to the Tables of Simple Interest, concerning their Nature, Construction, and Use.

The great Design of *Tables of Interest* (both *Simple* and *Compound*) is *Ease* and *Expedition* in practical *Calculations*. For, besides that the *Rules* expressed in *Words* for answering *Questions of Interest* are tedious and intricate, and the *Reason* no ways to be understood; the *Operations* themselves are, for the most part, very laborious; and consequently *Tables* which expedite and facilitate the *Practice* are indispensibly necessary.

204 The Nature, Construction, and Use

This being undeniably evident, the Question occurs, Whether *these Tables* are to be made in *Decimal* or *mixed Numbers* (i. e. such as express the *Money* in its common Denominations of *Pounds, Shillings, and Pence*)? The Answer to this can admit of no *Demurr* amongst those who understand the Doctrine of *Decimal Numbers*; they all know the *Excellency* and superiour Use of the *first Sort*, viz. *Decimal Tables*. But *Interest Tables* expressed in *common Money* are indulged to those who understand not *Decimals*, as *Crutches* to the *Lame*, and *Spectacles* to the *Weak-sighted*.

The *Numbers* in the *first* of these *Tables* of *Simple Interest* for *Days*, and in the *Second* for *Years*, being in *Arithmetical Proportion*, makes them capable of that *Perfection*, which no other *Tables* can pretend to.

These *Tables* are so contrived, That the *Interest* of any *Principal Sum* is easily found for any *Number* of *Days* or *Years* at any *Rate* from *one Pound* to *Ten*, with the *Halves* and *Quarters*. Having followed herein the *Rev. Mr. Brown* in his *Arithmetica Infinita*.

The *Construction* of these *Tables* is easy from the *Theorems* themselves, (and indeed the *Reason* of their *Construction* can be no otherways so easily conceiv'd.) Thus by *Theorem* the *first* of *Simple Interest*, viz. $tRP + P = A$ is the *first* and *second Table* constructed. For since the *Amount* less the *Principal*, is equal to the *Interest*, therefore the *Theorem* will be $tRP = \text{Interest}$. Now if $P = 1 \text{ l. } t = .002739 \text{ \text{£c.}}$ (the *Decimal* of a *Year* for *one Day*), and $R = \text{any Ratio of Interest}$, suppose *5 per Cent.*; then the *Simple Interest* of *one Pound* for *one Day*, at *5 per Cent.* is $.002739 \text{ \text{£c.}}$ $\times .05 \times 1 = .0013698 \text{ \text{£c.}}$ which being multiplied by the *nine Digits* severally constitute that part of the *Table* of *Interest* at *5 per Cent.* and thus the whole *first Table* is made. The *second Table* for *Years* is only the various *Ratio's* of *Interest* multiplied by the said *Nine Digits*; for since $t = 1 \text{ Year}$, and $P = 1 \text{ l.}$ it will be $tRP = R$ the *Interest* for the *first Year*, &c.

The *third Table* shews the *Rebate* or *Discount* to be made for *one Pound*, at the several *Rates per Cent.* for *Days*. The *Manner, Truth, and Reason* of its *Construction* is derived from *Theorem 2. of Simple Interest*, viz. $\frac{A}{tR - 1} = P$. For since the *Principal* or *present Worth* subducted from the *Amount* gives the *Rebate* or *Discount* of that *Amount*; therefore

fore the *Discount* of any *Amount* for any *Time* at any *Rate* (without *Regard* of the *present Value* or *principal Money*)

may be found by this *Theorem* $\frac{AtR}{tR + 1} = D = \text{Discount.}$

Hence if we put $A = 1 \text{ l.}$ $t = ,002739 \text{ \&c.}$ and $R =$ any *Ratio* of *Interest*, suppose 5 per Cent. then by this last *Theorem* we have the *Discount* of one *Pound* for one *Day* at the *Rate* of $5 \text{ per Cent. per Annum}$; For $AtR = 1 \times ,002739 \text{ \&c.} \times ,05 = ,00013698 \text{ \&c.}$ And $tR + 1 = 1,00013698 \text{ \&c.}$ then by *Division*; $,00013698 \text{ \&c.} \div 1,00013698 \text{ \&c.} (= ,00013697 \text{ \&c.})$ the *Discount*. If $t = 1 \text{ Year}$; then the *Annual Discount* of one *Pound* at 5 per Cent. will be found, by the above *Theorem*, thus; $AtR = ,05$ and $tR + 1 = 1,05$. Therefore by *Division*, $,05 \div 1,05 (= ,04761904 \text{ \&c.})$ the *Discount*. And thus is the *Discount* of any *Sum* at any *Rate* for any *Time* above one *Year* found at once by the above *Theorem*; and for any *Time* under a *Year* by the *Table of Discount for Days*, of which I have now taught the *Construction* in a *new* and *more rational Method* than any I have yet seen.

The Use of Table I, and II.

In order to understand how to make those two *Tables* universally usefull, the *Reader* is to observe, that if a *Number* consists of only one *Digit* with *Cyphers* affixed, as $10, 50, 700, 9000, 800000, \text{\&c.}$ 'tis called a *pure Number*; but those *Numbers* which consist of more than one, or wholly of *Digits*, As $370, 568, 7569, \text{\&c.}$ may be called *Mixed Numbers*. Now every *mixed Number* may be resolved into those *pure Numbers*, of which they are composed; thus the *mixed Number* 507 , may be resolved into the *Pure Numbers* $500, 60,$ and 7 ; so also 15890 is resolved into $10000, 5000, 800,$ and 90 .

Now then as to the *Use* of the *Tables*, observe these *Rules*;

I. If the *Number* of *Days, Years, \&c.* proposed, be a *mixed Number*, let it be resolved into *pure Numbers*.

II. With the *pure Numbers* severally enter the *Tables*, and take those *Decimal Numbers* which stand against the first *Figure* of each *pure Number*, in the *Column* marked *Numbers*.

III. Remove the *Decimal Point* in each such *Decimal Number*.

206 *The Nature, Construction, and Use*

Number, so many Places to the *Right-hand*, as there are *Cyphers* in the respective *pure* Numbers.

IV. Lastly, Add together all the *Decimal Numbers*, and find the *Value* thereof by the *Tables* for that purpose.

These things *premised*, the *Use* of the *Tables* will be obvious from the *Examples* of the following *Problems*.

Problem 1.

To find the *Interest* of any *Sum* of *Money* for a *Day*, or a *Year*, at any *Rate per Cent. per Annum*.

Example 1.

What is the *Interest* of 2746 *l.* at 5 *l.* 15 *s.* *per Cent.* for a *Day*?

			<i>Decimals.</i>
In Table 1. under	}	2000 —	,31506
the Rate 5 $\frac{3}{4}$. You		700 —	,11027
find against the <i>pure</i>		40 —	,00630
Numbers		6 —	,00094
The Answer is	—	—	,43257 = 8 <i>s.</i> 7 $\frac{3}{4}$ <i>d.</i>

Example 2.

What is the *Interest* of the same *Sum*, at the same *Rate* for a *Year*?

			<i>Decimals.</i>	
In Table 2.	}	2000 —	150,00000	} Under 5 $\frac{3}{4}$ <i>d.</i> <i>per Cent.</i>
You find against		700 —	40,25000	
		40 —	2,30000	
		6 —	0,34500	

The Answer in *Decimals* *l.* 192,895
Which is in *Money* = 192 *l.* 17 *s.* 10 $\frac{3}{4}$ *d.*

Problem 2.

To find the *Interest* of any *Sum* of *Money* for any *Num.* ber of *Days*.

Example.

What is the *Interest* of 265 *l.* for 149 *Days*, at the *Rate* of 3 *l.* 15 *s.* *per Cent.* &c.

Multi-

of Decimal Tables of Simple Interest. 207

Multiply the *Principal Sum* — 265 l.
By the given Number of *Days* — 149

The Product is the *mixed Number* 39485, with which resolved, enter the Table as before :

		<i>Decimals.</i>	
Thus in Table 1.	{ 30000 —	3,08200 }	
You find against	{ 9000 —	0,92466 }	Under 3 $\frac{1}{2}$ per Cent.
the <i>pure Num-</i>	{ 400 —	0,04109 }	
bers	{ 80 —	0,00822 }	
	{ 5 —	0,00051 }	

The Answer in *Decimals* — l. 4,05648

In *Money* 4 l. 1 s. 1 $\frac{1}{2}$ d.

The Method is the same for any *greater* Number of Days.

Problem. 3.

To find the *Interest* of any *Sum* forborne any Number of *Years* at any of the given *Rates per Cent.*

Example.

What is the *Interest* of 175 l. 15 s. forborne 13 *Years* at the *Rate* of 6 *per Cent.* &c.?

Multiply the <i>Principal Sum</i>	—	175,75
By the Number of <i>Years</i> given	—	13
The Product is the <i>mixed Number</i>	—	2284,75

Which resolved, as before, will stand thus,

	{ 2000, —	120,000 }	
In Table 2.	{ 200, —	12,000 }	
You find against	{ 80, —	4,800 }	Under 6 <i>per</i> <i>Cent.</i>
	{ 4, —	0,240 }	
	{ ,7 —	0,042 }	
	{ ,05 —	0,003 }	

The Answer in *Decimals* l. 137,085

The same in *Money* 137 l. 1 s. 8 d $\frac{1}{4}$.

N. B. The

208 *The Nature, Construction, and Use*

N. B. The Reader must observe, in resolving a *mixed* Number wherein are *Decimals*, to remove the Point one Place more to the *left* than are the *Number* of *Cyphers* in the *Decimal pure* Number, as in the *last* Example.

The Use of Table III. Of Discount.

In seeking the *Discount* for any *Sum* due at the End of any Number of *Days*, if the Number of *Days* be a *mixed* one, resolve them into *pure* Numbers as before taught; and even with them in the Table take the *Discount* of *1 l.* which add and multiply by the *Principal* Sum, the *Product* will be the *Discount* thereof.

Problem 4.

To find the *Discount* of any *Sum*, for any Number of *Days*, at any given *Rate* in the Table.

Example.

What is the *Discount* of 83 Pounds, 10 Shillings, for 235 Days at 4 per Cent. per Annum?

		<i>Decimals.</i>			
You find	200	—	,0214478	} Under 4 per Cent.	
In the Table	30	—	,0032769		
even with	5	—	,0005476		
The Sum is		—	,0252723		
Which multiplied by the Sum		835			
The Product is the Answer		2,110237	Gr.	= 2—2—2 $\frac{1}{4}$	

Problem 5.

To find the *Discount* of any *Sum* for a *Year*.

Example.

What is the *Discount* of 100 l. for one *Year*, at 5 per Cent?

In

In the Table under 5 per Cent. and against 365 Days is } ,047619 &c.

Which mul. by the Principal Sum 100

The Product is the Answer $\underline{\text{l. } 4,7619 \text{ &c.}} = 4-15-2\frac{1}{4}$

Now the Interest of 100 l. for one Year, at 5 per Cent. is $\underline{\text{5-0-0}}$

The Differ. therefore of *Discount* and *Interest*, is $\underline{\text{0-4-9}\frac{1}{4}}$

Whence 'tis evident, he who allows *Interest* for *Discount* wrongs himself considerably, which yet is very common among *Traders*; for so much Money ought to be paid, as, at *Interest*, would amount to the *Sum* due, in the *Time* proposed.

Example 2.

What is the *Discount* of 9342 l. at $4\frac{1}{2}$ per Cent. for a Year?

The *Discount* of 1 l. for 365 Days, at $4\frac{1}{2}$ per Cent. in the Table, is } ,043062, &c.

Which multiplied by the Principal Sum $\underline{9342}$

The Product is the Answer $\underline{\text{l. } 402,2852 \text{ &c.}}$

In Money 402 l. 5 s. 8 $\frac{1}{2}$. And thus proceed for other annual *Discounts*.

I must acknowledge this Table of *Discount* gives not the precise Truth, and yet differs but little from it; being sufficiently *exact* for any Use. None but a Table of the *Discount* for every Day, can be perfect; because every Day's *Discount* differs, being still less as the Number of Days increase.

This Table is perfectly true for all the Days express'd therein, and, as I said, may be used without much Error for any other.

			l.	s.	d.
Example in Prob. 4.	{	The true <i>Discount</i> is	—	2	1—11
		The <i>Discount</i> by this Table		2	2—2 $\frac{1}{4}$
		The <i>Int.</i> for the Time and Rate		2	3—0

TABLES of Simple Interest.

Table I. *The Interest of one Pound for Days.*

Table II. *The Interest of one Pound for Years.*

Both at any Rate per Cent. from one to ten Pounds with Halves and Quarters.

Table I. *The Interest of one Pound per Diem.*

Numb.	1 per Cent.	1 $\frac{1}{4}$ per C.	1 $\frac{1}{2}$ per C.	1 $\frac{3}{4}$ per C.
1	,00002740	,00003425	,00004110	,00004794
2	,00005480	,00006850	,00008220	,00009589
3	,00008220	,00010274	,00012329	,00014383
4	,00010959	,00013699	,00016438	,00019178
5	,00013698	,00017123	,00020548	,00023972
6	,00016438	,00020548	,00024657	,00028767
7	,00019178	,00023973	,00028767	,00033562
8	,00021918	,00027398	,00032877	,00038356
9	,00024657	,00030822	,00036986	,00043151
Month.		,00104166	,00125000	,00145833

Table II. *The Interest of one Pound per Annum.*

Numb.	1 per Cent.	1 $\frac{1}{4}$ per C.	1 $\frac{1}{2}$ per C.	1 $\frac{3}{4}$ per C.
1	0,01000000	0,01250000	0,01500000	0,01750000
2	0,02000000	0,02500000	0,03000000	0,03500000
3	0,03000000	0,03750000	0,04500000	0,05250000
4	0,04000000	0,05000000	0,06000000	0,07000000
5	0,05000000	0,06250000	0,07500000	0,08750000
6	0,06000000	0,07500000	0,09000000	0,10500000
7	0,07000000	0,08750000	0,10500000	0,12250000
8	0,08000000	0,10000000	0,12000000	0,14000000
9	0,09000000	0,11250000	0,13500000	0,15750000

Table

Decimal Tables of Simple Interest. 211

Table I. *The Interest of one Pound per Diem.*

Numb.	2 per C.	2 $\frac{1}{4}$ per C.	2 $\frac{1}{2}$ per C.	2 $\frac{3}{4}$ per C.
1	,00005480	,00006164	,00006849	,00007534
2	,00010959	,00012329	,00013699	,00015068
3	,00016438	,00018493	,00020547	,00022602
4	,00021918	,00024657	,00027397	,00030137
5	,00027397	,00030822	,00034246	,00037671
6	,00032876	,00036986	,00041095	,00045205
7	,00038356	,00043151	,00047945	,00052739
8	,00043835	,00049315	,00054794	,00060274
9	,00049315	,00055479	,00061644	,00067808
Month	,00186666	,00187500	,00208222	,00229186

Table II. *The Interest of one Pound per Annum.*

Numb.	2 per Cent.	2 $\frac{1}{4}$ per C.	2 $\frac{1}{2}$ per C.	2 $\frac{3}{4}$ per C.
1	0,02000000	0,02250000	0,02500000	0,02750000
2	0,04000000	0,04500000	0,05000000	0,05500000
3	0,06000000	0,06750000	0,07500000	0,08250000
4	0,08000000	0,09000000	0,10000000	0,11000000
5	0,10000000	0,11250000	0,12500000	0,13750000
6	0,12000000	0,13500000	0,15000000	0,16500000
7	0,14000000	0,15750000	0,17500000	0,19250000
8	0,16000000	0,18000000	0,20000000	0,22000000
9	0,18000000	0,20250000	0,22500000	0,24750000

Table I. *The Interest of one Pound per Diem.*

Numb.	3 per Cent.	3 $\frac{1}{4}$ per C.	3 $\frac{1}{2}$ per C.	3 $\frac{3}{4}$ per C.
1	,00008220	,00008904	,00009589	,00010274
2	,00016438	,00017808	,00019178	,00020548
3	,00024657	,00026712	,00028767	,00030822
4	,00032877	,00035616	,00038356	,00041096
5	,00041096	,00044520	,00047945	,00051363
6	,00049315	,00053424	,00057534	,00061644
7	,00057534	,00062328	,00067123	,00071917
8	,00065753	,00071232	,00076712	,00082192
9	,00073972	,00080137	,00086301	,00092465
Month	,00250000	,00270833	,00291666	,00312500

212 *Decimal Tables of Simple Interest.*

Table II. *The Interest of one Pound per Annum.*

<i>Numt.</i>	<i>3 per Cent.</i>	<i>3 $\frac{1}{4}$ per C.</i>	<i>3 $\frac{1}{2}$ per C.</i>	<i>3 $\frac{3}{4}$ per C.</i>
1	0,03000000	0,03250000	0,03500000	0,03750000
2	0,06000000	0,06500000	0,07000000	0,07500000
3	0,09000000	0,09750000	0,10500000	0,11250000
4	0,12000000	0,13000000	0,14000000	0,15000000
5	0,15000000	0,16250000	0,17500000	0,18750000
6	0,18000000	0,19500000	0,21000000	0,22500000
7	0,21000000	0,22750000	0,24500000	0,26250000
8	0,24000000	0,26000000	0,28000000	0,30000000
9	0,27000000	0,29000000	0,31500000	0,33750000

Table I. *The Interest of one Pound per Diem.*

<i>Days.</i>	<i>4 per Cent.</i>	<i>4 $\frac{1}{4}$ per C.</i>	<i>4 $\frac{1}{2}$ per C.</i>	<i>4 $\frac{3}{4}$ per C.</i>
1	,00010959	,00011644	,00012329	,00013014
2	,00021918	,00023288	,00024657	,00026027
3	,00032877	,00034931	,00036986	,00039041
4	,00043836	,00046575	,00049315	,00052055
5	,00054794	,00058219	,00061643	,00065068
6	,00065753	,00069863	,00073973	,00078082
7	,00076712	,00081507	,00086301	,00091096
8	,00087671	,00093151	,00098630	,00104109
9	,00098630	,00104794	,00110959	,00117123
<i>Month.</i>	,00333333	,00354166	,00375000	,00395833

Table II. *The Interest of one Pound per Annum.*

<i>Years.</i>	<i>4 per Cent.</i>	<i>4 $\frac{1}{4}$ per C.</i>	<i>4 $\frac{1}{2}$ per C.</i>	<i>4 $\frac{3}{4}$ per C.</i>
1	0,04000000	0,04250000	0,04500000	0,04750000
2	0,08000000	0,08500000	0,09000000	0,09500000
3	0,12000000	0,12750000	0,13500000	0,14250000
4	0,16000000	0,17000000	0,18000000	0,19000000
5	0,20000000	0,21250000	0,22500000	0,23750000
6	0,24000000	0,25500000	0,27000000	0,28500000
7	0,28000000	0,29750000	0,31500000	0,33250000
8	0,32000000	0,34000000	0,36000000	0,38000000
9	0,36000000	0,38250000	0,40500000	0,42750000

Table

Decimal Tables of Simple Interest. 213

Table I. *The Interest of one Pound per Diem.*

<i>Days.</i>	<i>5 per Cent.</i>	<i>5 $\frac{1}{4}$ per C.</i>	<i>5 $\frac{1}{2}$ per C.</i>	<i>5 $\frac{3}{4}$ per C.</i>
1	,00013698	,00014383	,00015068	,00015753
2	,00027397	,00028767	,00030137	,00031507
3	,00041096	,00043151	,00045205	,00047260
4	,00054794	,00057534	,00060274	,00063014
5	,00068493	,00071918	,00075342	,00078767
6	,00082192	,00086301	,00090411	,00094520
7	,00095890	,00100685	,00105479	,00110274
8	,00109589	,00115068	,00120548	,00126027
9	,00123288	,00129452	,00135616	,00141781
<i>Month.</i>	,00418666	,00437500	,00458333	,00479166

Table II. *The Interest of one Pound per Annum.*

<i>Years.</i>	<i>5 per Cent.</i>	<i>5 $\frac{1}{4}$ per C.</i>	<i>5 $\frac{1}{2}$ per C.</i>	<i>5 $\frac{3}{4}$ per C.</i>
1	0,05000000	0,05250000	0,05500000	0,05750000
2	0,10000000	0,10500000	0,11000000	0,11500000
3	0,15000000	0,15750000	0,16500000	0,17250000
4	0,20000000	0,21000000	0,22000000	0,23000000
5	0,25000000	0,26250000	0,27500000	0,28750000
6	0,30000000	0,31500000	0,33000000	0,34500000
7	0,35000000	0,36750000	0,38500000	0,40250000
8	0,40000000	0,42000000	0,44000000	0,46000000
9	0,45000000	0,47250000	0,49500000	0,51750000

Table I. *The Interest of one Pound per Diem.*

<i>Days.</i>	<i>6 per Cent.</i>	<i>6 $\frac{1}{4}$ per C.</i>	<i>6 $\frac{1}{2}$ per C.</i>	<i>6 $\frac{3}{4}$ per C.</i>
1	,00016438	,00017123	,00017808	,00018493
2	,00032876	,00034246	,00035616	,00036986
3	,00049315	,00051370	,00053424	,00055479
4	,00065753	,00068493	,00071232	,00073972
5	,00082192	,00085616	,00089041	,00092466
6	,00098630	,00102740	,00106849	,00110959
7	,00115068	,00119863	,00124657	,00129452
8	,00131507	,00136986	,00142465	,00147945
9	,00147945	,00154109	,00160274	,00166438
<i>Month.</i>	,00500000	,00520833	,00541666	,00562500

Table

Table II. *The Interest of one Pound per Annum.*

Years.	6 per Cent.	$6\frac{1}{4}$ per C.	$6\frac{1}{2}$ per C.	$6\frac{3}{4}$ per C.
1	0,06000000	0,06250000	0,06500000	0,06750000
2	0,12000000	0,12500000	0,13000000	0,13500000
3	0,18000000	0,18750000	0,19500000	0,20250000
4	0,24000000	0,25000000	0,26000000	0,27000000
5	0,30000000	0,31250000	0,32500000	0,33750000
6	0,36000000	0,37500000	0,39000000	0,40500000
7	0,42000000	0,43750000	0,45500000	0,47250000
8	0,48000000	0,50000000	0,52000000	0,54000000
9	0,54000000	0,56250000	0,58500000	0,60750000

Table I. *The Interest of one Pound per Diem.*

Days.	7 per Cent.	$7\frac{1}{4}$ per C.	$7\frac{1}{2}$ per C.	$7\frac{3}{4}$ per C.
1	,00019178	,00019863	,00020548	,00021233
2	,00038356	,00039726	,00041096	,00042466
3	,00057534	,00059589	,00061644	,00063699
4	,00076712	,00079452	,00082192	,00084932
5	,00095890	,00099315	,00102739	,00106164
6	,00115068	,00119178	,00123288	,00127397
7	,00134246	,00139041	,00143836	,00148630
8	,00153425	,00158904	,00164384	,00169863
9	,00172603	,00178767	,00184932	,00191096
Month.	,00584333	,00604166	,00625000	,00645833

Table II. *The Interest of one Pound per Annum.*

Years.	7 per Cent.	$7\frac{1}{4}$ per C.	$7\frac{1}{2}$ per C.	$7\frac{3}{4}$ per C.
1	0,07000000	0,07250000	0,07500000	0,07750000
2	0,14000000	0,14500000	0,15000000	0,15500000
3	0,21000000	0,21750000	0,22500000	0,23250000
4	0,28000000	0,29000000	0,30000000	0,31000000
5	0,35000000	0,36250000	0,37500000	0,38750000
6	0,42000000	0,43500000	0,45000000	0,46500000
7	0,49000000	0,50750000	0,52500000	0,54250000
8	0,56000000	0,58000000	0,60000000	0,62000000
9	0,63000000	0,65250000	0,67500000	0,69750000

Table

Table I. *The Interest of one Pound per Diem.*

Days.	8 per C.	8 $\frac{1}{4}$ per C.	8 $\frac{1}{2}$ per C.	8 $\frac{3}{4}$ per C.
1	,00021918	,00022603	,00023287	,00023973
2	,00043835	,00045205	,00046575	,00047945
3	,00065753	,00067808	,00069863	,00071918
4	,00087671	,00090411	,00093150	,00095890
5	,00109589	,00113014	,00116438	,00119863
6	,00131507	,00135616	,00139726	,00143835
7	,00153425	,00158219	,00163013	,00167808
8	,00175342	,00180822	,00186301	,00191781
9	,00197260	,00203424	,00209589	,00215753
Month.	,00866666	,00687500	,00708733	,00729186

Table II. *The Interest of one Pound per Annum.*

Years.	8 per Cent.	8 $\frac{1}{4}$ per C.	8 $\frac{1}{2}$ per C.	8 $\frac{3}{4}$ per C.
1	0,08000000	0,08250000	0,08500000	0,08750000
2	0,16000000	0,16500000	0,17000000	0,17500000
3	0,24000000	0,24750000	0,25500000	0,26250000
4	0,32000000	0,33000000	0,34000000	0,35000000
5	0,40000000	0,41250000	0,42500000	0,43750000
6	0,48000000	0,49500000	0,51000000	0,52500000
7	0,56000000	0,57750000	0,59500000	0,61250000
8	0,64000000	0,66000000	0,68000000	0,70000000
9	0,72000000	0,74250000	0,76500000	0,78750000

Table I. *The Interest of one Pound per Diem.*

Days.	9 per Cent.	9 $\frac{1}{4}$ per C.	9 $\frac{1}{2}$ per C.	9 $\frac{3}{4}$ per C.
1	,00024657	,00025342	,00026028	,00026712
2	,00049315	,00050684	,00052055	,00052424
3	,00073972	,00076027	,00078082	,00080137
4	,00098630	,00101370	,00104109	,00106849
5	,00123287	,00126712	,00130137	,00133561
6	,00147945	,00152055	,00156164	,00160274
7	,00172602	,00177397	,00182192	,00186986
8	,00197260	,00202739	,00208219	,00213699
9	,00221918	,00228082	,00234246	,00240410
Month.	,00750000	,00770833	,00791866	,00812500

Table

216 *Decimal Tables of Simple Interest.*

Table II. *The Interest of one Pound per Annum.*

<i>Years.</i>	<i>9 per Cent.</i>	<i>9 $\frac{1}{4}$ per C.</i>	<i>9 $\frac{1}{2}$ per C.</i>	<i>9 $\frac{3}{4}$ per C.</i>
1	0,09000000	0,09250000	0,09500000	0,09750000
2	0,18000000	0,18500000	0,19000000	0,19500000
3	0,27000000	0,27750000	0,28500000	0,29250000
4	0,36000000	0,37000000	0,38000000	0,39000000
5	0,45000000	0,46250000	0,47500000	0,48750000
6	0,54000000	0,55500000	0,57000000	0,58500000
7	0,63000000	0,64750000	0,66500000	0,68250000
8	0,72000000	0,74000000	0,76000000	0,78000000
9	0,81000000	0,83250000	0,85500000	0,87750000



T A-

TABLE III.

Of SIMPLE INTEREST.

The Rebate or Discount of one Pound for Days,
at the Rates of 2; $2\frac{1}{2}$; 3; $3\frac{1}{2}$; 4; $4\frac{1}{2}$;
5; 6; per Cent. per Annum.

Days.	2 per Cent.	$2\frac{1}{2}$ per C.	3 per Cent.	$3\frac{1}{2}$ per C.
1	,0000548	,0000685	,0000822	,0000959
2	,0001096	,0001370	,0001644	,0001917
3	,0001644	,0002054	,0002465	,0002876
4	,0002191	,0002739	,0003287	,0003834
5	,0002739	,0003424	,0004108	,0004792
6	,0003287	,0004108	,0004929	,0005750
7	,0003834	,0004792	,0005750	,0006708
8	,0004382	,0005477	,0006571	,0007666
9	,0004929	,0006161	,0007392	,0008623
10	,0005477	,0006845	,0008212	,0009580
20	,0010947	,0013680	,0016411	,0019141
30	,0016411	,0020506	,0024597	,0028685
40	,0021870	,0027322	,0032769	,0038210
50	,0027322	,0034139	,0040928	,0047716
60	,0032769	,0040928	,0049073	,0057205
70	,0038210	,0047716	,0057205	,0066676
80	,0043644	,0054496	,0065324	,0076128
90	,0049073	,0061266	,0073429	,0085563
100	,0054496	,0068027	,0081522	,0094980
110	,0059913	,0074779	,0089601	,0104379
120	,0065324	,0081522	,0097667	,0113760
130	,0070729	,0088255	,0105720	,0123123
140	,0076128	,0094980	,0113760	,0132468
150	,0081522	,0101695	,0121786	,0141796
160	,0086909	,0108401	,0129780	,0151106

TABLE III.

The Discount of one Pound for Days.

<i>Days.</i>	<i>4 per Cent.</i>	<i>4 $\frac{1}{2}$ per C.</i>	<i>5 per Cent.</i>	<i>6 per Cent.</i>
1	,0001096	,0001233	,0001370	,0001644
2	,0002191	,0002465	,0002739	,0003287
3	,0003287	,0003697	,0004108	,0004929
4	,0004382	,0004929	,0005477	,0006571
5	,0005477	,0006161	,0006845	,0008212
6	,0006571	,0007392	,0008212	,0009853
7	,0007665	,0008623	,0009580	,0011494
8	,0008759	,0009853	,0010947	,0013133
9	,0009853	,0011084	,0012314	,0014773
10	,0010947	,0012314	,0013680	,0016411
20	,0021870	,0024597	,0027322	,0032769
30	,0032769	,0036850	,0040928	,0049073
40	,0043644	,0049073	,0054496	,0065324
50	,0054496	,0061266	,0068027	,0081522
60	,0065234	,0073429	,0081522	,0097667
70	,0076128	,0085563	,0094980	,0113760
80	,0086909	,0097667	,0108401	,0129780
90	,0097667	,0109741	,0121786	,0145788
100	,0108401	,0121786	,0135135	,0161725
110	,0119112	,0133802	,0148448	,0177610
120	,0129800	,0145788	,0161725	,0193444
130	,0140465	,0157746	,0174966	,0209228
140	,0151006	,0169674	,0188172	,0224960
150	,0161725	,0181574	,0201342	,0240642
160	,0172321	,0193444	,0214477	,0256273

TABLE

T A B L E III.

The Discount of one Pound for Days.

Days.	2 per Cent.	$2\frac{1}{2}$ per C.	3 per Cent.	$3\frac{1}{2}$ per C.
170	,0092291	,0115098	,0137801	,0160399
180	,0097667	,0121786	,0145788	,0169674
190	,0103037	,0128465	,0153763	,0178932
200	,0108401	,0135135	,0161725	,0188172
210	,0113759	,0141796	,0169674	,0197395
220	,0119112	,0148448	,0177610	,0206601
230	,0124459	,0155091	,0185534	,0215789
240	,0129800	,0161725	,0193444	,0224959
250	,0135135	,0168350	,0201342	,0234114
260	,0140465	,0174966	,0209227	,0243251
270	,0145788	,0181574	,0217100	,0252370
280	,0151106	,0188172	,0224960	,0261473
290	,0156418	,0194762	,0232807	,0270558
300	,0161725	,0201342	,0240642	,0279627
310	,0167026	,0207914	,0248464	,0288679
320	,0172321	,0214477	,0256273	,0297714
330	,0177610	,0221031	,0264070	,0306732
340	,0182894	,0227577	,0271855	,0315734
350	,0188172	,0234114	,0279627	,0324718
360	,0193444	,0240642	,0287387	,0333686
361	,0193971	,0241294	,0288162	,0334582
362	,0194498	,0241946	,0288937	,0335478
363	,0195025	,0242598	,0289712	,0336374
364	,0195552	,0243251	,0290487	,0337269
365	,0196078	,0243902	,0291262	,0338164

T A B L E III.

The Discount of one Pound for Days.

<i>Days</i>	<i>4 per Cent.</i>	<i>4 ½ per C.</i>	<i>5 per Cent.</i>	<i>6 per Cent.</i>
170	,0182894	,0205286	,0227577	,0271855
180	,0193444	,0217100	,0240642	,0287387
190	,0203972	,0228885	,0253672	,0302869
200	,0214477	,0240642	,0266667	,0318302
210	,0224960	,0252370	,0279627	,0333686
220	,0235420	,0264070	,0292553	,0349022
230	,0245858	,0275743	,0305445	,0364309
240	,0256273	,0287387	,0318302	,0379547
250	,0266667	,0299003	,0331126	,0394737
260	,0277038	,0310592	,0343915	,0409879
270	,0287387	,0322153	,0356671	,0424974
280	,0297714	,0333686	,0369393	,0440021
290	,0308019	,0345192	,0382082	,0455021
300	,0318302	,0356671	,0394737	,0469974
310	,0328564	,0368122	,0407352	,0484880
320	,0338804	,0379547	,0419948	,0499740
330	,0349022	,0390444	,0432503	,0514553
340	,0359218	,0402314	,0445026	,0529320
350	,0369393	,0413657	,0457516	,0544041
360	,0379547	,0424974	,0469974	,0558717
361	,0380561	,0426104	,0471218	,0560182
362	,0381575	,0427234	,0472462	,0561647
363	,0382588	,0428364	,0473705	,0563111
364	,0383602	,0429493	,0474948	,0564575
365	,0384615	,0430622	,0476191	,0566028

The Nature, Construction, and Use of the Decimal Tables of Compound Interest.

What *Compound Interest* is, I have already shewn in the *Theoretical Part* of this Doctrine; and from the said *Theory* it also appears that Tables of *Compound Interest* are absolutely necessary for those who understand not *Logarithms* or *Algebra*; and therefore (though I have taught the Use of *Logarithms* after the *best Manner* in this Book) yet I have supplied the Reader with a *Set of Six Tables* for the Purposes of *Compound Interest*; I have framed them from the most compleat and approved *Calculations* of Mr. *John Smart*; his Book (which is *wholly* on *Tables of Interest*) having the best *Character* for Exactness, and the *Errata's* of the Press, no more than *four*.

As I intend *nothing* shall be wanting in any Part of this *System*, to make it *compleat*; so I have contrived these Tables to answer any *Question* of *Compound Interest*, for the *Rates* contained therein: For though they are not so large as the *Largest*, yet are they *larger* and more *universal* than any others, in any *mixed Pieces* of *Arithmetick* I have yet seen; I have chosen all the most *usual* and *necessary Rates* of *Interest*; and continued each *Annual Table* to 50 *Years* which is farther than is generally needful; and shall shew how they may be used for any *indefinite* Number of *Years* required; but first of their *Construction*, which is thus in the most *demonstrative* Manner deduced from the *Theorems* aforegoing, whence not only the *Manner*, but the *Reason* of their *Construction* (a Thing very *necessary*, though I know not where else to be met with) will be exceeding apparent.

The *Construction* of the *First Table* which shews the *Amount* of one Pound for *Days*, as also of the *Second Table*, which shews the same for *Years*, is made from *Theorem 1.* of *Compound Interest*, which is $PR^t = A$. Now if we put $P = 1$ l. then is the *Theorem* reduced to $R^t = A$. Consequently, if $R = 1,05$ l. per Cent. (or any other *Ratio*) and $t = 1, 2, 3, 4$, &c. *Years*;

Amount

212 The Nature, Construction, and Use

Amounts.

Then it will be $R^{\overset{t=1}{1}} = A = 1,05$ the *first* Year.
Which mult. by $R = 1,05$

The Product is $R^2 = A = 1,1025$ the *second* Year.
And again by $R = 1,05$

The Product is $R^3 = A = 1,157625$ the *third* Year.
And again by $R = 1,05$

The Product is $R^4 = A = 1,21550625$ the *fourth* Year.

And thus for the other Years subsequent in the Table.

Thus also if $R = 1,00013368$ the *Ratio* for 1 Day.
Mult. as before $R = 1,00013368$

The Prod. is $R^2 = A = 1,00026738$ the *Amount* for 2 D.
And again by $R = 1,00013368$

The Prod. is $R^3 = A = 1,00040110$ the *Amount* for 3 D.

Thus is found the *Amounts* for all the subsequent Days in the Table.

Here it may be proper to observe, that the *Amount* or *Interest* of any Sum, at the same *Rate*, is more at *Compound Interest* than at *Simple*, for any time *above a Year*; equal for *one Year*; but *less* for any time *less than a Year*. Though this seems *strange* in the last Assertion; yet the *Reason* is evident to any who understands and considers that *Simple Interest* is grounded on *Arithmetical*, but *Compound Interest* on *Geometrical Progression*.

The *Construction* of Table 3. is by *Theorem 2.* viz. $\frac{A}{R^t}$
 $= P$ the *present Worth* or *Value* of one Pound, which is here to be considered as the *Amount*; therefore if $A = 1\text{ l.}$
 $R = 1,05$ and $t = 1, 2, 3, 4, 5$, and *Years*, as before; 'tis evident that *Unity*, or 1, being divided by the Numbers in the *second* Table (designed by R^t) will give the Numbers in this *third* Table, or the *present Values* of 1 l. for the *Tabular Years*, and 5 *per Cent.* and so for any other *Rate of Interest*.

Example

Example at 5 per Cent.

Unity or 1	{ 1,05	The Quot. is	{ ,952381	The Present Value of 1 l. for	{ 1
divided by	{ 1,1025		{ ,907029		{ 2
	{ 1,157625		{ ,863837		{ 3
	{ 1,21550625		{ ,822702		{ 4
	{ 1,276281		{ ,783526		{ 5 Years, &c.

The Construction of Table 4. is from Theorem 1. of *Annuities*, &c. in *Arrears*, viz. $\frac{UR^t - U}{R - 1} = A$. Now as it is $U = 1$ l. and $R = 1,05$ as before, then the Theorem will be brought to $\frac{R^t - 1}{,05} = A$, the Amount of 1 l. Annuity for the Number of Years design'd by t . That is, from (R^t) the Numbers in the second Table, subtract Unity, or 1. The Remainder divided by ,05 (or $R - 1$) gives the Numbers in the fourth Table.

Example at 5 per Cent.

The Numbers in the 1 st Table less U.	{ ,05	Which divided by ,05 the Quotients are	{ 1,00000	The Amounts of 1 l. Annuity for	{ 1	Years &c.
Unity or 1. are	{ ,1025		{ 2,05		{ 2	
	{ ,157625		{ 3,1525		{ 3	
	{ ,21550625		{ 4,310125		{ 4	
	{ ,27628156		{ 5,525631		{ 5	

And thus you proceed for any other Rate of Interest.

The Construction of Table 5. is contained in Theorem 1. of the *present Worth* or Value of *Annuities*, which see; Now therein $U = 1$ l. and putting $R = 1,05$, and $t = 1, 2, 3, 4, 5$, &c. Years; that Theorem immediately becomes $\frac{R^t - 1}{,05R^t} = P$, the *present Worth* sought. But in Construction of Table 4. 'twas shewn that $\frac{R^t - 1}{,05}$ constituted the Numbers of that Table. Therefore 'tis manifest, if the Numbers in Table 4. be divided by R^t (that is, by 1,05¹, 1,05², 1,05³, 1,05⁴, &c.) the Quotients will be the Numbers of the fifth Table, at 5 per Cent. Which, in short, is only this;

Divide

224 The Nature, Construction, and Use

Divide the Numbers of the fourth Table, by the Numbers of the first Table, the Quotients make the fifth Table.

Table 1. Table 4. Table 5.

1,05) 1,00000 (=0,95238, &c. for the 1st Year.
 1,1025) 2,05000 (=1,85941, &c. for the 2d Year.
 1,157625) 3,15250 (=2,72324, &c. for the 3d Year.
 1,21550625) 4,310125 (=3,54595, &c. for the 4th Year.
 1,27628155) 5,525631 (=4,32947, &c. for the 5th Year, &c.

And thus for any other Rate of Interest.

The Construction of Table 6. is to be deduced from Theorem 2. of the present Worth of Annuities, &c. which see. Now since in this Case $P = 1$ l. therefore that Theorem will be reduced to this form, $RR^t - R^t = UR^t - U$; whence (at 5 per Cent.) 'twill be $,05 R^t = UR^t - U$; consequently $\frac{,05 R^t}{R^t - 1} = U$ the Annuity required; but this be-

ing just the Reverse of $\frac{R^t - 1}{,05 R^t}$, which make the Numbers of Table 5. 'tis plain, these two Theorems which constitute the Numbers of Table 5. and 6. multiplied together can make but 1. that is $\frac{,05 R^t}{R^t - 1} \times \frac{R^t - 1}{,05 R^t} = 1$.

Hence then if the Numbers of Table 5. be made Divisors, and Unity or 1. the constant Dividend, the Quotients shall be the Numbers which constitute the sixth Table, at 5 per Cent. and after the same Manner for any other Rate of Interest.

Example at 5 per Cent.

The Numbers of Table 5. at 5 per C. are	{	,9523809	} di-	vide	Unity,	the	Quotients	will be	{	1,05	} Annu-	ties 1 l. will	for	{ 1 }	} Years, &c.	
		1,8594103								,5378049						2
		2,7232480								,3637734						3
		3,5459505								,2787437						4
		4,3294767								,2277916						5

In like manner, when necessary, may other Tables be constructed from the Theorems; Here are as many Tables as any Book (that I have seen) contains, and more than are in most. My Aim in the Construction of these Tables is more to shew the

the young Artist the Rationale or Reason thereof, than the Manner how only; since the latter has been often done, the former not at all that I know of; at least, not in the natural Method by Deduction from the Theory it self, as I have here done it.

Quomodo factum est? Is a Question proper to *Mechanicks*; *Cur ita sit faciendum?* Beseems an Artist to enquire.

The Use of the following Tables.

The Use of all these Tables depends on this one obvious and easy *General Rule*,

Multiply the Tabular Number, which stands against the given Number of Days or Years, and under the given Rate of Interest, by the given Principal Sum; and the Product will satisfy the Question.

Example of 246 l. at 5 per Cent. for 30 Days, or Years.

In Table I. against 30 Days under 5 per C. stands 1,0040182
Which multiplied by the *Principal Sum* — 246

The Product is the *Amount* required; viz. l. 247,0684772

In Table II. Against 30 Years, at 5 per Cent. is 4,3219424
Which multiplied by — — — 246

The Product is the *Amount* required; viz. l. 1063,1978 &c.

In Table III. Against the same *Time* and *Rate*, is 0,2313775
Which multiplied by — — — 246

The Product is the *present Worth* required; l. 56,9189 &c.

In Table IV. For the given *Time* and *Rate*, is 66,4388475
Which multiplied by — — — 246

The Prod. is the *A.* of such an *Annuity*; l. 16343,9565 &c.

In Table V. For the given *Time* and *Rate*, is 15,372451
Which multiplied by — — — 246

The Prod. is the *present W.* of that *Ann.* l. 3781,6229 &c.

226 The Nature, Construction, and Use

In Table VI. For the given *Time* and *Rate*, is 50650514
 Which multiplied by ————— 246
 The Product is the purchased *Annuity*; l. 16,0026 &c.

Therefore by the Tables we immediately know, that 246 l. forborne 30 Days, at 5 per Cent. per Annum Compound Interest will amount to 247 l. 1 s. 4 $\frac{1}{4}$ d.

That 246 l. forborne 30 Years, at 5 per Cent. &c. will Amount to 1063 l. 3 s. 11 $\frac{1}{4}$ d.

That the present Worth of 246 l. due 30 Years hence, at the Rate of 5 per Cent. &c. is 56 l. 18 s. 3 $\frac{1}{4}$ d.

That the Amount of an Annuity of 246 l. per Annum, forborne or unpaid 30 Years, at 5 per Cent. &c. is 16343 l. 19 s. 1 $\frac{1}{4}$ d.

That the present Worth of an Annuity of 246 l. to continue 30 Years, at 5 per Cent. per Annum, is 3781 l. 12 s. 5 $\frac{1}{2}$ d.

That the Annuity which 246 l. will purchase, to continue 30 Years, reckoning 5 per Cent. Interest, is 16 l. per Annum.

If the Amount of any Sum be sought, for a Number of Days which are not in the first Table, and Years which are not in the second, observe this

Rule; Divide the given Number of Days, or Years, into two such Numbers as are in the Table, then multiply the Amounts pertaining to each, into each other; then shall the Product be the Amount for the Time required.

Example 1.

What will 523 l. amount to, in 194 Days, at 5 per Cent. per Annum? The two Parts of this Number in the Table, are 190, and 4; therefore,

In Table I. Against 190 Days, under 5 per C. is 1,0257228

And against 4 Days, at the same Rate, is 1,0005348

The Prod. is the Am. of 1 l. for 194 Days, viz. 1,0262714

Which multiply by the Principal Sum, viz. 523

This Product is the Answer ————— l. 536,7399840

In Money, 536 l. 14 s. 9 $\frac{1}{2}$ d.

Example

Example 2.

What is the *Amount* of 150 *l.* in 91 Years, at 5 per Cent.?

In Table II. Against 50 Years, under 5 per C. is 11,4674000

And against 41 Years, at 5 per C. is 7,3919881

The Prod. is the *Am.* of 1 *l.* for 91 Years, viz. 84,7668833

Which multiply by the *Principal* Sum, viz. 150

The Product is the Answer — l. 12715,032495

In Money 12715 *l.* 0 *s.* 7 $\frac{1}{2}$ *d.*

Example 3.

What will 523 *l.* amount to in 5 Years and 194 Days, at 5 per Cent.?

In Table II. against 5 Years, at 5 per Cent. is 1,2762816

And the *Am.* of 1 *l.* in 194 Days, as above, is 1,0262714

The Pr. is the *Am.* of 1 *l.* in 5 Years, and 194 D. 1,3098113

Which multiplied by the *Principal* Sum — 523

The Product is the Answer, viz. — l. 685,0313413

In Money 685 *l.* 0 *s.* 7 $\frac{1}{2}$ *d.*

N. B. The other Tables of *Compound Interest*, as they cannot in this Manner be extended, so they seldom require it.

I shall now present the Reader with a few Questions of a more *complex Nature*, and which frequently happen, in order to shew the more *extensive Use* of the Tables.

Question 1.

Suppose I have 790 *l.* to be paid me within 7 Years, in this Manner; at the End of the first Year 90 *l.* of two Years, 100 *l.* of four Years 200 *l.* and of seven Years 400 *l.* Quere what the *present Worth* of those several Payments is in *ready Money*, allowing 4 $\frac{1}{2}$ per Cent. *Compound Interest*?

228 The Nature, Construction, and Use

In Table III. the *present Worth* of 1 *l.* at $4\frac{1}{2}$ *per Cent.*

Due at the End of 1 Year, is — 0,9569378

Which multiply by the *Principal* — 90

The Product is the *present Worth* of 90 *l.* = 86,124402

Thus the *present Worth* of 100 *l.* due at } = 91,57299
the End of two Years, is found

Also, if 200 *l.* at the End of 4 Years = 167,71226

And of 400 *l.* at the End of 7 Years = 293,93140

The *Sum* of all these is — *l.* 639,341052

Which answers the Question, viz. 639 *l.* 6 *s.* 9 $\frac{1}{2}$ *d.*

Question 2.

A owes to *B* 455 *l.* to be paid in 14 Years, viz. at the End of every 2 Years 65 *l.* But he would agree to pay him in 7 Years, by equal Payments each Year; which *B* agrees to, and at the Rate of 6 *per Cent.* Compound Interest. Quere what the Annual Payment must be?

1. Find the *present Worth* (by Table III.) of the 7 Payments which were at first to be made, as *per Quest.* 1. which you will find to be 293 *l.* 5 *s.* 2 *d.*
2. Then find (by Table VI.) what *Annuity*, to continue 7 Years at the given Rate, 293 *l.* 5 *s.* 2 *d.* will purchase; which you will find to be 52 *l.* 10 *s.* 8 *d.* and is the Answer to the Question.

Question 3.

A has a Term of 7 Years in an *Estate* of 35 *l.* *per Annum.* *B* has a Term of 14 Years in the same *Estate* in *Reversion* after the 7 Years; and *C* has a farther Term of 20 Years in *Reversion* after the 21 Years. Quere the *present Values* of the several Terms, at the Rate of 5 *per Cent.* *per Annum*?

By Table V. the *present Value* of 35 *l.* *per Annum*, may be

	<i>l.</i>	<i>s.</i>	<i>d.</i>
found, for 41 Years, to be	—	—	605 — 6 — 0 $\frac{1}{2}$
for 21 Years, to be	—	—	448 — 14 — 9 $\frac{1}{2}$
for 7 Years, to be	—	—	202 — 10 — 5 $\frac{1}{2}$

Which

Which subtract from each other, it will appear,

	<i>l.</i>	<i>s.</i>	<i>d.</i>
That the <i>present Value</i> of <i>A's Term</i> is	202	10	5½
of <i>B's Term</i>	246	4	4
of <i>C's Term</i>	156	11	3
For these <i>Values</i> answer the Question	1. 605	6	0½

Question 4.

Which is most advantagious a *Term* of 15 Years in an *Estate* of 100 *l. per Annum*, or the *Reversion* of such an *Estate* for ever after the *Expiration* of the said 15 Years; computing at the *Rate* of 5 *per Cent. per Annum Compound Interest*?

An Estate of 100 <i>l. per Annum</i> , in <i>Fee</i>	}	<i>l.</i> 2000
<i>Simple</i> at 5 <i>per Cent.</i> is <i>Worth</i> —		
In <i>Table V.</i> the <i>present Value</i> of the same	}	<i>l.</i> 1037,9658
<i>Estate</i> , at the same <i>Rate</i> , for 15 Years, is		
The Difference is		<u><i>l.</i> 962,0342</u>

Now this *Difference* being the *Value* of the *Reversion*, it appears that the first *Term* of 15 Years is better than the *Reversion* for ever afterwards by 75,9316 *l.* = 75 *l.* 18 *s.* 7½ *d.* Answer.

Question 5.

A Person having 12 Years to come, in a *Lease* of an *Estate* of 60 *l. per Annum* for 40 Years, would know what *present Money* he must pay in order to *renew* or *complete* the *Lease* by adding 28 Years thereto, computing at 6 *per Cent. Compound Interest*?

By Table V. the <i>present Value</i> of 1 <i>l. per</i>	}	<i>l.</i> 15,046297
<i>Annum</i> , at 6 <i>per Cent.</i> for 40 Years, is		
By the same Table the <i>Value</i> of 1 <i>l. per</i>	}	<i>l.</i> 8,383844
<i>An.</i> at that <i>Rate</i> , for 12 Years to come, is		
		<hr/>
The Difference is	<i>l.</i>	6,662453
Which multiplied by		60

The *Product* is the Answer, viz. — *l.* 399,747180
In *Money*, 399 *l.* 14 *s.* 11 *d.*

Question

230 The Nature, Construction, and Use

Question 6.

A gives 1550 *l.* for an *Annuity* of 100 *l.* per *Annum* for 50 Years. *B* puts 1550 *l.* out at *Interest*. It is required to know which will amount to the greatest Sum at the End of the 50 Years, at the Rate of 6 *l.* per Cent. &c. Compound *Interest*?

By Table IV. the Amount of 100 *l.* *Annuity*, in 50 Years, at 6 per Cent. } *l.* 29033,59046
may be found to be — —

By Table II. it may be found, that the Amount of 1550 *l.* for that Time and Rate will be — — } *l.* 28551,23885

Hence *A's Annuity* is more than *B's* 1550 *l.* by *l.* 482,35161 at the End of 50 Years. The present Value of which Difference is found, by Table III. to be 26 *l.* 3 *s.* 8 $\frac{1}{2}$ *d.* and so much was *A's* Case better than *B's*.

Question 7.

What *Annuity* to continue 14 Years, may be purchased with 1000 *l.* due at the end of 5 Years; the *Annuity* to commence presently, at 5 *l.* per Cent?

By Table III. the present Worth of 1000 *l.* due 5 Years hence at 5 per Cent. } = *l.* 783,5262
may be found — —

By Table VI. it may be found, that the *Annuity* which 783,5262 *l.* will purchase for 14 Years, at the Rate of 5 per Cent. is } = *l.* 79,1518

In Money, 79 *l.* 3 *s.* 0 $\frac{1}{4}$ *d.* per *Annum*, the Answer.

Question 8.

For a Lease of certain Profits for 7 Years, *A*, makes two Offers, either to pay 150 *l.* as a *Fine*, and 300 *l.* per *Annum*; or 1700 *l.* *Fine*, without any *Rent*. *B*, bids 650 *l.* *Fine*, and 200 *l.* per *Annum*. And *C*, offers 200 *l.* *Fine*, and 405 *l.* per *Annum*. Quere which is the best Offer, and what the Difference, computing at 5 *l.* per Cent. &c. Compound *Interest*?

1. By

of the Dec. Tables of Comp. Interest. 231

1. By Table II. the Amount of 150*l.* in 7 Years, at 5 per Cent. may be found to be } *l.* 211,0659
 By Table IV. the Amount of 300 *l.* per Annum in 7 Years at the given Rate may be found } *l.* 242,6025

Therefore A's Offer, at the End of 7 Years would be } *l.* 2453,6684

2. By Table II. the Amount of 1700 *l.* in 7 Years (A's second Offer) at the said Rate, is found to be } *l.* 2392,0802

3. By Table II. the Amount of 650 *l.* in 7 Years, at the given Rate, will be found to be } *l.* 914,6189

By Table IV. the Amount of 200 *l.* per Annum in 7 Years, at that Rate, will be found to be } *l.* 1628,4016

Therefore B's Offer will, in 7 Years, amount to } *l.* 2543,0205

4. By Table II. the Amount of 200 *l.* in 7 Years, at the given Rate, will be found to be } *l.* 281,4212

By Table IV. the Amount of 405 *l.* per Annum for the given Time and Rate, will be found to be } *l.* 3297,5132

So that C's Offer, in 7 Years, will amount to *l.* 3578,9344

The Amounts therefore of the said Offers, at the End of the said Term, being thus known; the Present Worth of the several Amounts may be found by Table III. which are as follow.

	<i>l.</i>	<i>s.</i>	<i>d.</i>
The present Worth of A's first Offer will be	1885	18	03
A's second Offer —	1700	00	00
B's Offer —	1807	05	06
C's Offer —	2543	09	08

Therefore the present Worth of what C offers is more than — — A's first Offer, by 657—11—5
 A's second Offer, by 843—9—8
 B's Offer, by — 736—4—2

Which fully answers the Question.

N. B. This

232 The Nature, Construction, and Use

N. B. This Question might be more readily answered by finding the *present Worths* of the several offer'd *Annuities* (as per Table V.) and adding to them the several *Fines*; as the Reader may try at his *Leisure*.

Question 9.

What *Annuity* is sufficient to pay off a *Debt* of 50 *Millions* in 30 Years at 4 *l. per Cent. Compound Interest*?

In Table IV. against 30 Years, under 4 per C. is 0578301
Which multiply by the Debt — 50000000

The Product is the *Annuity* sought, viz. *l.* 2891505
per Annum.

So that supposing the *National Debt* to be 50 *Millions*, and the Interest paid to be 2 *Millions per Annum*, or 4 *l. per Cent.* then will a *Sinking Fund* of 891505 *l. per Annum*, clear the whole *Debt* in 30 Years.

N. B. By this Example appears the Necessity of continuing the *Tabular Numbers* to so many Places of *Decimals*.

Question 10.

Suppose one *Farthing* had been lent at *Compound Interest* at 5 per Cent. in the first Year of the *Christian Æra*, or Birth of *Christ*, and so continued to this present Year thereof 1734; Quere the *Amount* thereof?

N. B. Though this Question might be answered by Table II. as I have before shewn, yet I shall here use *Logarithms*, as most expeditious in this Case. For having said enough about the Use of the Tables, I here intend only to give the Reader a hint of the surprising Nature of Numbers in *Geometrical Proportion*.

Therefore, The *Logarithm* of the Rate 1,05 = 0,0211893
Multiplied by the Time — — — 1734

The Product is — — — 36,7422462

To which add the *Logarithm* of 1 *Farthing*, or the ,0010416 Part of a Pound, } = 7,0177288
viz. — — —

The Sum is the *Log.* of the *Amount* sought = 33,7599750

Now

The Use of the TABLE.

Suppose a Person of 50 Years of Age offers to sell his Life in an Estate of 46*l.* *per Annum*, what is the *Value* thereof in *Ready Money*.

The Age of 50, is Years Purchase	—	9,21
Which multiply by the <i>Annuity</i>	—	46

The Product is the Answer 423*l.* 13*s.* 2 $\frac{1}{4}$ *d.* = 423,66

If it happen that a Life and a *Reversion* for so many Years after, be offered; 'twill be necessary to reduce the Year's Purchase into Years certain, or Years of a *Lease*, by the *Fifth Table*; thus, Suppose I would find what Number of certain Years corresponds to 10,57 Years Purchase, and 6 *per Cent*. I look in *Table V.* under the *given Rate*, and I find the next *nearest Value* of *Annuity* of 1*l.* *per Annum* to 10,57; to be 10,4772597, opposite to which is 17 Years, which are to be added to the Years in *Reversion*, and then the Case is thus easily solved. *Example.*

What is the *present Worth* of an *Estate* of 78*l.* *per Annum* clear Rent for 20 Years after the *Death* of a Person 40 Years of Age, at 6 *per Cent*?

The Age of 40 Years is 10,57 Years purchase, which in *Table V.* gives 17 Years certain to come at 6 *per Cent*.

Then $20 + 17 = 37$ Years.

	Years.	l.
Therefore the <i>present Worth</i> of 1 <i>l.</i> } for 37 = 14,7367804		
<i>per Annum</i> for the given <i>Rate</i> —	} for 17 = 10,4772597	

The <i>present Worth</i> of 1 <i>l.</i> <i>per Annum</i> for 20 = 4,2595207	
Which multiply by the <i>Annuity</i> —	78

The Prod. is the Answer 232*l.* 4*s.* 11 *d.* = 232,2426146

TABLES

TABLES of COMPOUND INTEREST.

TABLE I.

The Amount of one Pound for Days; at the Rates of 2; 2 $\frac{1}{2}$; 3; 3 $\frac{1}{2}$; 4; 4 $\frac{1}{2}$; 5, and 6 per Cent. per Annum.

Days.	2 per C.	2 $\frac{1}{2}$ per C.	3 per C.	3 $\frac{1}{2}$ per C.
1	1,0000542	1,0000676	1,0000809	1,0000942
2	1,0001085	1,0001353	1,0001619	1,0001885
3	1,0001627	1,0002029	1,0002429	1,0002827
4	1,0002170	1,0002706	1,0003240	1,0003770
5	1,0002713	1,0003383	1,0004050	1,0004713
6	1,0003255	1,0004059	1,0004860	1,0005656
7	1,0003798	1,0004736	1,0005670	1,0006600
8	1,0004341	1,0005412	1,0006480	1,0007542
9	1,0004884	1,0006090	1,0007291	1,0008486
10	1,0005426	1,0006767	1,0008101	1,0009429
20	1,0010856	1,0013539	1,0016209	1,0018867
30	1,0016289	1,0020315	1,0024324	1,0028315
40	1,0021725	1,0027097	1,0032445	1,0037771
50	1,0027163	1,0033882	1,0040573	1,0047236
60	1,0032605	1,0040673	1,0048708	1,0056710
70	1,0038049	1,0047468	1,0056849	1,0066193
80	1,0043497	1,0054267	1,0064996	1,0075685
90	1,0048947	1,0061071	1,0073151	1,0085186
100	1,0054401	1,0067880	1,0081311	1,0094696
110	1,0059857	1,0074693	1,0089479	1,0104214
120	1,0065316	1,0081511	1,0097653	1,0113742
130	1,0070779	1,0088334	1,0105834	1,0123279
140	1,0076244	1,0095161	1,0114021	1,0132825
150	1,0081712	1,0101993	1,0122215	1,0142379
160	1,0087183	1,0108829	1,0130415	1,0151943

TABLE I.

The Amount of one Pound, Compound Interest.

<i>Days.</i>	<i>4 per C.</i>	<i>4½ per C.</i>	<i>5 per C.</i>	<i>6 per C.</i>
1	1,0001074	1,0001206	1,0001336	1,0001596
2	1,0002149	1,0002412	1,0002673	1,0003193
3	1,0003224	1,0003618	1,0004011	1,0004790
4	1,0004299	1,0004824	1,0005348	1,0006387
5	1,0005374	1,0006031	1,0006685	1,0007985
6	1,0006449	1,0007238	1,0008023	1,0009583
7	1,0007524	1,0008445	1,0009361	1,0011181
8	1,0008600	1,0009652	1,0010699	1,0012779
9	1,0009675	1,0010859	1,0012037	1,0014378
10	1,0010751	1,0012066	1,0013376	1,0015976
20	1,0021513	1,0024148	1,0026770	1,0031979
30	1,0032288	1,0036243	1,0040182	1,0048007
40	1,0043074	1,0048354	1,0053611	1,0064060
50	1,0053871	1,0060479	1,0067059	1,0080139
60	1,0064680	1,0072618	1,0080525	1,0096244
70	1,0075501	1,0084773	1,0094009	1,0112375
80	1,0086333	1,0096942	1,0107511	1,0128531
90	1,0097177	1,0109125	1,0121031	1,0144713
100	1,0108033	1,0121324	1,0134569	1,0160921
110	1,0118900	1,0133537	1,0148125	1,0177155
120	1,0129779	1,0145765	1,0161699	1,0193415
130	1,0140670	1,0158007	1,0175291	1,0209701
140	1,0151572	1,0170265	1,0188902	1,0226013
150	1,0162487	1,0182537	1,0202531	1,0242351
160	1,0173412	1,0194824	1,0216178	1,0258715

TABLE

TABLE I.

The Amount of one Pound, Compound Interest.

Days.	2 per C.	2 $\frac{1}{2}$ per C.	3 per C.	3 $\frac{1}{2}$ per C.
170	1,0092658	1,0115670	1,0138623	1,0161516
180	1,0098135	1,0122516	1,0146837	1,0171098
190	1,0103615	1,0129366	1,0155057	1,0180689
200	1,0109098	1,0136221	1,0163284	1,0190288
210	1,0114584	1,0143081	1,0171518	1,0199897
220	1,0120073	1,0149945	1,0179759	1,0209515
230	1,0125565	1,0156814	1,0188006	1,0219142
240	1,0131060	1,0163687	1,0196260	1,0228778
250	1,0136558	1,0170565	1,0204520	1,0238424
260	1,0142059	1,0177448	1,0212788	1,0248078
270	1,0147563	1,0184336	1,0221062	1,0257741
280	1,0153070	1,0191228	1,0229342	1,0267414
290	1,0158580	1,0198125	1,0237630	1,0277096
300	1,0164093	1,0205026	1,0245924	1,0286786
310	1,0169609	1,0211932	1,0254225	1,0296486
320	1,0175127	1,0218843	1,0262532	1,0306195
330	1,0180649	1,0225758	1,0270847	1,0315914
340	1,0186174	1,0232679	1,0279168	1,0325641
350	1,0191702	1,0239603	1,0287495	1,0335378
360	1,0197233	1,0246533	1,0295830	1,0345123
361	1,0197786	1,0247226	1,0296664	1,0346098
362	1,0198340	1,0247919	1,0297497	1,0347073
363	1,0198893	1,0248613	1,0298331	1,0348049
364	1,0199446	1,0249306	1,0299165	1,0349024
365	1,0200000	1,0250000	1,0300000	1,0350000

The

TABLE I.

The Amount of one Pound, Compound Interest.

<i>Days.</i>	<i>4 per C.</i>	<i>4 ½ per C.</i>	<i>5 per C.</i>	<i>6 per C.</i>
170	1,0184350	1,0207126	1,0229843	1,0275105
180	1,0195299	1,0219442	1,0243527	1,0291522
190	1,0206261	1,0231774	1,0257228	1,0307964
200	1,0217233	1,0244120	1,0270949	1,0324433
210	1,0228218	1,0256481	1,0284687	1,0340928
220	1,0239215	1,0268858	1,0298444	1,0357450
230	1,0250233	1,0281249	1,0312219	1,0373998
240	1,0261243	1,0293655	1,0326013	1,0390572
250	1,0272275	1,0306076	1,0339825	1,0407173
260	1,0283319	1,0318512	1,0353656	1,0423800
270	1,0294375	1,0330963	1,0367505	1,0440454
280	1,0305443	1,0343429	1,0381373	1,0457135
290	1,0316522	1,0355910	1,0395259	1,0473842
300	1,0327614	1,0368406	1,0409164	1,0490576
310	1,0338717	1,0383917	1,0423087	1,0507336
320	1,0349832	1,0393444	1,0437029	1,0524124
330	1,0360960	1,0405985	1,0450990	1,0540938
340	1,0372099	1,0418542	1,0464969	1,0557779
350	1,0383250	1,0431114	1,0478967	1,0574647
360	1,0394413	1,0443700	1,0492984	1,0591542
361	1,0395530	1,0444960	1,0494387	1,0593233
362	1,0396648	1,0446220	1,0495790	1,0594924
363	1,0397765	1,0447479	1,0497193	1,0596616
364	1,0398882	1,0448739	1,0498596	1,0598308
365	1,0400000	1,0450000	1,0500000	1,0600000

TABLE

TABLE II.

Of COMPOUND INTEREST.

The Amount of one Pound for Years, at the Rates of 2; $2\frac{1}{2}$; 3; $3\frac{1}{2}$; 4; $4\frac{1}{2}$; 5, and 6; per Cent. per Annum.

Years.	2 per C.	$2\frac{1}{2}$ per C.	3 per C.	$3\frac{1}{2}$ per C.
1	1,0200000	1,0250000	1,0300000	1,0350000
2	1,0404000	1,0506250	1,0609000	1,0712250
3	1,0612080	1,0768906	1,0927270	1,1087178
4	1,0824321	1,1038128	1,1255088	1,1475230
5	1,1040808	1,1314082	1,1592740	1,1876863
6	1,1261624	1,1596934	1,1948523	1,2292553
7	1,1486856	1,1886857	1,2298733	1,2722792
8	1,1716593	1,2184029	1,2667700	1,3168098
9	1,1950925	1,2488629	1,3047731	1,3628973
10	1,2189944	1,2800845	1,3439163	1,4105987
11	1,2433743	1,3120866	1,3842338	1,4599697
12	1,2682417	1,3448888	1,4257608	1,5110686
13	1,2936066	1,3785110	1,4685337	1,5639560
14	1,3194787	1,4129738	1,5125897	1,6186945
15	1,3458683	1,4482981	1,5579674	1,6753488
16	1,3727857	1,4845056	1,6047064	1,7339860
17	1,4002414	1,5216182	1,6528476	1,7946755
18	1,4282462	1,5596587	1,7024330	1,8574892
19	1,4568111	1,5986501	1,7535060	1,9225013
20	1,4859474	1,6386164	1,8061112	1,9897888
21	1,5156663	1,6795818	1,8602945	2,0594314
22	1,5459796	1,7215714	1,9161034	2,1315115
23	1,5768992	1,7646106	1,9735865	2,2061144
24	1,6084372	1,8087259	2,0327941	2,2833284
25	1,6406059	1,8539441	2,0937779	2,3632449

TABLE

TABLE II.

The Amount of one Pound; Compound Interest.

<i>Years.</i>	<i>4 per C.</i>	<i>4 ½ per C.</i>	<i>5 per C.</i>	<i>6 per C.</i>
1	1,0400000	1,0450000	1,0500000	1,0600000
2	1,0816000	1,0920250	1,1025000	1,1236000
3	1,1248640	1,1411661	1,1576250	1,1910160
4	1,1698586	1,1925186	1,2155063	1,2624769
5	1,2166529	1,2461819	1,2762816	1,3382256
6	1,2653190	1,3022601	1,3400956	1,4185191
7	1,3159318	1,3608618	1,4071061	1,5036303
8	1,3685691	1,4221006	1,4774554	1,5938481
9	1,4233118	1,4860951	1,5513282	1,6894790
10	1,4802443	1,5529694	1,6288946	1,7908477
11	1,5394541	1,6228530	1,7103393	1,8982980
12	1,6010322	1,6958814	1,7958563	2,0121965
13	1,6650735	1,7721961	1,8156491	2,1329283
14	1,7316764	1,8519449	1,9799316	2,2609039
15	1,8009435	1,9352824	2,0789282	2,3965582
16	1,8729812	2,0223701	2,1828746	2,5402517
17	1,9479005	2,1133768	2,2920183	2,6927728
18	2,0258165	2,2084787	2,4066192	2,8543392
19	2,1068492	2,3078603	2,5269502	3,0255995
20	2,1911231	2,4117140	2,6532977	3,2071355
21	2,2787681	2,5202111	2,7859626	3,3995636
22	2,3699188	2,6336520	2,9252607	3,6035374
23	2,4647155	2,7521663	3,0715238	3,8197497
24	2,5633042	2,8760138	3,2251000	4,0489346
25	2,6658363	3,0054344	3,3863549	4,2918707

TABLE

TABLE II.

The Amount of one Pound, Compound Interest.

Years.	2 per Cent.	2 $\frac{1}{2}$ per C.	3 per Cent.	3 $\frac{1}{2}$ per C.
26	1,6734181	1,9002927	2,1565912	2,4459585
27	1,7058864	1,9478000	2,2212890	2,5315871
28	1,7410242	1,9964950	2,2879276	2,6201719
29	1,7758446	2,0464073	2,3565655	2,7118779
30	1,8113615	2,0975675	2,4272624	2,8067937
31	1,8475888	2,1500067	2,5000803	2,9050314
32	1,8845405	2,2037569	2,5750827	3,0067075
33	1,9222314	2,2588508	2,6523352	3,1119423
34	1,9606760	2,3153221	2,7319053	3,2208603
35	1,9998895	2,3732051	2,8138624	3,3335904
36	2,0398873	2,4325353	2,8982783	3,4502661
37	2,0806850	2,4933487	2,9852266	3,5710254
38	2,1222987	2,5556824	3,0747834	3,6960113
39	2,1647447	2,6195744	3,1670269	3,8253717
40	2,2080396	2,6850638	3,2620377	3,9592597
41	2,2522004	2,7521904	3,3598989	4,0978338
42	2,2979444	2,8209952	3,4606958	4,2412579
43	2,3431893	2,8915500	3,5645167	4,3897020
44	2,3900531	2,9638080	3,6714522	4,5433416
45	2,4378542	3,0379032	3,7815958	4,7023585
46	2,4866112	3,1138508	3,8950437	4,8669411
47	2,5363435	3,1916971	4,0118950	5,0372840
48	2,5870703	3,2714895	4,1322518	5,2135889
49	2,6388117	3,3532768	4,2562194	5,3960645
50	2,6915880	3,4371087	4,3839060	5,5849268

TABLE II.

The Amount of one Pound, Compound Interest.

Years.	4 per Cent.	4 $\frac{1}{2}$ per C.	5 per C.	6 per Cent.
26	2,7724697	3,1406790	3,5556727	4,5493829
27	2,8833685	3,2820095	3,7334563	4,8223459
28	2,9987033	3,4296999	3,9201291	5,1116866
29	3,1186514	3,5840364	4,1161356	5,4183878
30	3,2433975	3,7453181	4,3219424	5,7434911
31	3,3731334	3,9138574	4,5380395	6,0881006
32	3,5080587	4,0899810	4,7649415	6,4533866
33	3,6483811	4,2740301	5,0031885	6,8405898
34	3,7943163	4,4663615	5,2533480	7,2510252
35	3,9460889	4,6673478	5,5160154	7,6860867
36	4,1039325	4,8773784	5,7918161	8,1472519
37	4,2680898	5,0968604	6,0814069	8,6360870
38	4,4388134	5,3262192	6,3854773	9,1542523
39	4,6163659	5,5658990	6,7047511	9,7035074
40	4,8010206	5,8163645	7,0399887	10,2857178
41	4,9930614	6,0781009	7,3919881	10,9028609
42	5,1927839	6,3516154	7,7615875	11,5570326
43	5,4004952	6,6374381	8,1496669	12,2504545
44	5,6165150	6,9361229	8,5571503	12,9854818
45	5,8411756	7,2482484	8,9850078	13,7646107
46	6,0748227	7,5744196	9,4342582	14,5904873
47	6,3178156	7,9152684	9,9059711	15,4659166
48	6,5705282	8,2714555	10,4012696	16,3938716
49	6,8333493	8,6436710	10,9213331	17,3775039
50	7,1066833	9,0326362	11,4674000	18,4201541

TABLE

TABLE III.

Of COMPOUND INTEREST.

The Present Worth of one Pound for Years, at the Rates of 2; $2\frac{1}{2}$; 3; $3\frac{1}{2}$; 4; $4\frac{1}{2}$; 5, and 6; per Cent. per Annum.

Years.	2 per C.	$2\frac{1}{2}$ per C.	3 per C.	$3\frac{1}{2}$ per C.
1	,9803921	,9756097	,9708738	,9661836
2	,9611687	,9518144	,9425959	,9335107
3	,9423223	,9285994	,9151417	,9019427
4	,9238454	,9059506	,8884870	,8714422
5	,9057308	,8838542	,8626088	,8419732
6	,8879713	,8622968	,8374843	,8135006
7	,8705601	,8412654	,8130915	,7859910
8	,8534903	,8207465	,7894092	,7594116
9	,8367552	,8007283	,7664167	,7337710
10	,8203483	,7811984	,7440939	,7089188
11	,8042630	,7621447	,7224213	,6849457
12	,7884931	,7435558	,7013799	,6617833
13	,7730325	,7254203	,6809513	,6394041
14	,7578750	,7077272	,6611178	,6177818
15	,7430147	,6904655	,6418619	,5968906
16	,7284458	,6736249	,6231669	,5767059
17	,7141625	,6571950	,6050164	,5572038
18	,7001593	,6411659	,5873946	,5383611
19	,6864307	,6255277	,5702860	,5201557
20	,6729713	,6102709	,5536758	,5025659
21	,6597758	,5953862	,5375493	,4855709
22	,6468390	,5808646	,5218925	,4691506
23	,6341559	,5666972	,5066917	,4532856
24	,6217214	,5528753	,4919337	,4379571
25	,6095308	,5393905	,4776056	,4231470

TABLE III.

The Present Worth of one Pound, Comp. Interest.

<i>Years.</i>	<i>4 per C.</i>	<i>4 ½ per C.</i>	<i>5 per C.</i>	<i>6 per C.</i>
1	,9615385	,9569378	,9523810	,9433962
2	,9245562	,9157299	,9070295	,8899964
3	,8889964	,8762966	,8638376	,8396193
4	,8548042	,8385613	,8227025	,7920937
5	,8219271	,8024511	,7835262	,7472582
6	,7903145	,7618957	,7462154	,7049605
7	,7599178	,7348285	,7106813	,6650571
8	,7306902	,7031851	,6768324	,6274124
9	,7025867	,6729044	,6446089	,5918985
10	,6755642	,6439277	,6139133	,5583948
11	,6495809	,6161987	,5846793	,5267875
12	,6245971	,5896639	,5568374	,4969694
13	,6005741	,5642716	,5303214	,4688390
14	,5774751	,5399729	,5050679	,4423010
15	,5552645	,5167204	,4810171	,4172651
16	,5339082	,4944693	,4581115	,3936463
17	,5133733	,4731764	,4362967	,3713644
18	,4936281	,4528004	,4155207	,3503438
19	,4746424	,4333018	,3957340	,3305130
20	,4563870	,4146429	,3768895	,3118047
21	,4388336	,3967874	,3589424	,2941554
22	,4219554	,3797009	,3418499	,2775051
23	,4057263	,3633501	,3255713	,2617973
24	,3901215	,3477035	,3100679	,2469786
25	,3751168	,3327306	,2953028	,2329986

TABLE

TABLE III.

The Present Worth of one Pound, Comp. Interest.

Years.	2 per C.	2 $\frac{1}{2}$ per C.	3 per C.	3 $\frac{1}{2}$ per C.
26	,5975793	,5262347	,4636947	,4088378
27	,5858620	,5133997	,4501891	,3950123
28	,5743746	,5008778	,4370768	,3816543
29	,5631123	,4886613	,4243464	,3687482
30	,5520709	,4767427	,4119868	,3562784
31	,5412460	,4651148	,3999871	,3442304
32	,5306333	,4537706	,3883370	,3325897
33	,5202287	,4427030	,3770263	,3213427
34	,5100282	,4319053	,3660449	,3104761
35	,5000276	,4213711	,3553834	,2999769
36	,4902232	,4110937	,3450324	,2898327
37	,4806109	,4010671	,3349829	,2800316
38	,4711872	,3912849	,3252262	,2705619
39	,4619482	,3817414	,3157536	,2614125
40	,4528904	,3724306	,3065568	,2525725
41	,4440102	,3633470	,2976280	,2440314
42	,4353041	,3544848	,2889592	,2357791
43	,4267688	,3458389	,2805429	,2278059
44	,4184008	,3374038	,2723718	,2201023
45	,4101968	,3291744	,2644386	,2126594
46	,4021537	,3211458	,2567365	,2054679
47	,3942684	,3133129	,2492588	,1985197
48	,3865376	,3056712	,2419988	,1918065
49	,3789584	,2982158	,2349503	,1853202
50	,3715279	,2909422	,2281071	,1790534

TABLE

TABLE III.

The Present Worth of one Pound, Comp. Interest.

<i>Years.</i>	<i>4 per C.</i>	<i>4½ per C.</i>	<i>5 per C.</i>	<i>6 per C.</i>
26	,3606892	,3184025	,2812407	,2198100
27	,3468166	,3046914	,2678483	,2073680
28	,3334775	,2915707	,2550936	,1956301
29	,3206514	,2790150	,2429463	,1845567
30	,3083187	,2670000	,2313775	,1741101
31	,2964603	,2555024	,2203595	,1642548
32	,2850579	,2444999	,2098662	,1549574
33	,2740942	,2339712	,1998762	,1461862
34	,2635521	,2238959	,1903548	,1379115
35	,2534155	,2142544	,1812903	,1301052
36	,2436687	,2050282	,1726574	,1227408
37	,2342969	,1961992	,1644356	,1157932
38	,2252854	,1877504	,1566054	,1092389
39	,2166206	,1796655	,1491479	,1030555
40	,2082890	,1719287	,1420457	,0972222
41	,2002779	,1645251	,1352816	,0917191
42	,1925749	,1574403	,1288396	,0865274
43	,1851682	,1506605	,1227044	,0816296
44	,1180464	,1441728	,1168613	,0770091
45	,1711984	,1379644	,1112965	,0726501
46	,1646139	,1320233	,1059967	,0685378
47	,1582826	,1263381	,1009492	,0646583
48	,1521948	,1208977	,0961421	,0609984
49	,1463411	,1156916	,0915639	,0575457
50	,1407126	,1107097	,0872037	,0542884

TABLE

TABLE IV.

Of COMPOUND INTEREST.

The Amount of one Pound per Annum, or Annuity, for Years; at the Rates of 2; $2\frac{1}{2}$; 3; $3\frac{1}{2}$; 4; $4\frac{1}{2}$; 5, and 6 per Cent. per Annum.

Years.	2 per Cent.	$2\frac{1}{2}$ per C.	3 per Cent.	$3\frac{1}{2}$ per C.
1	1,0000000	1,0000000	1,0000000	1,0000000
2	2,0200000	2,0250000	2,0300000	2,0350000
3	3,0604000	3,0756230	3,0909000	3,1062250
4	4,1216080	4,1525156	4,1836270	4,2149429
5	5,2040402	5,2563285	5,3091358	5,3624659
6	6,3081210	6,3877367	6,4684099	6,5501522
7	7,4342834	7,5474302	7,6624622	7,7794075
8	8,5829691	8,7361159	8,8923360	9,0516866
9	9,7546284	9,9545188	10,1591061	10,3684958
10	10,9497210	11,2033818	11,4638793	11,7313931
11	12,1687154	12,4834663	12,8077957	13,1419919
12	13,4120897	13,7955530	14,1920296	14,6019616
13	14,6803315	15,1404418	15,6177904	16,1130303
14	15,9739381	16,5189528	17,0863242	17,6769864
15	17,2934169	17,9319267	18,5989139	19,2956809
16	18,6392853	19,3802248	20,1568813	20,9710297
17	20,0120719	20,8647304	21,7615877	22,7050158
18	21,4123124	22,3863487	23,4144354	24,4996913
19	22,8405586	23,9460074	25,1168684	26,3571805
20	24,2973698	25,5446576	26,8703745	28,2796818
21	25,7833172	27,1832740	28,6764857	30,2694707
22	27,2989835	28,8628559	30,5367803	32,3289022
23	28,8449632	30,5844273	32,4528837	34,4604137
24	30,4218625	32,3490379	34,4264702	36,6665282
25	32,0302997	34,1577639	36,4592643	38,9498567

TABLE

TABLE IV.

The Amount of 1 l. Annuity, Compound Interest.

<i>Years.</i>	<i>2 per C.</i>	<i>2 ½ per C.</i>	<i>3 per C.</i>	<i>3 ½ per C.</i>
1	1,0000000	1,0000000	1,0000000	1,0000000
2	2,0400000	2,0450000	2,0500000	2,0600000
3	3,1216000	3,1370250	3,1525000	3,1836000
4	4,2464640	4,2781911	4,3101250	4,3746016
5	5,4163226	5,4707097	5,5256312	5,6370930
6	6,6329755	6,7168917	6,8019128	6,9753187
7	7,8982945	8,0191518	8,1420084	8,3938378
8	9,2142263	9,3800136	9,5491089	9,8974681
9	10,5827953	10,8021142	11,0265643	11,4913162
10	12,0061071	12,2882094	12,5778925	13,1807958
11	13,4863514	13,8411788	14,2067871	14,9716435
12	15,0258055	15,4640318	15,9171265	16,8699420
13	16,6268377	17,1599133	17,7129828	18,8821385
14	18,2919112	18,9321094	19,5986320	21,0150667
15	20,0235876	20,7840543	21,5785636	23,2759707
16	21,8245311	22,7193367	23,6574918	25,6725289
17	23,6975124	24,7417069	25,8403664	28,2128806
18	25,6454129	26,8550837	28,1323847	30,9056534
19	27,6712294	29,0635625	30,5390039	33,7599925
20	29,7780786	31,3714228	33,0659541	36,7855920
21	31,9692017	33,7831368	35,7192518	39,9927275
22	34,2479698	36,3033779	38,5052144	43,3922911
23	36,6178886	38,9370299	41,4304751	46,9958285
24	39,0826041	41,6891963	44,5019989	50,8155782
25	41,6459083	44,5652101	47,7270988	54,8645128

TABLE

TABLE IV.

The Amount of 1 l. Annuity, Compound Interest.

Years.	2 per Cent.	2 $\frac{1}{2}$ per C.	3 per Cent.	3 $\frac{1}{2}$ per C.
26	33,6709057	36,0117080	38,5530422	41,3131017
27	35,3443238	37,9120007	40,7096335	43,7590602
28	37,0512103	39,8598008	42,9309225	46,2906273
29	38,7922345	41,8562958	45,2188502	48,9107993
30	40,5680792	43,9027032	47,5754157	51,6226773
31	42,3794408	46,0022707	50,0026782	54,4294719
32	44,2270296	48,1502775	52,5027585	57,3345025
33	46,1115702	50,3540345	55,0778413	60,3412101
34	48,0338016	52,6128053	57,7301765	63,4531524
35	49,9944776	54,9282074	60,4620818	66,6740127
36	51,9943672	57,3014126	63,2759443	70,0076032
37	54,0342545	59,73339479	66,1742226	73,4576693
38	56,1149396	62,2272966	69,1594493	77,0288947
39	58,2372384	64,7829791	72,2342327	80,7249060
40	60,4019832	67,4025535	75,4012597	84,5502778
41	62,6100228	70,0876174	78,6632975	88,5095375
42	64,6822233	72,8398078	82,0231964	92,6073713
43	67,1594678	75,6608030	85,4838923	96,8486293
44	69,5026511	78,5523231	89,0484191	101,2383313
45	71,8927103	81,5161312	92,7198614	105,7816729
46	74,3305645	84,5540344	96,5014172	110,4840315
47	76,8171758	87,6678853	100,3965005	115,3509720
48	79,3535193	90,8595824	104,4083960	120,3882560
49	81,9405697	94,1310729	108,5406479	125,6018450
50	84,5794015	97,4843488	112,7968673	130,9979100

K k

TABLE

T A B L E IV.

The Amount of 1 l. Annuity, Compound Interest.

Years	4 per Cent.	4½ per Cent.	5 per Cent.	6 per Cent.
26	44,3117446	47,5706446	51,1134538	59,1563827
27	47,0842144	50,7113236	54,6691265	63,7057657
28	49,9675830	53,9933332	58,4025828	68,5281116
29	52,9662863	57,4230332	62,3227119	73,6397983
30	56,0849377	61,0070698	66,4388475	69,0581862
31	59,3283352	64,7523878	70,7607899	84,8016774
32	62,7014687	68,6662452	75,2988294	90,8897780
33	66,2095274	72,7562263	80,0637708	97,3431647
34	69,8579045	77,0302565	85,0669594	104,1837546
35	73,6522248	81,4966180	90,3203073	111,4347799
36	77,5983138	86,1639658	95,8363227	119,1208667
37	81,7022464	91,0413443	101,6281388	127,2681187
38	85,9703362	96,1382048	107,7095458	135,9042058
39	90,4091497	101,4644249	114,0950231	145,0584581
40	95,0255157	107,0303231	120,7997742	154,7619656
41	99,8265363	112,8466876	127,8397829	165,0476836
42	104,8195978	118,9247885	135,2317511	175,9505446
43	110,0123817	125,2764040	142,9933386	187,5075772
44	115,4128169	131,9138422	151,1430056	199,7580319
45	121,0293920	138,8499651	159,7001559	212,7435138
46	126,8705677	146,0982135	168,6851637	226,5081246
47	132,9453904	153,6726331	178,1194218	241,0986121
48	139,2632060	161,5879016	188,0253929	256,5645288
49	145,8337342	169,8593572	198,4266620	272,9584000
50	152,6670836	178,5030282	209,3479957	290,3359046

TABLE V.

Of COMPOUND INTEREST.

The Present Worth of one Pound per Annum, or
Annuity for Years, at the Rates of 2; $2\frac{1}{2}$; 3;
 $3\frac{1}{2}$; 4; $4\frac{1}{2}$; 5, and 6; per Cent. per Annum.

Years.	2 per Cent.	$2\frac{1}{2}$ per C.	3 per Cent.	$3\frac{1}{2}$ per C.
1	0,9803922	0,9756098	0,9708738	0,9661836
2	1,9415609	1,9274242	1,9134697	1,8996943
3	2,8838833	2,8560236	2,8285114	2,8016379
4	3,8077287	3,7619742	3,7170984	3,6730792
5	4,7134595	4,6458285	4,5797072	4,5150524
6	5,6014309	5,5081254	5,4171914	5,3285530
7	6,4719911	6,3493906	6,2302829	6,1145439
8	7,3254814	7,1701372	7,0196922	6,8739555
9	8,1622367	7,9708655	7,7851089	7,6076865
10	8,9825850	8,7520639	8,5302028	8,3166053
11	9,7868480	9,5142087	9,5256241	9,0015510
12	10,5753412	10,2577646	9,9540040	9,6633343
13	11,3483737	10,9831839	10,6349553	10,3027385
14	12,1062487	11,6909122	11,2960731	10,9205203
15	12,8492635	12,3813777	11,9379351	11,5174109
16	13,5777093	13,0550027	12,5611020	12,0941168
17	14,2918719	13,7121977	13,1661185	12,6513206
18	14,9920313	14,3533636	13,7535131	13,1896812
19	15,6784620	14,9788913	14,3237991	13,7098374
20	16,3514333	15,5891623	14,8774748	14,2124033
21	17,0112092	16,1845486	15,4150241	14,6979742
22	17,6580482	16,7654132	15,9369166	15,1671248
23	18,2922041	17,3321105	16,4436084	15,6204105
24	18,9139256	17,8849858	16,9355421	16,0583676
25	19,5234565	18,4243764	17,4131477	16,4815146

T A B L E V.

The Present Worth of 1l. Annuity, Comp. Interest.

<i>Years.</i>	<i>4 per Cent.</i>	<i>4 ½ per C.</i>	<i>5 per Cent.</i>	<i>6 per Cent.</i>
1	0,9615385	0,9569378	0,9523809	0,9433962
2	1,8860947	1,8726678	1,8594103	1,8333926
3	2,7750910	2,7489644	2,7232480	2,6730119
4	3,6298952	3,5875257	3,5459505	3,4651056
5	4,4518223	4,3899767	4,3294767	4,2123638
6	5,2421369	5,1578725	5,0756921	4,9173244
7	6,0020547	5,8927009	5,7863734	5,5823815
8	6,7327448	6,5958861	6,4632128	6,2097939
9	7,4353314	7,2687905	7,1078217	6,8016923
10	8,1108955	7,9127182	7,7217349	7,3600871
11	8,7604763	8,5289169	8,3064142	7,8868747
12	9,3850733	9,1185808	8,8632516	8,3838440
13	9,9856473	9,6828524	9,3925730	8,8526831
14	10,5631223	10,2228253	9,8986409	9,2949840
15	11,1183868	10,7395457	10,3796500	9,7122491
16	11,6522949	11,2340151	10,8377695	10,1058953
17	12,1656680	11,7071914	11,2740662	10,4772597
18	12,6592961	12,1599918	11,6895869	10,8276035
19	13,1339385	12,5932936	12,0853208	11,1581165
20	13,5903253	13,0079365	12,4622103	11,4699213
21	14,0291589	13,4047239	12,8211527	11,7640767
22	14,4511142	13,7844248	13,1630026	12,0415818
23	14,8558405	14,1477749	13,4885739	12,3033790
24	15,2469619	14,4954784	13,7986418	12,5503570
25	15,6220787	14,8282089	14,0939445	12,7833562

T A B L E

TABLE V.

The Present Worth of 1l. Annuity, Comp. Interest.

Years.	2 per C.	2 ½ per C.	3 per Cent.	3 ½ per C.
26	20,1210358	18,9506111	17,8768420	16,8903523
27	20,7068978	19,4640109	18,3270315	17,2853645
28	21,2812724	19,9648887	18,7641082	17,6670188
29	21,8443847	20,4535499	19,1884546	18,0357670
30	22,3964556	20,9302926	19,6004413	18,3920454
31	22,9377015	21,3954074	20,0004285	18,7362758
32	23,4683348	21,8491780	20,3887655	19,0688656
33	23,9885636	22,2918809	20,7657918	19,3902082
34	24,4985917	22,7237863	21,1318367	19,7006842
35	24,9986193	23,1451573	21,4872200	20,0006612
36	25,4888425	23,5562511	21,8322525	20,2904938
37	25,9694534	23,9573181	22,1672354	20,5709254
38	26,4406406	24,3486030	22,4924616	20,8410874
39	26,9025888	24,7303444	22,8082151	21,1024999
40	27,3554792	25,1027751	23,1147719	21,3550723
41	27,7994895	25,4661220	23,4123999	21,5991037
42	28,2347936	25,8206068	23,7013592	21,8348828
43	28,6615623	26,1664457	23,9819021	22,0626887
44	29,0799631	26,5038495	24,2542739	22,2827910
45	29,4901599	26,8330239	24,5187125	22,4954503
46	29,8923136	27,1541696	24,7754490	22,7009181
47	30,2865820	27,4674826	25,0247078	22,8994378
48	30,6731196	27,7731537	25,2667066	23,0912443
49	31,0520780	28,0713695	25,5016569	23,2765645
50	31,4236059	28,3623117	25,7297640	23,4556179

TABLE

TABLE V.

The Present Worth of 1 l. Annuity, Comp. Inter.

<i>Years.</i>	<i>4 per Cent.</i>	<i>4 $\frac{1}{2}$ per C.</i>	<i>5 per Cent.</i>	<i>6 per Cent.</i>
26	15,9827678	15,1466115	14,3751853	13,0031663
27	16,3295844	15,4513028	14,6430336	13,2105342
28	16,6630618	15,7428735	14,8981272	13,4061644
29	16,9837132	16,0218885	15,1410735	13,5907211
30	17,2920318	16,2888885	15,3724510	13,7648312
31	17,5884921	16,5443209	15,5928104	13,9290861
32	17,8735500	16,7888909	15,8026766	14,0840435
33	18,1476441	17,0228621	16,0025491	14,2302297
34	18,4111962	17,2467580	16,1929039	14,3681412
35	18,6646116	17,4610124	16,3741942	14,4982465
36	18,9082803	17,6660406	16,5468516	14,6209872
37	19,1425771	17,8622398	16,7112872	14,7367804
38	19,3678625	18,0499902	16,8678926	14,8460192
39	19,5844831	18,2296557	17,0170406	14,9490747
40	19,7927721	18,4015844	17,1590862	15,0462969
41	19,9930500	18,5661095	17,2943678	15,1380160
42	20,1856250	18,7235498	17,4232074	15,2245434
43	20,3707931	18,8742103	17,5459118	15,3061730
44	20,5488395	19,0183831	17,6627732	15,3831821
45	20,7200378	19,1563474	17,7740697	15,4558321
46	20,8846517	19,2883707	17,8800663	15,5243699
47	21,0429342	19,4147088	17,9610155	15,5890282
48	21,1951289	19,5356066	18,0771576	15,6500266
49	21,3414700	19,6512981	18,1687215	15,7075723
50	21,4821826	19,7620078	18,2559253	15,7618610

TABLE

TABLE VI.

Of COMPOUND INTEREST.

The Annuity which one Pound will purchase for
any Number of Years; at the Rates of 2;
2 $\frac{1}{2}$; 3; 3 $\frac{1}{2}$; 4; 4 $\frac{1}{2}$; 5, and 6 per Cent. per
Annum.

Years.	2 per Cent.	2 $\frac{1}{2}$ per C.	3 per Cent.	3 $\frac{1}{2}$ per C.
1	1,0200000	1,0250000	1,0300000	1,0350000
2	,5150495	,5188272	,5226108	,5264005
3	,3467547	,3501372	,3535304	,3569342
4	,2626238	,2658179	,2690271	,2722511
5	,2121584	,2152469	,2183546	,2214814
6	,1785258	,1815499	,1845975	,187668
7	,1545120	,1574954	,1605054	,1635445
8	,1365098	,1394674	,1424564	,1454767
9	,1225154	,1254569	,1284339	,1314460
10	,1113265	,1142588	,1172305	,1202414
11	,1021779	,1051060	,1080775	,1110920
12	,0945596	,0974871	,1004621	,1034840
13	,0881183	,0910483	,0940295	,0970516
14	,0826020	,0855365	,0885263	,0915707
15	,0778255	,0807665	,0837666	,0868251
16	,0736501	,0765990	,0796109	,0826848
17	,0699698	,0729278	,0759525	,0790431
18	,0667021	,0696701	,0727087	,0758168
19	,0637818	,0667606	,0698139	,0729103
20	,0611557	,0641471	,0672157	,0703610
21	,0587847	,0617873	,0648718	,0680366
22	,0566314	,0596466	,0627474	,0659321
23	,0546681	,0576964	,0608139	,0640188
24	,0528511	,0559128	,0590174	,0622728
25	,0512204	,0542759	,0574279	,0606740

TABLE

TABLE VI

The Annuity which one Pound will purchase,
Compound Interest.

Years.	4 per Cent.	4 $\frac{1}{2}$ per C.	5 per C.	6 per Cent.
1	1,0400000	1,0450000	1,0500000	1,0600000
2	,5301961	,5339976	,5378049	,5454369
3	,3603485	,3637734	,3672086	,3741098
4	,2754901	,2787437	,2820118	,2885915
5	,2246271	,2277916	,2309748	,2373964
6	,1907619	,1938784	,1970157	,2033626
7	,1666096	,1697015	,1728198	,1791350
8	,1485279	,1516097	,1547218	,1610359
9	,1344930	,1375745	,1406901	,1470222
10	,1232909	,1263788	,1295046	,1358680
11	,1141490	,1172482	,1203890	,1267929
12	,1065522	,1096662	,1128254	,1192770
13	,1001437	,1032754	,1064558	,1129601
14	,0946690	,0978203	,1010240	,1075849
15	,0899411	,0931138	,0963423	,1029628
16	,0858200	,0890154	,0922699	,0989521
17	,0821985	,0854176	,0886991	,0954448
18	,0789933	,0822369	,0855462	,0923565
19	,0761386	,0794073	,0827450	,0896209
20	,0735818	,0768761	,0802426	,0871846
21	,0712801	,0746006	,0779961	,0850046
22	,0691988	,0725457	,0759705	,0830456
23	,0673091	,0706825	,0741368	,0812785
24	,0655868	,0689870	,0724709	,0796790
25	,0640121	,0674390	,0709545	,0782267

TABLE

TABLE VI.

*The Annuity, which one Pound will purchase,
Compound Interest.*

<i>Years.</i>	<i>2 per Cent.</i>	<i>2 $\frac{1}{2}$ per C.</i>	<i>3 per Cent.</i>	<i>3 $\frac{1}{2}$ per C.</i>
26	,0496992	,0527688	,0559383	,0592054
27	,0482931	,0513769	,0545642	,0578524
28	,0469897	,0500879	,0532932	,0566027
29	,0457784	,0488913	,0521147	,0554454
30	,0446499	,0477776	,0510193	,0543713
31	,0435964	,0467390	,0499989	,0533724
32	,0426106	,0457683	,0490466	,0524415
33	,0416865	,0448594	,0481561	,0515724
34	,0408187	,0440068	,0473220	,0507597
35	,0400022	,0432056	,0465393	,0499984
36	,0392329	,0424516	,0458038	,0492842
37	,0385068	,0417409	,0451116	,0486133
38	,0378206	,0410701	,0444593	,0479821
39	,0371711	,0404362	,0438439	,0473878
40	,0365558	,0398362	,0432624	,0468273
41	,0359719	,0392679	,0427124	,0462982
42	,0354173	,0387288	,0421917	,0457983
43	,0348899	,0382169	,0416981	,0453254
44	,0343879	,0377304	,0412299	,0448777
45	,0339096	,0372675	,0407852	,0444534
46	,0334534	,0368268	,0403625	,0440511
47	,0330179	,0364067	,0399605	,0436692
48	,0326018	,0360060	,0395778	,0433065
49	,0322040	,0356235	,0392131	,0429617
50	,0318032	,0352581	,0388655	,0426337

TABLE VI.

The Annuity which one Pound will purchase,
Compound Interest.

Years.	4 per Cent.	4 $\frac{1}{2}$ per C.	5 per Cent.	6 per Cent.
26	,0625674	,0660214	,0695643	,0769044
27	,0612385	,0647195	,0682919	,0756972
28	,0600130	,0635208	,0671225	,0745926
29	,0588799	,0624146	,0660455	,0735796
30	,0578301	,0613915	,0650514	,0726489
31	,0568554	,0604435	,0641321	,0717522
32	,0559486	,0595632	,0632804	,0710023
33	,0551036	,0587445	,0624900	,0702729
34	,0543148	,0579819	,0617554	,0695984
35	,0535773	,0572705	,0610717	,0689739
36	,0528869	,0566058	,0604345	,0683948
37	,0522396	,0559840	,0598398	,0678574
38	,0516319	,0554017	,0592842	,0673581
39	,0510608	,0548557	,0587646	,0668938
40	,0505235	,0543431	,0582782	,0664915
41	,0500174	,0538616	,0578223	,0660589
42	,0495402	,0534087	,0573947	,0656834
43	,0490899	,0529824	,0569933	,0653331
44	,0486645	,0525807	,0566163	,0650061
45	,0482625	,0522020	,0562617	,0647005
46	,0478821	,0518447	,0559282	,0644149
47	,0475219	,0515073	,0556142	,0641477
48	,0471807	,0511886	,0553184	,0638977
49	,0468571	,0508872	,0550397	,0636636
50	,0465502	,0506021	,0547767	,0634443

C H A P. X.

The Use of DECIMALS in Vulgar, Duodecimal, and Sexagesimal Fractions.

Vulgar Fractions in Decimals.

I HAVE already shewn the Method of finding the Decimal equivalent to any *Vulgar Fraction*, in *Reduction*; What I propose here, is to shew with how much greater Ease and Pleasure any of the Operations of *Vulgar Fractions* are wrought by *Decimal Numbers*. I shall exemplify the Matter in the *common Rules* as follows.

Addition.

Example 1. What is the Sum of $\frac{7}{9}$ and $\frac{2}{7}$ of a Pound?

Add $\left\{ \begin{array}{l} \text{The Decimal of } \frac{7}{9} = ,7777 \\ \text{The Decimal of } \frac{2}{7} = ,2857 \end{array} \right\} \begin{array}{l} \text{By the general} \\ \text{Decimal Table,} \\ \text{P.} \end{array}$

The Sum of both is $= \overline{1.1,2062} = 1\text{ l. } 4\text{ s. } 1\frac{1}{2}\text{ d.}$ Anf.

Example 2. What is the Value of $\frac{1}{16}$ and $\frac{5}{6}$ of a Shilling?

Add $\left\{ \begin{array}{l} \text{The Decimal of } \frac{1}{16} = ,0625 \\ \text{To the Dec. of } \frac{5}{6} = ,8333 \end{array} \right\} \begin{array}{l} \text{By the said Ta-} \\ \text{ble.} \end{array}$

Their Sum is $= \overline{0,8958} = 10\frac{1}{4}\text{ d.}$ Anf.

260 *The Use of Decimals in the*

Example 3. What is $\frac{2}{13}$, $\frac{3}{13}$ and $\frac{7}{13}$ of a Yard?

$$\text{Add } \left\{ \begin{array}{l} \text{The Decimal of } \frac{2}{13} = ,1538 \\ \text{And the Dec. of } \frac{3}{13} = ,2307 \\ \text{And the Dec. of } \frac{7}{13} = ,5384 \end{array} \right\} \text{ By the Tab.}$$

The Sum, of Course, is $\frac{12}{13} = 0,9239 = 2 \text{ F. } 9 \text{ In. An.}$

Example 4. What is the Value of $\frac{2}{3}$, $\frac{4}{5}$, $\frac{2}{9}$, and $\frac{3}{4}$ of $\frac{5}{6}$ of a Hundred Weight Averdupois?

$$\text{Add } \left\{ \begin{array}{l} \text{The Decimal of } - \frac{2}{3} = ,6666 \\ \text{The Decimal of } - \frac{4}{5} = ,8000 \\ \text{The Decimal of } - \frac{2}{9} = ,2222 \\ \text{The Dec. of } \frac{3}{4} \text{ of } \frac{5}{6} = \frac{5}{8} = ,6250 \end{array} \right\} \text{ By the Table.}$$

The Sum of all is $- \text{C. } 2,3138 = 2 : 1 : 7 : 2 \text{ C. Q. lb. oz.}$

Example 5. What Number of Years, do $476\frac{7}{11}$, $36\frac{9}{16}$, $21\frac{10}{11}$, $7\frac{13}{14}$, $1\frac{14}{17}$ and $\frac{17}{19}$ of a Year, make?

$$476\frac{7}{11} = 476,6363 \text{ Years.}$$

$$36\frac{9}{16} = 36,5625$$

$$21\frac{10}{11} = 21,9090$$

$$7\frac{13}{14} = 7,9285$$

$$1\frac{14}{17} = 1,8234$$

$$\frac{17}{19} = 0,8944$$

Those mixed Fractions being set down in order, and the Decimals of the Fractional Parts being found in the General Table and set down opposite thereto; add them, and their Sum will be

545,7541 Years.

That is, 545 Years, 9 Months, 3 Weeks, and 2 Days nearly.
Subtraction.

Subtraction.

Example 1. What is $\frac{5}{6}$ less $\frac{3}{8}$ of a Pound Sterling?

From	—	—	$\frac{5}{6} = ,833$	}	By the general Decimal Table.
Subtract	—	—	$\frac{3}{8} = ,375$		
There remains					
			$0,4583 = 9s. 2d. Ans.$		

Example 2. What is the Value of $\frac{2}{3}$ of $\frac{7}{8}$ less $\frac{3}{4}$ of $\frac{5}{9}$ of a Rod?

From	—	—	$\frac{2}{3}$ of $\frac{7}{8} = \frac{7}{12} = ,583$	}	By the Table.
Subtract	—	—	$\frac{3}{4}$ of $\frac{5}{9} = \frac{5}{12} = ,416$		
There remains					
			$0,167 = 0 Y. 2 F. 9 In.$		

Example 3. What is $14\frac{2}{9}$, less $\frac{5}{7}$ of a Pound Troy?

From	—	—	$14\frac{2}{9} = 14,2222$	}	By the Table.
Take	—	—	$\frac{5}{7} = 0,7142$		
There remains					
			$13,5080 = 13 lb. 6 oz. 2 pw.$		

Example 4. What is $170\frac{17}{19}$, less $159\frac{2}{19}$ C. Weight, Averdupois?

From	—	—	$170\frac{17}{19} = 170,8947$	}	By the Table.
Take	—	—	$159\frac{2}{19} = 159,4736$		
There remains the Answer					
			$11,4211 C.$		

Multi.

Multiplication.

Example 1. What is $\frac{4}{7}$ of a Yard multiplied by $\frac{2}{9}$ of a Yard?

$$\begin{array}{r} \text{Multiply} \quad \frac{4}{7} = .5714 \\ \text{By} \quad \frac{2}{9} = .2222 \\ \hline 2) .11428 \end{array}$$

The Product is 1 Fg. 20,3 Inq. = .12697

Example 2. Suppose a Piece of Timber $14\frac{3}{7}$ Feet in Length, $\frac{8}{11}$ of a Foot Wide, and $\frac{1}{2}$ of $\frac{5}{8}$ of a Foot thick, What is the Solid Content of that Piece?

$$\begin{array}{r} \text{Multiply the Length} \quad 14\frac{3}{7} = 14.4285 \\ \text{By the Width} \quad \frac{8}{11} = .7272 \\ \hline \text{The Product will be found to be} \quad 10.49348 \\ \text{That multiplied by the Thick-} \\ \text{ness, which is } \frac{1}{2} \text{ of } \frac{5}{8} = \frac{5}{16} \quad \left. \vphantom{\frac{1}{2} \text{ of } \frac{5}{8}} \right\} = .3125 \end{array}$$

The Product is the Content sought = 3,27818, &c.

That is, 3 Feet, 480 Inches, 46 $\frac{1}{2}$ Quarters, solid Content the Answer.

Example 3. There is a Cistern $10\frac{1}{6}$ Feet in Length, $4\frac{5}{9}$ Deep, and $5\frac{4}{12}$ Wide; Quere how many Corn Gallons it will hold?

Multi-

Multiply the Length — $10\frac{1}{6} = 10,1\bar{6}$

By the Width — $5\frac{4}{11} = 5,3\bar{6}$

That Product will be — $54,526\bar{3}$

Multiply that by the Depth — $4\frac{5}{9} = 4,5\bar{5}$

The Product is the Solid Content
in Feet and Parts } $= 248,3976 \text{ \&c.}$

Multiply that by the Gallons in
a Solid Foot; viz. $6\frac{13}{32}$ } $= 6,40625$

The Product is the Number of
Corn Gallons sought, viz. } $= 1591,2969 \text{ \&c.}$

Note ; When there is given any Number of pure Fractions to be multiplied into one another ; You may multiply the *Numerators* and *Denominators* into one another, and the Products will be a Fraction, whose Value in *Decimals* you may find as before taught,

Thus, { Multiply $\frac{1}{9}, \frac{2}{7}, \frac{5}{11}, \frac{9}{17}$ all into one another.
Then $1 \times 2 = 2, 2 \times 5 = 10, 10 \times 9 = 90$
And $9 \times 7 = 63, 63 \times 11 = 693, 693 \times 17 = 11781$ } $\frac{10}{1309}$
But $\frac{10}{1309} = 0,00764$ The Answer

Division.

Example 1. Divide $\frac{5}{7}$ by $\frac{1}{17}$

Thus

264 . The Use of Decimals in the

Thus $\frac{1}{17} = 0,0588$, $,714285 = \frac{5}{7}$ (12,14754 = The Answer.)

588

1262

1176

..868

588

2805

2352

4437

4116

3211

2940

2714

2352

362

Example 2. What is $\frac{16}{19}$ divided by $\frac{2}{5}$ of $\frac{8}{15}$?

Thus $\frac{2}{5}$ of $\frac{8}{15} = \frac{16}{75} = ,213$, $,842105263 = \frac{16}{19}$

21 842105263

,192 ,7578947368 (3,947368

576

1818

1728

..909

768

1414

1344

..707

576

1313

1152

1616

1536

..808

There is somewhat remarkable and uncommon in the Work of this Question, viz. First, that the 1st, 3d, 5th, &c. Remainders, with the Numbers taken down, consist of 2 Pair of the same Figures; Secondly, That the 2d, 4th, &c. Remainders, with the Figures taken down, are just half the others; Thirdly, That where the Remainders of Pairs are prime Numbers, the next is a Remainder is a Pair of even Numbers, and then goes on as before, as at the 5th and 6th.

Ex.

Example 3. What is $2\frac{7}{18}$ of a Pound, divided by $\frac{10}{11}$?

Thus $\frac{10}{11} = ,90$ $2,388 = 2\frac{7}{18}$

$$\begin{array}{r}
 ,00 \quad 23 \\
 ,9 \overline{) 2,365} \quad (2,627 = 2\text{ l. } 12\text{ s. } 6\frac{1}{2}\text{ d.} \\
 \underline{18} \\
 56 \\
 \underline{54} \\
 25 \\
 \underline{18} \\
 70 \\
 \underline{63} \\
 7
 \end{array}$$

(The Answer.)

Example 4. What is it per C. Weight, when $21\frac{7}{11}$ C. costs $67\frac{7}{16}$ Pounds?

Thus $21\frac{7}{11} = 21,63$ $67,4375 = 67\frac{7}{16}$

$$\begin{array}{r}
 21 \quad 674375 \\
 21,42 \overline{) 66,763125} \quad (3,11685\text{ l.} \\
 \underline{6426} \\
 2503 \\
 \underline{2142} \\
 3611 \\
 \underline{2142} \\
 14692 \\
 \underline{12852} \\
 18405 \\
 \underline{17136} \\
 12690 \\
 \underline{10710} \\
 1980
 \end{array}$$

That is $3,11685\text{ l.} = 3\text{ l. } 2\text{ s. } 4\text{ d. per C. the Answer.}$

M m

Extraction

Extraction of Roots.

Example 1. What is the Square Root of $\frac{25}{36}$?

The Decimal of $\frac{25}{36} = .694444 = \frac{5}{6} =$ the Root
(found.

$$\begin{array}{r} 64 \\ 163 \overline{) 544} \\ \underline{489} \\ 1663 \overline{) 5544} \\ \underline{4989} \\ 555 \end{array}$$

Example 2. Required the Square Root of the Surd
Fraction $\frac{13}{17}$?

The Decimal of $\frac{13}{17} = .7647058$ &c. the Root of which
extracted will be $.8744726$ &c. the Answer.

Example 3. Extract the Square Root of $58\frac{7}{9}$

The Decimal of the mixed Fraction $58\frac{7}{9} = 58,7$

Then extract thus; $58,7 (7,66 = 7\frac{2}{3} =$ the Root
(found.

$$\begin{array}{r} 42 \\ 146 \overline{) 977} \\ \underline{876} \\ 1526 \overline{) 19177} \\ \underline{9156} \\ 1021 \end{array}$$

Exam-

Example 4. Extract the Cube Root of $\frac{3}{16}$

This is best (as being vastly soonest and easiest) done by *Logarithms*, thus,

From the *Logarithm* of the Numerator 3 = 0,4771212

Subtract the *Logar.* of the Denominator 16 = 1,2041200

There remains the *Lagar.* of the Dec. .1875 = 9,2730012

Add to the *Index* of that *Logarithm* 20 — ,29,2730012

One Third of which *Logarithm* is
the *Logarithm* of the Root sought } ,572358 = ,9,7576670

The Cube Root then of $\frac{3}{16}$ is ,572358 which was to be found.

Example 5. What is the Cube Root of $512\frac{13}{15}$?

First, From the *Logarithm.* of the }
Numerator of the Fractional Part } — 13 = 1,113943

Subtract the *Logarith.* of the Denominator 15 = 1,176091

There remains the *Logarithm* of the Dec. ,86 = ,9,9378;2

To which add the *integral* Part 512

The Sum is the equivalent *mixed* Dec. $512,86 = 2,710004$

A Third of whose *Logarithm* is
the *Logarithm* of the Cube Root } 8,0045 = 0,903334
sought — — — —

The Cube Root therefore of $512\frac{13}{15}$ is 8,0045

Note. The Use of *Decimals* is not only very obvious in all Parts of the Doctrine of *Vulgar Fractions*, but absolutely necessary in *Extraction of Roots*; which sometimes else cannot be done.

The Use of DECIMALS in Duodecimal Arithmetick.

Duodecimals are a Sort of *Fractions* made use of in *Measurement*; Where one Foot is the *Integer*; The Foot is divided into twelve Parts or *Inches*; one of these, into twelve others, and so on dividing by twelve. Whence as ten is the *Common Denominator* in *Decimals*, so twelve is the *Common Denominator* in *Duodecimal Fractions*.

The Notation and Reading of *Duodecimals* is

Thus { Feet, Primes, Seconds, Thirds, Fourths, &c.
 15 : 69 : 10 : 66 : 11 &c.

Now because this kind of *Arithmetic* is useful to Persons concern'd in *Building, Measuring, &c.* and the most useful Parts, viz. *Multiplication, Division, and Extraction*, being by far the most difficult; I thought it very proper (and hope it will be very acceptable) to shew how those Operations may be most easily and speedily perform'd by *Decimal Arithmetick*.

To that End I have made the following Table for the ready converting any *Duodecimals* into *Decimals*, and the contrary. The Use of which, to those who understand any *Decimal Tables* at all, is very obvious and easy.

The Duodecimal Table.

Duodecimals.	The Decimal Parts.			
	Primes. '	Seconds "	Thirds. '"	Fourths. '"'
1	,083333	,006944	,000578	,000048
2	,166666	,013888	,001157	,000096
3	,25	,020833	,001736	,000144
4	,333333	,027777	,002314	,000192
5	,416666	,034722	,002893	,000241
6	,5	,041666	,003472	,000289
7	,583333	,048611	,004051	,000337
8	,666666	,055555	,004629	,000384
9	,75	,0625	,005208	,000435
10	,833333	,069444	,005787	,000482
11	,916666	,076388	,006365	,000530

This Table, as I said, being so easy, needs no Instructions for its Use; nor shall I pretend to say *Decimals* are of any Service in the Rules of *Addition* and *Subtraction* of *Duodecimals*.

But

But their extream Utility in the aforefaid Operations of *Multiplication*, *Division*, and *Extraction* of Roots, of *Duodecimals*; will be undeniably evident by the enfuing Examples.

Multiplication.

Example 1. What is 9 F, 10' multiplied by 8 F. 08'?

$$\begin{array}{r}
 \text{Multiply the Decimal of } 9 : 10 = 9,83 \\
 \text{By the Decimal of } 8 : 08 = 8,6 \\
 \hline
 9) 5900 \\
 \quad 6355 \\
 \hline
 \quad 78666 \\
 \hline
 \text{The Product is the } \left. \begin{array}{l} \text{Feet} \\ \text{Answer, viz.} \end{array} \right\} 85 : 2 : 8 = 85,222 \text{ Feet.}
 \end{array}$$

Example 2. What is the Product of 40 F. 09' : 10'' : by 11' : 09''?

$$\begin{array}{r}
 \text{Multiply the Decimal of } 40 : 09 : 10 = 40,8194 \\
 \text{By the Decimal of } \quad \quad \quad 11 : 09 = 9,7918 \\
 \hline
 \end{array}$$

In fuch Cafes where the *Decimals* run far, and terminate in *Repetends*, 'tis best to multiply by the *contracted*, or *inverted* Way, heretofore taught, thus;

$$\begin{array}{r}
 40,81944 \\
 9,79180 \\
 \hline
 3673750 \\
 285736 \\
 36737 \\
 468 \\
 272 \\
 \hline
 \end{array}$$

Answer. 39 F. 11' : 07'' : 06''' : 06 = 39,96903 Feet.

Example

Example 3. What is 175 Feet, 00' : 04'' : by 08'''?

Multiply the Decimal of 175 : 00 : 04 : 00 = 175,027
By the Decimal of — — 08 = ,000386

1050168

1400222

5250833

The Product is 2 F
the Answer, viz. 3 0 : 00 : 09 : 08 : 07 = ,067560721

Example 4. { Multiply 17 : 09 : 02 : 06 = 17,76736
By — — 6 = 6

The Product is the Answer 106 F. 07' : 03'' = 106,60416

Example 5. What is the Square of 12 Feet, 09' : 07'' : 10'''?

Thus { 12 ; 09 : 07 : 10 } = 12,804390
12 : 09 : 07 : 10 } = 93408,21
Inverted

12804390
2560878
1024351
5121
384
115

Answer 163 Feet, 11' 05'' 01''' 09''' = 163,95239

Division.

Division.

Example 1. Divide 12 Feet, 10' : 07" : by 3.

Thus, the *Decimal* of 12 Feet, 10' : 07" is 12.88194

Then 3) $12,88194 \cdot (4,2939814 = 4 : 03 : 06 : 04$

$$\begin{array}{r}
 12 \\
 \hline
 \cdot \cdot 8 \\
 6 \\
 \hline
 28 \\
 27 \\
 \hline
 11 \\
 9 \\
 \hline
 29 \\
 27 \\
 \hline
 24 \\
 24 \\
 \hline
 \cdot \cdot 4 \\
 3 \\
 \hline
 14 \\
 12 \\
 \hline
 24
 \end{array}$$

Example 2. Divide 14 Feet, 07' : 09" : by 06' : 04"

The *Decimal* of $\left\{ \begin{array}{l} 14:07:09 = 14,64588 \\ 06:04 = ,527 \end{array} \right.$

Then $\begin{matrix} ,527) & 14,64583 \\ & 52 & 146458 \end{matrix}$

475) 13,18125 (27,75 = 27 Feet, 09"

$$\begin{array}{r} 950 \\ \hline \bullet 3681 \\ 3325 \\ \hline \bullet 3562 \\ 3325 \\ \hline 2375 \\ 2375 \\ \hline \dots \end{array}$$

Exam-

Example 3. Divide 5 Feet, by 1 Foot, 02' : 03" : 11"

The Decimal of 1 Foot, 02' : 03" : 11" = 1,193865

Then 1,193865) 5,000000 (4,188079 =
 4 775460 (4 F. 02' : 03" : 01"

• 224540

119386

105154

95509

• 9645

9550

• 94

83

11

10

1

Feet

Example 4. { Divide 32 : 10 : 11 : 06 = 32,913194
 { By 8 : 01 : 10 : 11 = 8,159142

Thus 8,159142) 32,913194 (4,033904 =

32636568 (4 F. 00' : 04" : 10"

• 276626

244774

• 31852

24477

• 7375

7343

1808 32

2588 32

• • •

• • •

• • •

• • •

• • •

• • •

Extraction

Extraction of Roots.

Example 1. What is the Side of the Square of

F. $163 : 11 : 05 : 01 : 09 ?$

The Decimal of those Duodecimals is 163,95239, the Square Root of which extracted either by Logarithms or in the com-

mon way, gives the Side $12 : 09 : 07 : 10$ for Answer.

Example 2. What is the Side of the Cube, whose Solid

Content is $1 : 10 : 07 : 06 : 10 ?$

By Logarithms

Thus, the Decimal $1 : 10 : 07 : 06 : 10 = 1,885898$

Then the Logarithm of $1,885898 = 0,2755182$

One Third of that is $1,23549 = 0,0918394$

But 1,23549 is the Decimal of $1 : 02 : 09 : 10 : 11$ Which is the Length of the Side of the Cube proposed.

As in Vulgar, so in Duodecimal Fractions, the Extraction of Roots, can be performed, no way so well as by Decimal Parts; and the other Laborious Operations are hereby rendered easy and concise.

The Use of Decimals in Sexagesimal Arithmetick.

Sexagesimals are those Fractions which have 60 for their common Denominator; and are chiefly used in Computations of Motion and Time.

Hence this Kind of Arithmetick is proper to Astronomy, which, as it is a Science of Motion, and Time, makes use thereof in all its Calculations: Hereby it is the Astronomer calculates the Motion, Place, Magnitude, Distance, Time, Aspects, and other Phenomena of the Heavenly Bodies; the Sun, Moon, Planets, Comets, and Stars.

The Notation and Reading of *Sexagesimals* is in this Manner following ;

Viz. \S Signs, Degrees, Minutes, Seconds, Thirds, &c.
 \mathcal{C} 05 : 26 : 57 : 53 : 47 &c.

And as 60 is one *Degree* of *Motion*, so it is one *Hour* of *Time* ; hence *Sexagesimals* properly so called, begin only at *Minutes*, and go to *Seconds*, *Thirds*, &c. forwards, in both *Motion*, and *Time* ; though in common, it comprehends any *Division* of either.

But as all *Astronomical* Calculations are made from *Sexagesimal* Numbers, already computed and disposed into Tables of various Sorts ; if I would shew or demonstrate the Use of *Decimals*, and their Preference to *Sexagesimal Numbers* in these Kind of Computations ; I must first suppose those *Sexagesimal* Tables, made into *Decimal* ones ; and if such a thing were once done, I believe 'twould be no very hard Task to make good the Proposition asserted.

The Reader need only judge of this by the following Example of *Addition* in both *Species*.

<i>Sexagesimally.</i>					<i>Decimally.</i>
S	D	M	S		S
03	: 21	: 57	: 49	=	3,73211
11	: 29	: 47	: 58	=	11,99331
10	: 18	: 59	: 37	=	10,63311
09	: 25	: 17	: 43	=	9,84317
08	: 29	: 59	: 57	=	8,99996
11	: 18	: 43	: 49	=	11,62433
Sum 08	: 24	: 46	: 53	=	8,82601

In this *Specimen* I think 'tis easy to observe how concise, simple, natural and easy the Operation by *Decimals* is if compared with the *Sexagesimal* Process ; which therefore I think must needs prove the Preference and Excellency of those Tables in *Decimals*.

But since none as yet have said any thing about this Affair, nor have we any *Astronomical* Tables in *Decimals*, I shall give a *Specimen* thereof in the *Mean Motions* of all the *Planets* for one whole *Year*, *Day*, *Hour*, and *Minute*, in the Table subjoin'd.

Planets

Planets.	A Year.	A Day.	An Hour.	A Minute.
	S.	S.	S.	S.
Sun	11,99025	0,03285	0,001367	0,000018
Moon	4,31271	0,43921	0,018292	0,000294
Saturn	0,40709	0,00111	0,000046	0,000000
Jupiter	1,01096	0,00276	0,000110	0,000000
Mars	6,37574	0,01749	0,000728	0,000009
Venus	7,49255	0,05340	0,00222	0,000036
Mercury.	1,79005	0,13639	0,005683	0,000092

Such then is the Form, and such would be the Difference of *Decimal* and *Sexagesimal Tables*; The Numbers here are *homogeneous*, all of one Sort; in them, they are *heterogeneous*, or consist of diverse Sorts; here they are *Uniform* and to be wrote as *integral* Numbers, there they are ranged in a different Form and in diverse *Classes*, as all mixed Numbers are; besides the great Ease and Facility of Working *Decimal* in Comparison of *Sexagesimal* Numbers, as I before observed. Upon all these Accompts, and several others I might mention, A Set of *Astronomical Tables* in *Decimal Numbers* must certainly be much more Useful, and every way preferable to the present *Sexagesimal Tables*.

After having turn'd your *Sexagesimal Numbers* into *Decimals*, they are to be worked in the same Manner as *Duodecimals* through all the *Rules*, as is there taught; and therefore needs not be here again repeated. Only, I would here observe, that the Rules of *Multiplication* and *Division*, which are here often necessary, cannot be perform'd without a great deal of Difficulty, or a long and tedious Process, whereas by *Decimals* 'tis done with the utmost Facility and Expedition.

To this End, I have taken Care that the Reader should not want large and sufficient Tables for the expeditious turning of his *Sexagesimals* into *Decimals*, and the contrary; the like of which are not to be found elsewhere, that I know of.

C H A P. XI.

The Use and Management of DECIMALS
(after a new Manner) by Logarithms.

THERE may chance to happen to the Reader a double Advantage in this Chapter; for first, he may here perceive, not only the common, but an *entire new Management of Decimals by Logarithms*; and *secondly*, he may here as well as any where learn the whole noble and excellent *Art of Logarithmical Arithmetick*, if he has not learnt it already; for *Decimals and Integers* having the same essential Properties, the *Logarithms* of both are the same, and differ only in their *Indexes*.

But that the *young Student* may the better understand how to vary and adjust the *Index* of the *Logarithm*, I have in the following Table given *all Variety of Cases* that can happen to a Number, its Logarithm, and Index, under the various Conditions, and Denominations of *Whole Number, Mixed Number, Pure Decimal, Repeating Decimals, Decimals with Cyphers prefixed, &c.* as follow.

			Index Logar.
Whole Numbers	—	5243	= 3,7195799
Mixed Numbers	—	{ 524,3	= 2,7195799
		{ 52,43	= 1,7195799
		{ 5,243	= 0,7195799
A Perfect Decimal	—	,5243	= 9,7195799
Decimals with Cyphers prefixed		{ ,05243	= 8,7195799
		{ ,005243	= 7,7195799
		{ ,0005243	= 6,7195799
A Single Repetend	—	3,	= 0,5228787
Mixed Single Repetends	{	43,3 or 43	= 1,6368221
		243,3 or 243	= 2,3862016
		5243,3 or 5243	= 3,7196075
Compound Repetends	—	{ 4,3	= 0,6378333
		{ 2,43	= 0,3860408
		{ 3,243	= 0,7196234

Com-

		Index	Logar.
Compound Repetends with Cyphers	{	,00003	= 5,5228787
		,00043	= 6,6378333
		,00243	= 7,3860408
		,05243	= 8,7196234
Mix'd Compound Repetends	{	243	= 2,3863818
		52,43	= 1,7196000
The same with Cyphers	{	,000243	= 6,3863818
		,005243	= 7,7196000

From this general Scheme, the following Observations may be made relating to the *Logarithm*, and determining its *Index* for any Kind of Number.

Observation 1. That the *Index* of the *Logarithm* of any whole Number, is always one less than the Number of Places of Figures in the whole Number.

Observation 2. That the *Logarithm* of any Number, whether *Integral*, *Mix'd*, or wholly *Decimal*, is the very same; only the *Index* differs and must be adjusted solely in regard of the *integral* Part of the Number; as *per Observ. 1.*

Observation 3. That if there be no *integral* Part but the Number is entirely *Decimal*, and the first left-hand Figure be one of the *nine Digits*, the *Index* is (0).

Observation 4. That if the Number be entirely *Decimal*, and have any Number of Cyphers prefixed, the *Index* (being in this Case dotted on both sides) must be such as, when subtracted from 9, the Remainder may express the Number of Cyphers prefixed.

Observation 5. That any *Repetend*, or Set of *Circulating* Numbers, whether *Whole*, or *Decimal*, observe all the Rules of *terminate* Numbers aforementioned, relating to the *Index*; but the *Logarithm* is different.

Observation 6. That the *Logarithm* varies, according as the same Figures are either *terminate* or *repetends*; and again as those *Repetends* make either a *Part*, or the *Whole Number*; or thus, the *Logarithm* is bigger or less as the first Figure of the *Repetend* is so.

As to what concerns the *Adding* and *Subtracting* of *Indexes*, that may be thoroughly understood by the following Table of all the *Varieties* that can happen in that Affair.

Addi.

278 The Use and Management of Decimals

Addition.

$$\begin{array}{r} 1. \text{ To } - 2,5132176 \\ \text{Add } - 1,8061800 \\ \hline \text{Sum } = 4,3193976 \end{array}$$

$$\begin{array}{r} 2. \text{ To } - ,3,3916407 \\ \text{Add } - ,5,2041200 \\ \hline \text{Sum } = ,18,5957607 \end{array}$$

$$\begin{array}{r} 3. \text{ To } - 2,2671717 \\ \text{Add } - ,8,1414498 \\ \hline \text{Sum } = 0,4086215 \end{array}$$

$$\begin{array}{r} 4. \text{ To } - ,8,5132176 \\ \text{Add } - 3,8061800 \\ \hline \text{Sum } = 2,3193976 \end{array}$$

$$\begin{array}{r} 5. \text{ Add } \left\{ \begin{array}{l} ,9,2671717 \\ ,5,5132176 \\ ,8,8061800 \end{array} \right. \\ \hline \text{Sum } = ,3,5865693 \end{array}$$

Subtraction.

$$\begin{array}{r} 1. \text{ From } - 4,3193976 \\ \text{Subtract } 1,8061800 \\ \hline \text{Rem. } = 2,5132176 \end{array}$$

$$\begin{array}{r} 2. \text{ From } - ,18,5957607 \\ \text{Take } - ,3,3916407 \\ \hline \text{Rem. } = ,5,2041200 \end{array}$$

$$\begin{array}{r} 3. \text{ From } - 0,4086215 \\ \text{Take } - ,8,1414498 \\ \hline \text{Sum } = 2,2671717 \end{array}$$

$$\begin{array}{r} 4. \text{ From } - 0,4086215 \\ \text{Take } - 2,2671717 \\ \hline \text{Sum } = ,8,1414498 \end{array}$$

$$\begin{array}{r} 5. \text{ From } - ,7,4086215 \\ \text{Take } - ,9,2671717 \\ \hline \text{Sum } = ,8,1414498 \end{array}$$

To understand the better what concerns the Ordering and Adjusting the *Indices*, in the foregoing Examples, I have sub-join'd the following *Scheme* of the Number of Cyphers, and their corresponding *Indices*.

Numb. of Cy. 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, &c.
Their Indic. 9, 8, 7, 6, 5, 4, 3, 2, 1, 0, 19, 18, 17, 16, 15, &c.

Hence observe, In *adding Indices*, 1. If both be *Affirmative*, their *Sum* is *Affirmative*. 2. If both be *Negative*, and the *Sum* be under 10, add 10 thereto; but if above 10, or just 10, cast away 10; the *Remainder* is *negative*. 3. If one *Index* be *affirmative*, and the other *negative*; the *Sum* if under 10 is *negative*: if just 10, or above 10, cast 10 away; the *Remainder* is *affirmative*.

In

In *Subtracting Indices*, observe, 1. If they are both *Affirmative*, and the Higher be the *Greater*, the Remainder is *Affirmative*; if the Lower be the *Greater*, the Remainder is *Negative*, (10 being added to the *higher*.) 2. If one or both be *Negative*, and the *higher* smaller than the *lower*, add 10 to it; than if the *higher* be of *greater Value*, the Remains are *Affirmative*; if not, they are *Negative*.

In order to understand the *Art* of Logarithms, and the dexterous Management of Numbers (more particularly *circulating Decimal Numbers*) thereby, 'twill be absolutely necessary to understand, and that perfectly well, the following *Logarithmetical Problems*.

Problem 1. To find the *Arithmetical Complement* of any given *Logarithm*.

Rule. Begin at the Left-hand to subtract (*mentally*) each Figure from 9, and the last of all from 10.

Exam. What is the *Arithm. Compl.* of the Log. 3,8649262
Answer (*per Rule*) is = 6,1350738

Problem 2. To find the *Logarithm* of any *terminate Number* under 10000000.

Rule. Take the *Logarithm* out of the Tables to the *four first Figures* of any given Number of above *four Places*, and also the next *greater Logarithm*; then take the *Difference* of those two *Logarithms*, and multiply it by the remaining Figures of the given Number; from the Product cut off so many Places of Figures to the Right-hand, as were the remaining Figures above *four*; then add the other Part of the Product to the *Logarithm* of the *four Figures* first taken out of the *Canon*; that Sum is the *Logarithm* sought.

Example. Required the *Logarithm* of 1012659?

The *Logarithm* of — { 1012 = 3,0051805
 1013 = 3,0056094

Their Difference — — — 4289

Multiply by the remaining Figures — — — 659

The Product (with 3 Places cut off) — — — 2826,451

Which add to the *Logarithm* of 1012 = 3,0051805

The Sum is the *Logarithm* — — — 6,0054631 sought.

Problem

280 *The Use and Management of Decimals*

Problem 3. A *Logarithm* being given, to find the Number belonging to the same.

Rule. Seek the next less *Logarithm* to the given one, in the Tables, and its four Figures are the first four of the Number required. Then take the Differences of the given *Logarithm* and the next less, and also the next greater and next less; Add to the first Difference, so many Cyphers as you seek Figures more than four. Divide That by the second Difference, and the Quotient annexed to the four Figures first found compleats the Number required.

Example. Required the Number of the given *Logarithm* 6,0054631?

The *Logarithm* next less is — 1012 = 3,0051805

The *Logarithm* next greater is — 1013 = 3,0056094

The Diff. of the given Log. and next less is = 2826

The Diff. of the next less, and next greater is = 4289

Then say, As 4289 : 2826 :: 1000 : 659, which annex to the first four Figures 1012, they compleat 1012659 the Number sought for the given *Logarithm*.

Problem 4. To find the *Logarithm* of any terminate Decimal Number.

Rule. Seek the *Logarithm* for it as though it were a whole Number, and then adapt the Index as before taught.

Thus the *Logarithm* of 1012,659 is 3,0054631, and of ,1012659 = 9,0054631, &c.

Problem 5. To find the *Logarithm* of a single Repetend, or circulating Digit.

Rule. To the tabular *Logarithm* of the Digit, add the Arithmetical Complement of the *Logarithm* of 9, the Sum is the *Logarithm* sought.

Example. Required the *Logarithm* of 6?

To the Tabular *Logarithm* of 6 = 0,7781512

Add the Arith. Complement of the Log. 9 = 0,0457575

The Sum is the *Logarithm* sought of 6 = 0,8239087

In

In this Manner I have calculated the *Logarithms* of all the Nine Digits perpetually circulating, and disposed them ready for Use in the annexed Table.

Rep. Digits. Logarithms.

1	=	0,0457575
2	=	0,3467875
3	=	0,5228787
4	=	0,6478175
5	=	0,7447275
6	=	0,8239087
7	=	0,8908555
8	=	0,9488475
9	=	1,0000000

Problem 6. To find the *Logarithm* of any pure Compound Repetend.

Rule. To the *Tabular Logarithm* of the Number (as terminate,) add the *Arithmetical Complement* of the Logarithms of so many 9's, as are Places of the Repetend; the Sum is the *Logarithm* of the given Repetend.

Example 1. Required the *Logarithm* of the Compound Repetend 24?

To the *Tabular Logarithm* of — 24 = 1,3802112
 Add the *Arithmetical Complement* of 99 = 0,0043648
 The Sum is the *Logarithm* of — 24 = 1,3845760

Example 2. Required the *Logarithm* of 36,5?

To the *Tabular Logarithm* of — 36,5 = 1,5622929
 Add the *Arithmetical Complement* of 999 = 0,0004345
 The Sum is the *Logarithm* of — 36,5 = 1,5627274

Example 3. Required the *Logarithm* of 3746?

To the *Tabular Logarithm* of — 3746 = 3,5735678
 Add the *Arithmetical Complement* of 9999 = 0,0000434
 The Sum is the *Logarithm* of — 3746 = 3,5736112

Example 4. Required the *Logarithm* of 200,60?

To the *Tabular Logarithm* of — 200,60 = 2,3023309
 Add the *Arith. Comp.* of the Log. of 99999 = 0,0000043
 The Sum is the *Logarithm* of 200,60 = 2,3023352

282 *The Use and Management of Decimals*

Note. In all the foregoing *Examples* (and in those which follow) the *Indexes* of the *Arithmetical Complements* are omitted; and this must be observed by the Learner in all Operations of this kind.

Problem 7. To find the *Logarithm* of any *mixed Repetend*, either *Single* or *Compound*.

Rule. From the given mix'd *Repetend*, subtract its *terminate* Part; Then to the *Logarithm* of the Remainder add the *Arithmetical Complement* of the *Logarithm* of so many *Nines*, as there are Figures in the *Repetend*, the Sum will be the *Logarithm* sought.

Example 1. Requir'd the *Logarithm* of 2,6?

From the given <i>Repetend</i>	—	2,6	
Subtract the <i>terminate</i> Part	—	2	
		<hr/>	
Then to the <i>Logarithm</i> of	—	2,4	= 0,3802112
Add the <i>Arithm. Comp.</i> of the Log. of 9		9	= 0,0457575
		<hr/>	
The Sum is the <i>Logarithm</i> of	—	2,6	= 0,4259687

Example 2. Requir'd the *Logarithm* of 57,23?

From the given <i>Repetend</i>	—	57,23	
Subtract the <i>terminate</i> Part	—	572	
		<hr/>	
Then to the <i>Logarithm</i> of	—	51,51	= 1,7118915
Add the <i>Arith. Compl.</i> of the Log. of 9		9	= 0,0457575
		<hr/>	
The Sum is the <i>Logarithm</i> of		57,23	= 1,7576490

Example 3. Requir'd the *Logarithm* of 2,733?

From the given <i>Repetend</i>	—	2,733	
Subduct the <i>terminate</i> Part	—	27	
		<hr/>	
Then to the <i>Logarithm</i> of	—	2,726	= 0,4355258
Add the <i>Arith. Compl.</i> of the Log. of 99		99	= 0,0043648
		<hr/>	
The Sum is the <i>Logarithm</i> of	—	2,733	= 0,4398906

Example

Example 4. Requir'd the *Logarithm* of 725,6?

From	—	—	725,6	
Subduct	—	—	7	
<hr/>				
To the <i>Logarithm</i> of	—		724,9	= 2,8602781
Add the <i>Arit. Compl.</i> of the <i>Log.</i> of			999	= 0,0004345
<hr/>				
The Sum is the <i>Logarithm</i> of	—		725,6	= 2,8607126

Example 5. Requir'd the *Logarithm* of 26892,7?

From	—	—	26892,7	
Subtract	—	—	26	
<hr/>				
To the <i>Logarithm</i> of	—		26890,1	= 4,4295924
Add the <i>Arit. Comp.</i> of the <i>Log.</i> of			9999	= 0,0000434
<hr/>				
The Sum is the <i>Logarithm</i> of	—		26892,7	= 4,4296358

In the like Manner may the *Logarithm* of any other *Mix'd Repetend* be found, so far as the *Canon* of *Logarithms* (you use) will permit.

Problem 8. Between two Numbers given, to find any Number of *mean Proportionals* required.

Rule. Subtract the *Logarithm* of the lesser Number from the *Logarithm* of the greater; divide the Remainder by a Number greater by one than the Number of *Means sought*; this Quotient add to the *Logarithm* of the lesser Number; the Sum is the *Logarithm* of the first Mean; to which the said Quotient is to be added again for the *Logarithm* of the second Mean; and thus proceed for as many *Means* as you please.

284 *The Use and Management of Decimals*

Example. Between 8 and 56 to find *four Mean Proportional Numbers*.

The Logarithm of 56 is	—	1,7481880
The Logarithm of 8 is	—	0,9030900
The Remainder or Difference is	—	0,8450980
One fifth Part (for four <i>Means</i>) is	—	0,1690196
To which add the Logarithm of 8	—	0,9030900
The Sum is the Log. of the <i>first Mean</i> 11,809	—	1,0721096
To which add again	—	0,1690196
The Logarithm of the <i>second Mean</i> 27,42	—	1,2411292
Add again	—	0,1690196
The Logarithm of the <i>third Mean</i> is 25,71	—	1,4101488
Add again	—	0,1690196
Logarithm of the <i>fourth and last Mean</i> 37,94	—	1,5791684

This Problem I have chiefly inserted for their Sakes who would hereby learn to calculate *Tables of Compound Interest*; The Numbers in the Table of Amounts of 1*l.* being only Mean Proportionals between the Logarithm of Rate and the last Year's Amount in the Table.

Multiplication of all Kinds of Decimals by Logarithms.

Rule. { To the *Logarithm* of the *Multiplicand*,
Add the *Logarithm* of the *Multiplier*;
The Sum is the *Logarithm* of the *Product*.

<i>Example 1.</i> Multiply	—	12,4 = 1,0934217
By	—	3,6 = 0,5563025
The Product	—	44,64 = 1,6497242

<i>Example 2.</i> Multiply	—	36,5 = 1,5622929
By	—	,00019 = ,6,2787536
Product	—	,006935 = ,7,8410465

Example

Example 3. Multiply — — ,762 = ,9,8819550
 By — — 570 = 2,7558748
 Product — — 434,34 = 2,6378298

Example 4. Multiply — — ,0097 = ,7,9867717
 By — — ,00021 = ,6,3222193
 Product — — ,000002037 = ,4,3089910

Example 5. Multiply — — 26,4 = 1,4216039
 By — — 8 = 0,8239087
 Product — — 176 = 2,2455126

Example 6. Multiply — — 2,73 = 0,4361626
 By — — 2,8 = 0,4259687
 Product — — 7,28 = 0,8621313

Example 7. Multiply — — ,473 = ,9,6748611
 By — — 63 = 1,8037053
 Product — — 30,1 = 1,4785664

Example 8. Multiply — — 57,24 = 1,7576996
 By — — 2,788 = 0,4398906
 Product — — 177,61236 = 2,1975902

Example 9. Multiply — — 8, = 0,8239087
 By — — ,8 = ,9,7447275
 Product — — 3,70 = 0,5686362

Example 10. Multiply — — 36,8 = 1,5627274
 By — — 2,4 = 0,3845760
 Product 88,5734218 &c. = 1,9473034

Example 11. Multiply — — 21,23 = 1,3265407
 By — — 42,0 = 1,6234581
 Product — — 900,718 = 2,9499988

Example

286 The Use and Management of Decimals

Example 12. Multiply — ,00420 = ,76234581
 By — ,00008 = ,59488475
 Product ,0000003735 &c. = ,35723056

Division of all Kinds of Decimals by Logarithms.

Rule { From the *Logarithm* of the *Dividend*,
 Subtract the *Logarithm* of the *Divisor*;
 The *Remainder* is the *Logarithm* of the *Quotient*.

Example 1. Divide — 44,64 = 1,6497242
 By — 12,4 = 1,0934217
 The Quotient — 36 = 0,5563025

Example 2. Divide — 310 = 2,4913617
 By — 4,275 = 0,6309361
 Quotient — 72,51457 = 1,8604256

Example 3. Divide — 434,34 = 2,6378298
 By — ,762 = ,98819550
 Quotient — 570 = 2,7558748

Example 4. Divide — ,006935 = ,78410465
 By — 36,5 = 1,5622929
 Quotient — ,00019 = ,62787536

Example 5. Divide — ,000002073 = ,43089910
 By — ,00021 = ,63222193
 Quotient — ,0097 = ,79867717

Example 6. Divide — 176 = 2,2455126
 By — 6, = 0,8239087
 Quotient — 26,4 = 1,4216039

Example 7. Divide — 7,28 = 0,8621314
 By — 2,6 = 0,4259687
 Quotient — 2,73 = 0,436162

Example 7

Example 8. Divide — 30,1 = 1,4785664
 By — 63, = 1,8037053
 Quotient — ,473 = ,9,6748611

Example 9. Divide — 177,61236 = 2,1975902
 By — 2,753 = 0,4398906
 Quotient — 57,24 = 1,7576996

Example 10. Divide — 3,70 = 0,5686362
 By — ,8 = ,9,7447275
 Quotient — 6, = 0,8239087

Example 11. Divide — 800,718 = 2,9499988
 By — 42,0 = 1,6234581
 Quotient — 21,23 = 1,3265407

Example 12. Divide ,000000243208 = ,3,3861506
 By — ,0002172 = ,6,3370052
 Quotient ,00111,98 = ,7,0491454

The Golden Rule in Decimals by Logarithms.

Example 1. Direct Proportion.

If 2 C. 3 qr. 21 lb. of Sugar — 2,9375 = 0,4679778
 Cost 6 l. 1 s. 8 d. — 6,083 = 0,7841316
 What costs 12 C. 2 qrs. — 12,5 = 1,0969100
 1,8810416

Answer, 25 l. 17 s. 8 $\frac{1}{4}$ d. — 25,8864583 = 1,4130638

Example 2. Inverse Proportion.

If Wheat be 6 s. 4 d. per Bushel, — 6,3 = 0,8016325
 And the Pen. white Loaf weigh 7 $\frac{1}{4}$ oz. 7,75 = 0,8893017
 What must it weigh, when } 1,6909342
 Wheat is 3 s. 10 d. per Bushel? } — 3,83 = 0,5835766

Answer, 12 oz. 16 pwt. 2 gr. = 12,8043 Gr. = 1,1073576

Notz,

288 The Use and Management of Decimals

Note, Either of those Questions may (and that most conveniently) be wrought at once, viz. by *One Addition* of the two Logarithms now added, and the *Arithmetical Complement* of that *Logarithm* that is subtracted; for to *subtract* a *Logarithm*, or *add* its *Arithmetical Complement*, produces the same Effect, or is the same Thing.

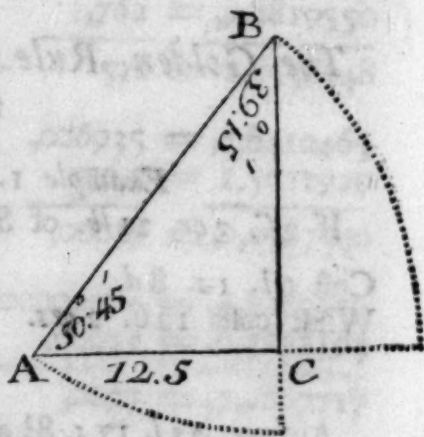
Example 3. Direct Proportion at one Operation.

If $\frac{1}{2}$ C. of Tobacco	—	—	.55 = 0,3010299
Cost 4l. 12s. 8d.	—	—	4,63 = 0,6658935
What cost 7 Pounds?	—	—	,0625 = 8,7958800
Answer, 00l. 11s. 7d.	—	—	.57918 = 9,7628034

Example 4. Inverse Proportion at one Operation.

If 8 Rods in Width,	—	8 = 0,9030900
Require 20 in Length, to make an Acre;	—	20 = 1,3010300
What Length does 12,5 Rods in } Width require for an Acre?	—	12,5 = 8,9030899
Answer, 12,8 Rods	—	12,8 = 1,1072099

This Method with working at once with Arithmetical Complement is to be advised to the expert Geometrician in his Trigonometrical Calculations, as much the best.



Example 5.

As the Sine of the Angle ABC	39° 15'	= 0,1987985
Is to the Side given AC	—	12,5 = 1,0969100
So is the Sine of the Angle BAC	50° 45'	= 9,8889612
To the Side sought BC	—	15,3 = 1,1846697

Extraction

Extraction of Roots in Decimals by Logarithms.

To extract the Root of any Number, do thus;

If it be the	{	Square Root	{	Divide the Lo-	{	2
		Cube Root		garithm of the		3
		Biquadrate Root		given Number		4
		Surfsolid Root		by —		5

Then the second, third, fourth, &c. Part of the Logarithm thus divided, shall be the Logarithm of the Root sought.

Example 1. What is the *Square Root* of the Number 2830,24?

The Logarithm thereof is — 3,4518232
 Half of which, is the Logarithm of 53,2 = 1,7259116
 The Root sought.

Example 2. Requir'd the *Square Root* of 13,2?

The Logarithm of the given Number 13,2 = 1,1205739
 Half, is the Logar. of the Root 3,6331 &c. = 0,5602869

Example 3. Requir'd the *Square Root* of 14,6?

The Logarithm of — 14,6 = 1,1663314
 Half, is the Logar. of the Root 3,8297 &c. = 0,5831657

Example 4. What is the *Cube Root* of 1,728?

The Logarithm of — 1,728 = 0,2375437
 One Third, is the Log. of the *Cube Root* 1,2 = 0,0791812

Example 5. What is the *Biquadrate Root* of 175,62?

The Logarithm of — 175,62 = 2,2445526
 One 4th, is the Log. of the R. 3,64019 &c. = 0,5611381

Example 6. Quere the *Surfsolid Root* of 31,25?

The Logarithm of — 31,25 = 1,4948500
 One Fifth, is the Log. of the Root 1,990 &c. = 0,2989700

Note ; When the *Index* is *Negative*, add to it 10 for the *Square Root*, 20 for the *Cube* ; 30 for the *Biquadrate* ; 40 for the *Sur-solid Root*, &c. and then divide as before ; as in the following *general Example*.

Example 7. What are the several Roots of ,27589 ?

The Logarithm of	—	,27589 =	,9,4407132
An Half, is the <i>Square Root</i>	,52523 &c. =	,9,7203566	
A Third, is the <i>Cube Root</i>	,6509 &c. =	,9,8135710	
A Fourth, is the <i>Biquadrate Root</i>	,7247 &c. =	,9,8601783	
A Fifth, is the <i>Sur-solid Root</i>	,7729 &c. =	,9,8881426	

C H A P. XII.

The Use of DECIMALS in Algebra ; exemplified in the Resolution of thirty four Select, Pleasant, and Useful Algebraick Questions.

NOTWITHSTANDING *Algebra* has the Glory and Reputation of being one of the Topmost Branches of the Tree of human Arts and Sciences ; yet must it be acknowledged that (as sublime and useful a Science as it is) it would answer no great Purpose of practical Knowledge, were not the Art of *Decimal Arithmetick*, on every Occasion, called in to its Assistance. *Vulgar Fractions* and *Algebra* together, may be view'd as the *Blind leading the Blind* ; And Whole Numbers miserably help the *lame Dog over the Stile*.

'Tis *Decimals* therefore which in all Cases (not studied and stated on purpose, but) which contingently or occasionally happen, can only speak out plainly and intelligibly the re-cluse Meaning of an *Algebraick Equation* or *Theorem*.

This

This I shall make appear by the Resolution of the most curious pleasant and useful Questions, which I have selected from the best *Algebraick Authors* extant?

Question 1. The Sum ($= s = 67$) of any two Numbers, and their *Difference* ($= d = 30$) being given to find those Numbers.

Let	1	$a =$ the Greater, and $e =$ the Lesser Number.	
Then	2	$a + e = s = 67$	
And	3	$a - e = d = 30$	
$2 + 3$	4	$2a = s + d = 97$	
$4 \div 2$	5	$a = \frac{s+d}{2} = 48,5$ the greater Numb.	} sought.
$2 - 3$	6	$2e = s - d = 37$	
$6 \div 2$	7	$e = \frac{s-d}{2} = 18,5$ the lesser Number	

Question 2. The Sum ($= s = 15$) and *Product* ($= p = 15$) of any two Numbers given, to find those Numbers?

Then {	1	$a + e = s = 15$	} Quere a , and e .
	2	$ae = p = 15$	
$1 \odot 2$	3	$aa + 2ae + ee = ss = 225$	
2×4	4	$4ae = 4p = 60$	
$3 - 4$	5	$aa - 2ae + ee = ss - 4p = 165$	
$5 \div 2$	6	$a - e = \sqrt{ss - 4p} = 12,845$ &c.	
$1 + 6$	7	$2a = s + \sqrt{ss - 4p} = 27,845$	
$7 \div 2$	8	$a = \frac{s + \sqrt{ss - 4p}}{2} = 13,921$ &c. Great.	} Numb. sought.
$1 - 6 \div 2$	9	$e = \frac{s - \sqrt{ss - 4p}}{2} = 1,038$ &c. Lesser	

292 *The Use of Decimals in Algebra.*

Question 3. What two Numbers are those whose Sum is $= 40 = s$, and the Greater divided by the Lesser shall quote $50 = q$?

Here $\left\{ \begin{array}{l|l} 1 & a + e = s = 40 \\ 2 & \frac{a}{e} = q = 50 \end{array} \right\} \text{ Quere } a, \text{ and } e?$

$2 \times e$ $3 \quad a - qe = 50e$

$1 - 3$ $4 \quad e = \frac{s}{q} - qe = 40 - 50e$

$4 + qe$ $5 \quad qe + e = s = 40$

$5 \div q + 1$ $6 \quad e = \frac{s}{q+1} = 0,7843 \text{ Ec. Lesser}$

$1 - 6$ $7 \quad a = \frac{qs}{q+1} = 39,2156 \text{ Ec. Greater}$

} Numb. sought.

Question 4. What two Numbers are those, whose Sum is $= 8 = s$, and the Sum of their Squares $37\frac{5}{9} = z$?

Here $\left\{ \begin{array}{l|l} 1 & a + e = s = 8 \\ 2 & aa + ee = z = 37\frac{5}{9} \end{array} \right\} \text{ Quere } a, \text{ and } e?$

$1 \ominus 2$ $3 \quad aa + 2ae + ee = ss = 64$

$3 - 2$ $4 \quad 2ae = ss - z = 26\frac{4}{9}$

$2 - 4$ $5 \quad aa - 2ae + ee = 2z - ss = 11\frac{1}{9}$

$5 \div 2$ $6 \quad a - e = \sqrt{\frac{2z - ss}{2}} = 3\frac{2}{3}$

$1 + 6$ $7 \quad 2a + s + \sqrt{2z - ss} = 11\frac{1}{9}$

$7 \div 2$ $8 \quad a = \frac{s + \sqrt{2z - ss}}{2} = 5,8 \text{ Greater}$

$1 - 6 \div 2$ $9 \quad e = \frac{s - \sqrt{2z - ss}}{2} = 2,8 \text{ Lesser}$

} Number sought.

Question 5. If the Sum of any two Numbers be $50 = s$, and the Difference of their Squares be $273 = x$, What are the said Numbers?

Here $\left\{ \begin{array}{l|l} 1 & a + e = s = 50 \\ 2 & aa - ee = x = 273 \end{array} \right\} \text{ Quere } a, e?$

$2 \div 1$ $3 \quad a - e = \frac{x}{s} = 5,46$

$1 + 3$ $4 \quad 2a = \frac{ss + x}{2s} = 55,46$

$4 \div 2$ $5 \quad a = \frac{ss + x}{2s} = 27,73 \text{ the Greater}$

$1 - 3 \div 2$ $6 \quad e = \frac{ss - x}{2s} = 22,27 \text{ the Lesser}$

} Number sought.

Question

Question 6. Suppose the Difference of two Numbers be $30 = d$, and the Product $512 = p$. Quere those Numbers?

Here {	1	$a - e = d = 30$	} Quere a, e .
	2	$ae = p = 512$	
1 \ominus 2	3	$aa - 2ae + ee = dd = 900$	
2 \times 4	4	$4ae = 4p = 9048$	
3 $+$ 4	5	$aa + 2ae + ee = dd + 4p = 2948$	
5 $\sqrt{}$ 2	6	$a + e = \sqrt{dd + 4p} = 54,29$	
6 $+$ 1 \div 2	7	$a = \frac{d + \sqrt{dd + 4p}}{2} = 42,18$ Greater	} Numb. sought.
6 $-$ \div 2	8	$e = \frac{\sqrt{dd + 4p} - d}{2} = 12,18$ Lesser	

Question 7. Suppose the Difference of two Numbers be the same with the Quotient of the Greater divided by the Lesser, viz. $= 18$. Quere those Numbers?

Here {	1	$a - e = d = 18$	} Quere a, e .
	2	$\frac{a}{e} = q = 18$	
2 \times e	3	$a = qe = 18e$	
1 $+$ e	4	$a = d + e = 18 + e$	
3, 4	5	$qe = d + e = 18 + e$	
5 $-$ e	6	$qe - e = d = 18$	
6 \div q $-$ 1	7	$e = \frac{d}{q - 1} = 1,058$ the Lesser	} Number sought.
1 $-$ 7	8	$a = \frac{qd}{q - 1} = 19,058$ the Greater	

Question 8. The Difference of two Numbers $= 5 = d$, and the Sum of their Squares $= 52 = z$, being given; to find those Numbers.

Here {	1	$a - e = d = 5$	} Quere a, e .
	2	$aa + ee = z = 55$	
1 \ominus 2	3	$aa - 2ae + ee = dd = 25$	
2 $-$ 3	4	$2ae = z - dd = 30$	
2 $+$ 4	5	$aa + 2ae + ee = 2z - dd = 85$	

294 *The Use of Decimals in Algebra.*

$$\begin{array}{l|l}
 5 \text{ w } 2 & 6 \quad a + e = \sqrt{2z - dd} = 9,219 \\
 1 + 6 \div 2 & 7 \quad a = \frac{d + \sqrt{2z - dd}}{2} = 7,109 \text{ Greater} \\
 6 - 1 \div 2 & 8 \quad e = \frac{\sqrt{2z - dd} - d}{2} = 2,109 \text{ Lesser}
 \end{array}
 \left. \vphantom{\begin{array}{l} 6 \\ 7 \\ 8 \end{array}} \right\} \text{Number sought.}$$

Question 9. The Difference of any two Numbers $= 12 = d$, and the Difference of their Squares $= 279 = x$, being given; to find those Numbers?

$$\begin{array}{l|l}
 \text{Here } \left\{ \begin{array}{l} 1 \quad a - e = d = 12 \\ 2 \quad aa - ee = x = 279 \end{array} \right. & \left. \vphantom{\begin{array}{l} 1 \\ 2 \end{array}} \right\} \text{Quere } a, e? \\
 2 \div 1 & 3 \quad a + e = \frac{x}{d} = 23,25 \\
 1 + 3 & 4 \quad 2a = \frac{da + x}{d} = 35,25 \\
 4 \div 2 & 5 \quad a = \frac{da + x}{2d} = 17,625 \text{ Greater} \\
 3 - 5 & 6 \quad e = \frac{x - dd}{2d} = 5,625 \text{ Lesser}
 \end{array}
 \left. \vphantom{\begin{array}{l} 5 \\ 6 \end{array}} \right\} \text{Number sought.}$$

Question 10. The Product of any two Numbers being $= 573 = p$, and the Quotient of the Greater divided by the Lesser $= 75$; Quere those Numbers?

$$\begin{array}{l|l}
 \text{Here } \left\{ \begin{array}{l} 1 \quad ae = p = 573 \\ 2 \quad \frac{a}{e} = q = 75 \end{array} \right. & \left. \vphantom{\begin{array}{l} 1 \\ 2 \end{array}} \right\} \text{Quere } a, e? \\
 1 \times 2 & 3 \quad na = qp = 42975 \\
 3 \text{ w } 2 & 4 \quad a = \sqrt{qp} = 207,304 \text{ Sc. Greater} \\
 1 \div 2 & 5 \quad ee = \frac{p}{q} = 7,64 \\
 5 \text{ w } 2 & 6 \quad e = \sqrt{\frac{p}{q}} = 2,764 \text{ Sc. Lesser}
 \end{array}
 \left. \vphantom{\begin{array}{l} 4 \\ 5 \\ 6 \end{array}} \right\} \text{Number sought.}$$

Question

Question 11. The Product of any two Numbers $= 100 = p$, and the Sum of their Squares $= 1000 = z$; *Quere* those Numbers?

Here {	1	$ae = p = 100$	} <i>Quere</i> a, e ?
	2	$aa + ee = z = 1000$	
1 \times 2	3	$2ae = 2p = 200$	
2 + 3	4	$aa + 2ae + ee = z + 2p = 1200$	
4 \div 2	5	$a + e = \sqrt{z + 2p} = 34,641$	
2 - 3	6	$aa - 2ae + ee = z - 2p = 800$	
6 \div 2	7	$a - e = \sqrt{z - 2p} = 28,284$	
5 + 7 \div 2	8	$a = \frac{\sqrt{z + 2p} + \sqrt{z - 2p}}{2} = 31,462 \text{ Gr.}$	} <i>Numb. sought.</i>
5 - 7 \div 2	9	$e = \frac{\sqrt{z + 2p} - \sqrt{z - 2p}}{2} = 3,178 \text{ Lef.}$	

Question 12. The Product of any two Numbers $= 10 = p$, and the Difference of their Squares $= 20 = x$, being given; thence to find those Numbers?

Here {	1	$ae = p = 10$	} <i>Quere</i> a, e ?
	2	$aa - ee = x = 20$	
1 \odot 2	3	$aaee = pp = 100$	
2 \odot 2	4	$aaaa - 2aaee + eeee = xx = 400$	
3 \times 4	5	$4aaee = 4pp = 400$	
4 + 5	6	$aaaa + 2aaee + eeee = xx + 4pp = 800$	
6 \div 2	7	$aa + ee = \sqrt{xx + 4pp} = 28,284$	
2 + 7	8	$2aa = x + \sqrt{xx + 4pp} = 48,284$	
8 \div 2	9	$aa = \frac{x + \sqrt{xx + 4pp}}{2} = 24,142$	
9 \div 2	10	$a = \sqrt{\frac{x + \sqrt{xx + 4pp}}{2}} = 4,917 \text{ Sc. G.}$	} <i>N.</i>
7 - 2 \div 2 \div 2	11	$e = \sqrt{\frac{\sqrt{xx + 4pp} - x}{2}} = 2,035 \text{ Sc. L.}$	

Question

296 The Use of Decimals in Algebra.

Question 13. The Quotient of any two Numbers $= 10 = q$, and the Sum of their Squares $= 57 = z$, being given; to find those Numbers?

Here	$\left\{ \begin{array}{l} 1 \times e \\ 3 \ominus 2 \\ 2 - 4 \\ 4 + qqe \\ 6 \div qq + 1 \\ 2 - 7 \\ 8 \ominus 2 \\ 7 \ominus 2 \end{array} \right.$	1	$\frac{a}{e} = q = 10$	} Quere a, e ?
		2	$da + ee = z = 57$	
		3	$a = qe = 10e$	
		4	$aa = qqee = 100ee$	
		5	$ee = z - qqee = z - 100ee$	
		6	$qqee + ee = z = 57$	
		7	$ee = \frac{z}{qq + 1} = ,5643$	
		8	$ad = \frac{qqz}{qq + 1} = 56,43$	
		9	$a = \sqrt{\frac{qqz}{qq + 1}} = 7,512 \text{ Gr. Great.}$	} Numb. sought.
		10	$e = \sqrt{\frac{z}{qq + 1}} = ,7512 \text{ Gr. Les.}$	

Question 14. Suppose the Quotient of two Numbers $= 20 = q$, and the Difference of their Squares $= 100 = x$; Thence to find the Numbers.

Here	$\left\{ \begin{array}{l} 1 \times e \\ 3 \ominus 2 \\ 2 + ee \\ 4, 5 \\ 6 - ee \\ 7 \div qq - 1 \\ 2 + 8 \\ 9 \text{ w } 2 \\ 8 \text{ w } 2 \end{array} \right.$	1	$\frac{a}{e} = q = 20$	} Quere a, e ?
		2	$aa - ee = x = 100$	
		3	$a = qe = 20e$	
		4	$aa - qqee = 400ee$	
		5	$aa = x + ee = 100 + ee$	
		6	$qqee = x + ee = 100 + ee$	
		7	$qqee - ee = x = 100$	
		8	$ee = \frac{x}{qq - 1} = ,2506 \text{ Gr.}$	
		9	$aa = \frac{qqx}{qq - 1} = 100,2506, \text{ Gr.}$	
		10	$a = \sqrt{\frac{qqx}{qq - 1}} = 10,012 \text{ Gr. Greater}$	} Numb. sought.
	11	$e = \sqrt{\frac{x}{qq - 1}} = 0,5006 \text{ Gr. Lesser}$		

Question

Question 15. Suppose the Sum of the Squares of any two Numbers $= 300 = z$, and the Difference of the said Squares $= 250 = x$, to find the Numbers?

$$\begin{array}{l|l|l} \text{Here } \left\{ \begin{array}{l} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{array} \right. & \begin{array}{l} aa + ee = z = 300 \\ aa - ee = x = 250 \\ 2aa = z + x = 550 \\ aa = \frac{z+x}{2} = 275 \\ a = \sqrt{\frac{z+x}{2}} = 16,583 \text{ \&c. Greater} \\ e = \sqrt{\frac{z-x}{2}} = 5 \text{ \&c. Lesser} \end{array} & \begin{array}{l} \text{Quere } a, e? \\ \\ \\ \\ \text{Number} \\ \text{sought.} \end{array} \end{array}$$

Thus any two of those Six Things (viz. Sum, Difference, Product, Quotient, Sum of the Squares, and Difference of the Squares, of any two Numbers) being given; 'twill be easy to find the Numbers themselves and all the other Particulars.

I have chose to give the Analytical Process of the Work of each Question at large, that the Young Student may see the Manner of Investigating Theorems; and by viewing the frequent Divisions and Extractions, may the more clearly perceive the great Use, or rather, the absolute Necessity of Decimals, in order to express the Equations in Numbers.

But in those Questions which follow, I have only exhibited the Theorem or Equation which answers them, and given the Solution of each in Decimal Numbers.

Question 16. There are two Numbers a, e . The Sum of their Squares is $aa + ee = z = 97$ The Greater is to the Less as $12 = b$ is to $7 = d$ That is $a : e :: b : d$ Quere a, e ?

$$\text{Theorem. } e = \sqrt{\frac{zdd}{dd + bb}} \text{ } \left. \vphantom{\frac{zdd}{dd + bb}} \right\} \text{ to be resolved.}$$

First Multiply the Sum of the Squares $z = 97$

By the Square of d $dd = 49$

The Product is $zdd = 4763$ Dividend.

Then to the Square of d $dd = 49$

Add the Square of b $+ bb = 144$

The Sum of both is $dd + bb = 193$ Divisor.

Q q

By

298 *The Use of Decimals in Algebra.*

By which Divide xdd , } — $\frac{xdd}{dd + bb} = 24,679$ &c.
The Quotient will be

The Square Root of which is $\sqrt{\frac{xdd}{dd + bb}} = e = 4,966$

Then as $\left\{ \begin{matrix} d : e :: b : a \\ 7 : 4,966 :: 12 : 8,513 \end{matrix} \right\}$ Thus $\left\{ \begin{matrix} a = 8,513 \text{ &c.} \\ e = 4,966 \text{ &c.} \end{matrix} \right.$

Question 17. There are three Numbers in continued Proportion, viz. — — — $a. m. e.$

The Sum of the Extreams — $a + e = 37\frac{2}{5} = s$

And the Mean — $m = 13$. Quere a, e ?

Theorem. $a = \frac{s + \sqrt{ss - 4mm}}{2}$ } to be solved.

First, The Square of the Sum is — $ss = 1398,76$

And the Square of $m \times 4$ is — $4mm = 676$

Which substracted, there remains, $ss - 4mm = 722,76$

The Square Root whereof is $\sqrt{ss - 4mm} = 26,885$

To which add the Sum of the Extreams $s = 37,4$

That Sum is — $s + \sqrt{ss - 4mm} = 64,285$

The half of that is the First Number $a = 32,142$

Then as $\left\{ \begin{matrix} a : m :: m : e \\ 32,142 : 13 :: 13 : e \end{matrix} \right\}$ $e = 5,257$

The Sum of which is the Proof $a + e = 37,4$

Question 18. There are three Numbers in continued Proportion, viz. — — — $a : m :: m : e$

Their Sum is — $a + m + e = s = 31\frac{2}{7}$ } Quere a, m, e ?

And the Sum of their Sq. $aa + mm + ee = z = 763$

Theorem. $m = \frac{ss - z}{2s}$ } to be solved.

First,

First, from the Square of their Sum $ss = 978,7955$ &c.
Subtract the Sum of their Squares $z = 763$

The Remainder is $ss - z = 215,7955$
Which divided by twice the Sum $2s = 62,871428$
Gives the Quotient $m = 3,446$ &c.

Then $31,285714 - 3,446 = 27,839 = a + e$
Which may be found as in the Theorem of the last Question.

Question 19. Suppose three Numbers a, b, c in Musical Proportion, viz. As $a : c :: a - b : b - c$, and any Two of them being given, to find the Third,

Theorem 1. $a = \frac{cb}{2c - b}$ Finds a , if b, c , be given.

Theorem 2. $b = \frac{2ac}{c + a}$ Finds b , if a, c , be given.

Theorem 3. $c = \frac{ba}{2a - b}$ Finds c , if a, b , be given.

Suppose $b = 13$, and $c = 10$ To find a ?

Multiply $b = 13$

By $c = 10$

The Product $cb = 130$

Divide by twice c , less b , $2c - b = 7$ $130 \div 7 = 18,571428$

The Quotient is $a = 18,571428$ the Number sought. And so for either of the other.

Question 20. Suppose four Numbers, a, b, c, d , in Musical Proportion, viz. $a : d :: a - b : c - d$; and any Three of these given, to find the fourth,

Theorem 1. $a = \frac{db}{2d - c}$ Finds a , if b, c, d , be given.

Theorem 2. $b = \frac{2da - ca}{d}$, Finds b , if a, d, c , be given.

Theorem 3. $c = \frac{2da - db}{a}$, Finds c , if a, b, d , be given.

Theorem 4. $d = \frac{ca}{2a - b}$, Finds d , if a, b, c , be given.

300 The Use of Decimals in Algebra.

Question 21. Suppose it was required to divide any Number ($20 = s$) into *Extreme* and *Mean Proportion*; That is, into two such Parts, a , and e , that $aa = ae + ee = se$. Quere a , e ?

Theorem, $a = \sqrt{ss + \frac{1}{4}ss} - \frac{1}{2}s$

Square the given Number $ss = 400$
Add thereto $\frac{1}{4}$ of the said Square $\frac{1}{4}ss = 100$

The Sum is $ss + \frac{1}{4}ss = 500$

The square Root thereof is $\sqrt{ss + \frac{1}{4}ss} = 22,3606 \text{ \&c.}$
From which subtract $\frac{1}{2}$ the given Numb. $\frac{1}{2}s = 10$

There Remains the *Greater Part* $a = 12,3606 \text{ \&c.}$

Which subtracted from the given Numb. $\}$ $e = 7,6393 \text{ \&c.}$
There remains the *Lesser Part*

Note, 'Tis impossible to answer this Question in *Whole Numbers*.

Question 22. What is the *Canon* or *Theorem* for *Extracting the square Root*?

Suppose $a + e = \text{Root}$; Then the *Canon* is this, viz.
 $aa + 2ae + ee = \text{Square.}$

Extract the Sq. Root of $655,36 = aa + 2ae + ee$

From the Numb. subst. $400 = aa$ ($a = 20$)

There Remains $255,36 = 2ae + ee$ $\left\{ \begin{array}{l} a = 20 \\ e = 5 \end{array} \right.$

Divide that by $2a = 40$ $255,36 \div 40 = 5 = e$ $\left\{ \begin{array}{l} a + e = 25 \\ = a, \text{ anew.} \end{array} \right.$

Then subtract $225 = 2ae + ee$

There Remains $30,36 = 2ae + ee$ anew.

Which divid by. $2a = 50$ $30,36 \div 50 = e$ anew

Then again subtract $30,36 = 2ae + ee$ $\left\{ \begin{array}{l} 25 = a \\ 6 = e \\ 25,6 = a + e \end{array} \right.$

Question 23. What is the *Canon* or *Theorem* for extracting the *Cube Root*?

Suppose $a + e = \text{Root}$, Then $aaa + 3aae + 3aee + eee$ is the *Canon* or *Rule* for *Extracting any Cube Root*.

Requir'd

Requir'd the C. Root of $1953,125 = aaa + 3aae + 3aee + eee$
 Subtract the Cube $1000 = aaa$ ($a=10$. 1st.

There remains $953,125 = 3aae + 3aee + eee$

Div. by $3aa + 3a = 330$ $953,125(2=e$

Then $600 = 3aae$

And $120 = 3aee$

Lastly $8 = eee$

The Sum of all is $728 = \{ 3aae + 3aee + eee \text{ subst. from the Remainder.}$

There remains $225,125 = 3aae + 3aee + eee$, answ.

Div. by $3aa + 3a = 468$ $225,125(5=e$ { Then $a = 10$

Then $216,000 = 3aae$ { $e = 2$

And $9,000 = 3aee$ { $a + e = 12$

Lastly $0,125 = eee$ { $= a$, answ.

The Sum of all is $225,125 = 3aae + 3aee + eee$ to be
 ... (subst. from the last Rem.

Hence the Root is $10 + 2 + .5 = 12,5$

Note, From hence appears the *Rationale* of the Method of extracting the Cube Root; for the Precepts there are only the Words expressing the order and combination of the Symbols of this Canon.

Question 24. What is the Manner of *Extraction* by *Converging Series*, or Theorems raised thereby?

There are several Kinds of *Converging Series* for this Purpose, but Mr. *Ward's* I take to be the best, which is thus.

Let $aa = G$. Quere a ?

Let $1 \mid r + e = a$ The Root sought.

$1 \text{ @ } 2 \mid 2 \mid rr + 2re + ee = aa = G$

$2 - r \mid 3 \mid 2re + ee = G - rr = D$ The Dividend.

Then $4 \mid \frac{D}{2r + e} = e$, or, $\frac{D}{r + \frac{1}{2}e} = e$ { Theorems for the Sq. Root.

Let

302 *The Use of Decimals in Algebra.*

Let $aaa = G$. Quere a .

Put	1	$r + e = a$ The Root sought
1 \ominus 3	2	$rrr + 3rre + 3ree + eee = aaa = G$
2 — rrr	3	$3rre + 3ree + eee = G - rrr$
3 \div 3 ^r	4	$re + ee + \frac{eee}{3^r} = \frac{G - rrr}{3^r} = D$, the Dividend.
Reject $\frac{eee}{3^r}$	5	$re + ee = D \frac{eee}{3^r}$ being of small or no Value.
5 \div	6	$\frac{D}{r + e} = e$. The Theorem for the Cube.

If $aaaa = G$ Then $\frac{G - r^4}{2rr} = D$ and $\frac{D}{r + 3e} = e$

The Theorem for the Biquadrate Root.

If $aaaaa = G$. Then $\frac{G - r^5}{5r^3} = D$. And $\frac{D}{r + 2e}$

The Theorem for the Sur-solid Root.

And in the same Manner proceed for any other Root.

Note ; To work by these Theorems, r must be taken less than the Root ; otherwise, if r be taken greater than the true Root, it will be $rr - G$ instead of $G - rr$, and $\frac{D}{2r - e} = e$ and the same in other Theorems for the other Roots.

After the same Manner you raise Theorems for all kind of affected Equations.

Suppose $aaa + 24a = 587914$ Quere a ?

Put	1	$r + e = a$ Put $r = 80$	} The powers of e above ee are all to be rejected.
1 \ominus 3	2	$rrr + 3rre + 3ree = aaa$	
1 \times 24	3	$24r + 24e = 24a$	
2 in Numb.	4	$512000 + 19200e + 240ee = aaa$	
3 in Numb.	5	$1920 + 24e = 24a$	
4 + 5	6	$513920 + 19224e + 240ee = 587914$	
5 — 513920	7	$19224e + 240ee = 73994$	
7 \div 240	8	$80,1e + ee = 308,31 = D$ the Dividend	
8 \div	9	$e = \frac{D}{80,1 + e}$ See the Operation.	

80,1)

$$\begin{array}{r}
 80,1) 308,31 = D(3,7 = e \\
 + e = 3 \quad 2493 \\
 \hline
 1 \text{ Divisor } 83,1) 59,01 \\
 + e = ,7 \quad 5866 \\
 \hline
 2 \text{ Divisor } 83,8) \cdot 35
 \end{array}
 \quad
 \left.
 \begin{array}{l}
 r = 80 \\
 e = 3,7 \\
 r + e = 83,7 = a
 \end{array}
 \right\}$$

Here 83,7 is a new r for a *second Operation*; but being involved will be found too big, or *greater* than the *true Root*; Therefore it must be made $r - e = a$ the Root.

Thus	1	$r - e = a$ the Root sought.
1 \odot 3	2	$rrr - 3re + 3ee = aaa$
1 \times 24	3	$24r - 24e = 24a$
2 in Numb.	4	$586376,253 - 21017,07e + 251,1ee = ana$
3 in Numb.	5	$2008,8 - 24e = 24a$
4 + 5	6	$588385,053 - 21041,07e + 251,1ee = 587$
6 - 587914	7	$21041,07e - 251,1ee = 471,053 \quad (914$
7 \div 251,1	8	$83,7955e - ee = 1,87595778 = D$
8 \div	9	$e = \frac{D}{83,7955 - e}$

Operation 2.

$$\begin{array}{r}
 83,7955) 1,87595778(,022392736 \\
 - e = 02 \quad 1,675510 \\
 \hline
 1 \text{ Divisor } 83,7755) ,2001477 \\
 - e = ,002 \quad 1675470 \\
 \hline
 2 \text{ Divisor } 83,7735) ,03290078 \\
 - e = ,0003 \quad ,02513196 \\
 \hline
 3 \text{ Divisor } 83,7732 \quad ,00776882
 \end{array}$$

From hence 'tis sufficient to work by Contracted Division to find the Remainder of the Root.

$$\begin{array}{r}
 \text{Now } r = 83,7 \quad 304 \\
 \text{And } e = 0,022392736 \quad 251 \\
 \hline
 \text{Then } r - e = 83,677607264 = a \quad 53 \\
 \text{The Root of } 587914 \text{ requir'd} \quad 49
 \end{array}$$

4 &c.

And

304 The Use of Decimals in Algebra.

And if this Root be not exact enough, it may be made a New r , to work as before, in a third Operation.

And thus may the Root of any *Affected Equation* be found.

Question 25.

Suppose { Any Body A
 Pass over a given Space c
 In a given Time f
 And any other Body B
 Pass over a Space d
 In the Time g
 Also their Distance of Place e
 And the Interval of Time, in which they begin to move, be h
 Thence to determine x The Distance they pass, ere the hindmost Body overtakes the foremost, if they both tend the same way; or before they meet, if their Motions be opposite.

First, If they both tend the same Way, and A begin to move first, and is nearest the Place they tend to;

$$\text{The Theorem is } x = \frac{hdc + fed}{fd - ge} \quad 1.$$

$$\text{But if } B \text{ begin to move first, the Theorem is } x = \frac{hdc - fed}{fd - ge} \quad 2.$$

$$\text{If they both begin to move in the same Moment; the Theorem will be thus } x = \frac{fed}{fd - ge} \quad 3.$$

For if $h = 0$, then the Member of the Equation that has it, vanisheth.

Secondly. If the *Moveable Bodies* meet, and x , as before, be the Distance of the farthest Body, from the Place of *Ren-counter*, or Meeting; then $e - x$ will be the Distance of the other Body. Call the Body at farthest Distance A , and the other B .

Then,

Then, if A moves first, } $x = \frac{cdb + cg^2}{fd + gc}$ 4.
the Theorem is —

But, if B moves first, } $x = \frac{cdb + cg^2}{fd + gc}$ 5.
the Theorem is —

If they both move at the } $x = \frac{g^2}{fd + gc}$ 6.
same time, the Theorem
is — —

These Six Theorems answer most of the curious (and some of them very useful) Questions that are usually proposed concerning the Motion of two Bodies.

Question 26. Suppose the Sun (A) in the Beginning of Virgo, and eight Days after the Moon (B) is in the Beginning of Gemini; Quere the Place of the next New Moon?

Here are given $\left\{ \begin{array}{l} c = 0,03285 = 00 : 59 : 08 \\ f = 1, \\ d = 0,4892 = 13 : 10 : 35 \\ g = 1, \\ e = 3, \\ b = 8, \end{array} \right\}$ Quere x ?

By Theorem 1, work as follows;

Multiply — $d = 0,4892$ ☽'s given Motion.

By — $c = 0,03285$ ☉'s given Motion.

The Product is $dc = 0,01607$

Which mult. by $b = 8$ Difference of Time.

That Prod. is $dcb = 0,12856$

Again multiply $d = 0,4892$ ☽'s given Motion,

By — $fe = 3$ Difference of Place mul.
(multiplied into f .)

The Prod. is $fed = 1,4676$

To which add $dcb = 0,12856$

Sum is $bdc + fed = 1,59616$ The Dividend.

Then from — $fd = 0,4892$

Subtract — $gc = 0,03285$

Remains $fd - gc = 0,45635$ The Divisor.

R r

By

306 *The Use of Decimals in Algebra.*

By which divide, } *Signs*
 The Quotient is } $x = 3,49769 = 3 : 14 : 55 : 05$
 To which add the *Moon's* pref. Pl. $= 2 : 00 : 00 : 00$
 The Sum is the Place of the }
 Next New Moon sought } viz. In $5 \}$ $14 : 55 : 05$

Question 27. If a Ship *B* sail from the *Equator* percifely North, at the Rate of $7\frac{1}{2}$ Degrees in 3 Days, and another Ship *A*, 8 Days after set sail on the same Meridian the same Way, in Latitude $36 : 30$, and runs $8 : 45$ in five Days; 'tis required to tell in what Degree of North Latitude *B* will overtake *A*?

By Theorem 2. Thus;

Multiply the given Motion of <i>A</i>	—	$c = 8,75$
By the given Motion of <i>B</i>	—	$d = 7,5$
The Product is	—	$dc = 65,625$
Multiply that by the Interval of Time	—	$b = 8$
The Product is the Subtrahend	—	$bdc = 525,000$
Then multiply the given Interval of Places	$e = 36,5$	
By the given Motion of <i>B</i>	—	$d = 7,5$
The Product is	—	$ed = 273,75$
Which multiplied by <i>A</i> 's given Time	—	$f = 5$
The Product is	—	$fed = 1368,75$
From which subtract the Subtrahend	—	$bdc = 525,00$
Remains the Dividend	—	$fed - bdc = 843,75$
Then from $7,5 \times 5$	—	$fd = 37,5$
Subtract $8,75 \times 3$	—	$gc = 26,25$
There remains the Divisor	—	$fd - gc = 11,25$
By which Divide ; the Quotient is x , } the Latitude sought, viz.	—	$x = 75^{\text{th}} \text{ Deg.}$

Question 28. *A* challenges *B* to run a Race with him, provided he will give him 30 Rod in a 100 ; now the Velocity of *B*'s Running to that of *A*, is as $7\frac{1}{4}$ to $5\frac{1}{2}$. Quere which of the two beat?

By

By Theorem 3, work thus ;
 Multiply the Velocity of B — $d = 7,75$
 By the given Interval of Distance — $e = 30$
 The Product (because $f = 1$) is the Dividend $fed = 232,5$

Then from — $fd = 7,75$
 Subtract — $gc = 5,5$
 There Remains the Divisor $fd - gc = 2,25$
 Then — $2,25 \overline{) 232,5} (103 = x = 103,3 \text{ Rods.}$

Hence A beat B , since above the 100 Rods were pass'd ere B came up with, or could overtake him.

Question 29. Suppose the Hour and Minute Hand of a Horologium, or Clock, be now both in Conjunction at 12, Quere the Place of their next Conjunction ?

If you proceed by the same Theorem 3, you will find it to be at 1 : 09 Hours = 1 : 5 : 27 : 16 : 21 : 49, &c. the last five Places repeating *ad infinitum*. Hence we may observe, that though there really is a certain Moment of Time in which the Minute-Hand is precisely in Conjunction with the Hour-Hand, yet 'tis impossible to determine or represent that Moment of Time either in whole Numbers, or Decimal Fractions ; But by Vulgar Fractions we know it is $1,1$ Hours, that is just one Eleventh Part of an Hour after one a Clock.

Question 30. From London to Chichester is 60 Miles ; A Post-Boy (A) sets out from London, and goes $8\frac{1}{4}$ Miles in $2\frac{1}{2}$ Hours ; Another Post-Boy (B) $1\frac{1}{2}$ Hour after sets out from Chichester, and rides 9 Miles in $3\frac{1}{4}$ Hours. I demand how far A will have gone before he meets B ?

This Question is answered by Theorem 4, thus ;
 Multiply A 's given Space — $c = 8,75$
 By the Interval of the Times — $b = 1,5$
 Then multiply that Product — $cb = 13,125$
 By the given Space of B — $d = 9$
 The Product is — $cdh = 118,125$
 R r 2 Again

Again multiply	—	$e =$	8,75	
By the given Time of B	—	$g =$	3,25	
And that Product	—	$cg =$	28,4375	
Mult. by the Interv. of Distance	—	$e =$	60	
To that Product	—	$cg^2 =$	1706,25	
Add that above, viz.	—	$cdh =$	118,125	
The Sum is the Dividend	$cdh + cge =$		1824,375	
Then to the Product above,	—	$cg =$	28,4375	
Add the Prod. of $(f \times d = 2,5 \times 9)$	$fd =$		22,5	
The Sum is the Divisor	$fd + cg =$		50,9375	
By which dividing; the Quotient is the Dist. of A's Journey	$x =$		36,208	} Anf.
Then the Distance B will have pass'd is	$=$		23,791	

And thus proceed for answering Questions by *Theorems* the fifth and sixth. By these *Theorems* several other pretty *Problems* proposed, may be resolv'd. by any one versed in those Matters.

Question 31. This present Year of our Lord, the Cycle of the Sun is $6 = e$, and the Cycle of the Moon (call'd the Prime or Golden Number) is $5 = d$; *Quere* the Year of the Dionysian Æra or Period?

Here is given $\left\{ \begin{array}{l} e = 6 \\ d = 5 \\ e - d = z = 1 \end{array} \right. \left. \begin{array}{l} \text{Let } x \text{ be the} \\ \text{Year of the Pe-} \\ \text{riod sought;} \end{array} \right\}$

The *Theorem* is $59, x z + 3, x d - 2, x e = x$.

For, multiply the Difference of the Cycles $z = 1$

By the Number — — — — — 59, x

Then that Product is — — — — — = 59, x

To which add $d \times 3, x$ — — — — — = 15, 8

From that Sum — — — — — = 74, 8

Subtract $e \times 2, x$ — — — — — = 12, 8

There remains the Year of the Period = 62 sought.

This

This Theorem I contrived my self; and inserted it here as being a *Decimal* one.

Question 32. Let A , and B , be two *spherical Bodies perfectly elastick*, and let (a) denote the *Velocity* of A , and (b) = the *Velocity* of B ; then the *Motion* of $A = aA$, and the *Motion* of $B = bB$; lastly let x = the *Increase* of *Motion* communicated by the *Impact* or *Stroke*, to one Body; and the *Decrease* or *Loss* of *Motion* in the *percipient* or *striking* Body.

Let A follow B , and let it be required to determine the *Celerity* of each Body after the *Stroke* or *Impulse*.

If A and B tend both the same Way, the Theorem for

$$\left\{ \begin{array}{l} A's \text{ Celerity is } x = \frac{aA - aB + 2bB}{A + B} \\ B's \text{ Celerity is } x = \frac{2aA - bA + bB}{A + B} \end{array} \right.$$

But if they meet, the Theorems will be altered thus, for

$$\left\{ \begin{array}{l} A, \quad x = \frac{aA - aB - 2bB}{A + B} \\ B, \quad x = \frac{2aA + bA - bB}{A + B} \end{array} \right.$$

Example. Suppose two Bodies of the same sort, A of $5\frac{1}{2}$ Pounds, and 9 Degrees of *Velocity*; and B of $6\frac{1}{2}$ Pounds, and 4 Degrees of *Velocity*; tend the same Way; Quere their *Celerities* after the *Impulse*?

Here $A = 5,3. a = 9. B = 6,5. b = 4.$ Then,

From the *Motion* of A — — — $aA = 48$
 Subtract the *Velocity* of A into B — — — $aB = 58,5$
 There remains *negative*, — — — $aA - aB = - 10,5$
 To which add twice the *Motion* of B — $2bB = + 52$
 There Remains the *Dividend* — — — $= + 41,5$
 Then $A + B = 11,8) + 41,5 (3,49 = A's \text{ Celerity}.$
 In like Manner may be found $8,46 = B's \text{ Celerity}.$

Note; If either *Celerity* come out *Negative*, it signifies the *Motion* of that Body, after the *Impulse*, to be *contrary* to what it was before.

I have

310 The Use of Decimals in Algebra.

I have inserted this Question and Theorem, for the Sake of any such Persons as would wish to have always a Theorem by them for the ready determining the Celerity of Bodies in Motion by Calculation, and the rather because this Doctrine of Motion is the principal Basis of a good Part of the modern mechanical Philosophy.

Question 33. Says A, I've an Acre of Land to inclose; says B, I've ten Thousand such Acres as those, which lie in a Square; but the Form you design must have the same Fencing as goes round all mine. Quere the Length and Breadth of A's Acre of Land?

Let $\begin{cases} a = 1 = \text{The Area of A's Plot of Land.} \\ d = 100 = \text{The Side of B's square Plot.} \\ x = \text{The Side of A's Plot to be found.} \end{cases}$

The Theorem is $x = \sqrt{dd - a} : + d$.

From the Square of d $dd = 10000$

Subtract the given Area $- a = 1$

There remains $- dd - a = 9999$

The Sq. Root thereof is $\sqrt{dd - a} = 99,995$

To which add $- d = 100$

The Sum is one Side of the Area $x = 199,995 = \text{the Length.}$

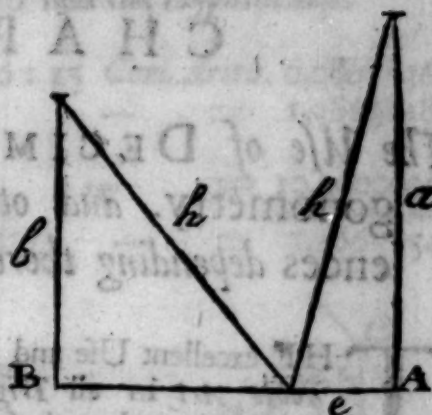
And the other Side is $- \frac{a}{x} = 0,005 = \text{the Breadth.}$

Thus twice their Sum is $= 4b = 400$ the Perimeter of Both.

Question 34. Suppose the Tower A 160 Feet high, and another Tower B 124 Feet high, at the Distance $AB = 150$ Feet; 'tis required to set a Ladder in some Point (e) in the Line A, of such a length, as from thence it may reach the Tops of both the Towers: Quere the Point e , and the length of such a Ladder?

Let

$a=160$ the Tower A
 $b=124$ the Tower B
 Let $c=150$ the Distance BA
 $e=$ the Point's Dist.
 $h=$ the Ladder sought.



The Theorem for the Distance (e) $\frac{bb + cc - aa}{2c} = e$

Then for the Length of the Ladder (h) $\sqrt{aa + ee} = h$

Thus, to the Square of B's height $bb = 15376$

Add the Square of the Distance $cc = 22500$

Then from that Sum $bb + cc = 37876$

Subtract the Square of A's height $aa = 25600$

There remains the Dividend $bb + cc - aa = 12276$

Which divided by $2c = 300$, the

Quotient is the Distance (e) $e = 40,92$

Then the Length of the Ladder is $\sqrt{aa + ee} = h = 165,149$
 &c.

C H A P. XIII.

The Use of DECIMALS in Plain Trigonometry, and other Mathematical Sciences depending thereon.

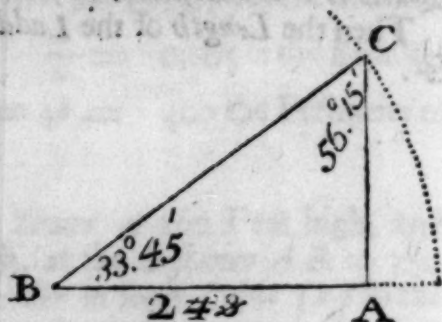
THE excellent Use and indispensable Necessity of this noble Art in all Trigonometrical Calculations, is evident enough to those who are versed therein. Nothing with any *Exactness, Ease, or Expedition* can be done therein without it; and as *Trigonometry* is the Foundation (yea the very Essence) of *Navigation, Fortification, Altimetry, Longimetry*, and is of Use also in divers Cases of *Astronomy, Surveying, Dialling, &c.* 'tis manifest the Use and Knowledge of *Decimal Arithmetick* is so requisite in all those Arts and Sciences, that without its Assistance a Person can make but a gloomy and fruitless Progress in the Study of them.

I shall therefore illustrate the Use of *Decimals* in the Resolution of all the Cases of *Right-lined Trigonometry* (for that only is to be understood in this Chapter) both in the Doctrine of *Right-angled*, and *Oblique-angled Triangles*, as follows.

Right-angled Triangles.

Case I.

Given $\left\{ \begin{array}{l} \text{The two Angles} \\ B \text{ and } C, \\ \text{The Base } B A, \end{array} \right.$
To find the Cathetus and Hypotenuse,



The Analogy to find the Cathetus.

As the Sine of the Angle C $56^{\circ} 15'$ Com. Arith. 0.801536
Is to the Base B A 24.3 — = 1.3863818

So is the Sine of the Angle 33.45 — = 0.7447390

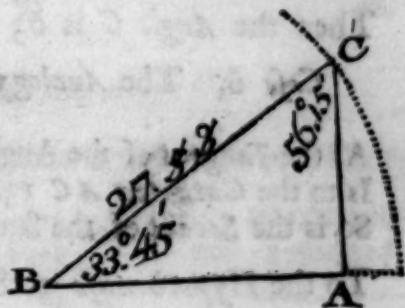
To the Cathetus or Perpendicular 16.26 = 1.2112744
Case

Case 2. The Analogy to find the Hypotenuse.

As the Sine of the Angle C $56^{\circ} : 15'$ Com. Arith. 0,0801536
Is to the Base B A 24,3 — = 1,3863818
So is Radius 90° — — = 10,0000000
To the Hypotenuse B C 29:27 — = 1,4665354

Case 3.

Given { The two Angles B
and C,
And the Hypotenuse
B C ;
To find the Base and Cathetus.



The Analogy for finding the Base.

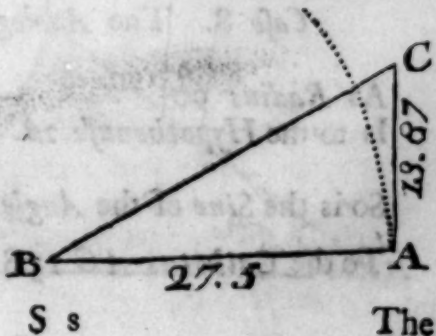
As Radius 90° — — = 10,0000000
Is to the Hypotenuse B C 27:32 — = 1,4398906
So is the Sine of the Angle C $56^{\circ} : 15'$ — = 9,9198464
To the Base B A, 22:89 — = 1,3597370

Case 4. The Analogy to find the Cathetus.

As the Radius 90° — — = 10,0000000
Is to the Hypotenuse B C 27:32 — = 1,4398906
So is the Sine of the Angle B $33^{\circ} : 45'$ — = 9,7447390
To the Cathetus, or A C 15:29 — = 1,1846296

Case 5.

Given { The Base B A,
The Cathetus A C ;
To find the Angles, and the
Hypotenuse.



The *Analogy* to find the *Angle B*.

As the *Base B A* 27:5 — = 1,4393327

Is to the *Radius* 90 — = 10,0000000

So is the *Perpendicular* 13:87 — = 1,1420765

To the *Tangent* of the *Angle B* 26 : 46 = 9,7027438

Then the *Angle C* is 63 : 14.

Case 6. The *Analogy* to find the *Hypothenufe*.

As the *Tangent* of the *Angle B* 26 : 46 *C. A.* 0,2972562

Is to the *Cathetus A C* 13:87 — = 1,1420765

So is the *Secant* of the same *Angle B* — = 10,0492225

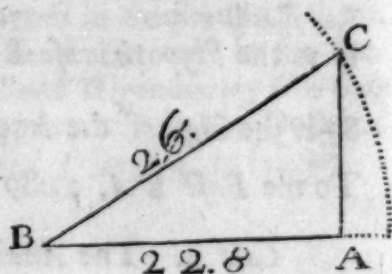
To the *Hypothenufe* 30:8 — = 1,4885552

Note. The *Secant* of any *Angle* is the *Arithmetical Complement* of the *Co-fine* of the said *Angle*, added to *Radius* 10,0000000.

Case 7.

Given { The *Base B A*,
 { The *Hypothenufe B C*;
To find the *Angles*, and the *Cathetus A C*,

The *Analogy* for the *Angle C*.



As the *Hypothenufe B C* 26 — = 1,4259687

Is to *Radius* 90 — = 10,0000000

So is the *Base B A* 22:8 — = 1,3579348

To the *Sine* of the *Angle C* 58:45 = 9,9319661

Wherefore the *Angle B* 31:15.

Case 8. The *Analogy* to find the *Cathetus*.

As *Radius* 90 — = 10,0000000

Is to the *Hypothenufe* 26 — = 1,4259687

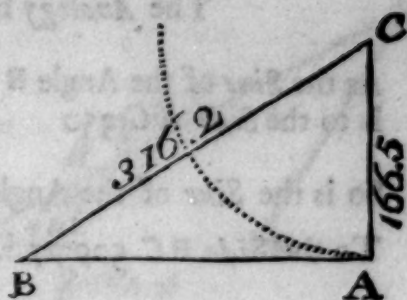
So is the *Sine* of the *Angle B* 31:15 = 9,7149776

To the *Cathetus A C* 13:83 = 1,1409463

Case

Case 9.

Given { The Hypotenuse BC
The Cathetus AC,
To find the Angles, and the Base
BA.



The Analogy to find the Angle C.

As the Cathetus AC 166:5 — Arith. Com. 7,7788467

Is to the Radius 90 — — 10,0000000

So is the Hypotenuse BC, 316:2 = 2,5000369

To the Secant of the Angle C, 58:15 10,2788836

Then the other Angle B will be 31:45

Case 10. The Analogy for finding the Base.

As the Radius 90 — — 10,0000000
Is to the Cathetus 166:5 — — = 2,2211533

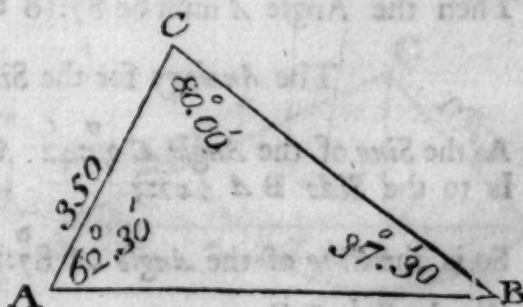
So is the Tangent of the Angle C 58:15 = 10,2084365

To the Base BA 268:927 — = 2,4295898

Oblique-angled Triangles.

Case 1.

Given { The Angles
A, B, and
C,
The Side A
C,
To find the other
two Sides,



The Analogy for the Side B C.

As the Sine of the Angle B $37:30'$ Arith. Com. 0,2155529
Is to the Side A C 350 — = 2,5440680

So is the Sine of the Angle A $62:30'$ — = 9,9479289

To the Side B C 509:97 — = 2,7075498

The Analogy for the Side A B.

As the Sine of the Angle B $37:30'$ Arith. Com. 0,2155529
Is to the Side A C 350 — = 2,5440680

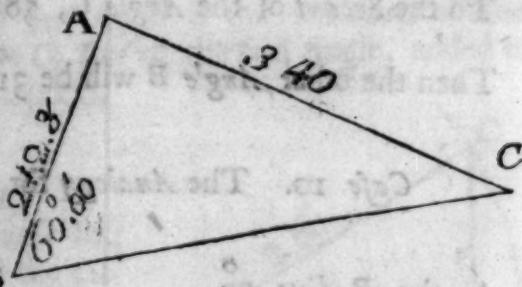
So is the Sine of the Angle C $80:00'$ — = 9,9933515

To the Side A B 566:2 — = 2,7529724

Case 2.

Given { Two Sides
A C and
A B,
And an Angle oppo-
site B,

To find the other
Side and Angles.



The Analogy for the Angle C.

As the Side A C 340 — Arith. Comp. 7,4685211

Is to the Sine of the Angle B $60:00'$ — = 9,9375306

So is the Side B A $2x2:3$ — = 2,3265407

To the Sine of the Angle C $32:42'$ — = 9,7325924

Then the Angle A must be $87:18$ therefore,

The Analogy for the Side B C.

As the Sine of the Angle C $32:42'$ Com. Arith. 0,2674076
Is to the Side B A $2x2:3$ — = 2,3265407

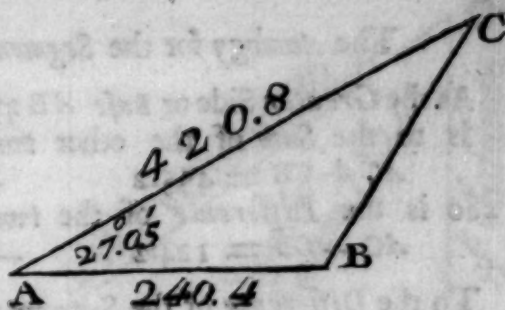
So is the Sine of the Angle A $87:18'$ — = 9,9995176

To the Side B C $39:21$ — = 2,5934659

Case

Case 3.

Given { Two Sides AC
and AB ,
And the Angle
included A ,
To find the other Side
and Angles.



The Analogy for the Angles.

As the Sum of the two Sides 661:2 Com. A . = 7,1796672
Is to the Difference of the Sides 180:4 = 2,2562365
So is Tangent of $\frac{1}{2}$ Sum of the unknown } = 10,6179795
Angles B and C 76:27 —

To the Tangent of $\frac{1}{2}$ their Difference 48:32 = 10,0538832

Then, To half the Sum — 76:27 of two Ang. B, C .
Add half the Difference 48:32

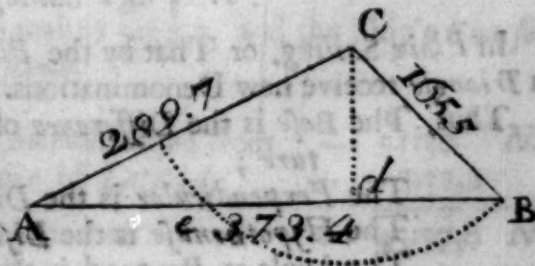
The Sum is — 124:59 = Greater Ang. B .
Subtract the $\frac{1}{2}$ Differ. Remains 27:55 = Lesser Angl. C .

The Analogy for the Side BC .

As the Sine of the Angle C 27:55 Arith Com. 0,3295808
Is to the Sine of the Angle A 27:05 = 9,6582842
So is the Side AB 240,4 — = 2,3809345
To the Side BC 23,37 — — = 2,3687995

Case 4.

Given { All the three
Sides, AB ,
 AC , and
 CB ,
To find the Angles.



The

The Analogy for the Segments *Ad*, and *dB*.

As the Greater Side or Base *AB* 373:4 *Ar. Com.* 7,4278258

Is to the Sum of the other two Sides } = 2,6582023

$$AC + CB = 455.2$$

So is the Difference of the two Sides } = 2,0941216

$$AC - CB = 124.2$$

To the Difference of the Segments of the } = 2,1801497

$$\text{Base } Ae = 151:4$$

Then from the *Grea. Side* or Base = 373:4

Subst. the Diff. of the Segments *Ae* = 151:4

There will remain *eB* — = 222

The half of which is *Bd* = *ed* = 111 the Lesser Segm.

Also to *Ae* added *ed*, the Sum is *Ad* = 262:4 the Greater Seg.

The whole *Oblique Triangle ACB* being thus resolved into the two *Right-angled Triangles ACd*, and *BCd*, the Angles *A*, *B*, and *C* are found by the seventh Case of *Right-angled Triangles* foregoing.

Having thus pass'd through all the Cases of *Right* and *Oblique-angled Plain Triangles*, in each of which the absolute Necessity of *Decimal Numbers* to express the Length of the *Sides* sought, is sufficiently evident; I shall next shew, in brief, the Application of the foregoing Doctrine of *Plain Trigonometry* to several *Arts Mathematical*; intending thereby to convince those who purpose to learn them, of the Necessity of their first learning *Decimal Arithmetick*.

The Use of Decimals in Navigation exemplified in all Kinds of Sailing.

1. Plain Sailing.

In *Plain Sailing*, or That by the *Plain Chart*, the Parts of a *Triangle* receive new Denominations.

Thus, The Base is the Difference of *Longitude*-or *Departure*;

The *Perpendicular* is the Difference of *Latitude*;

The *Hypothenuse* is the Distance the Ship has run;

The Angle at *Perpend.* is the *Course* of the Ship;

And the Angle at Base the Complement of the *Course*.

Admit

Admit a
Ship sails
from the
Lat. North

51:30 on
the Rumb.

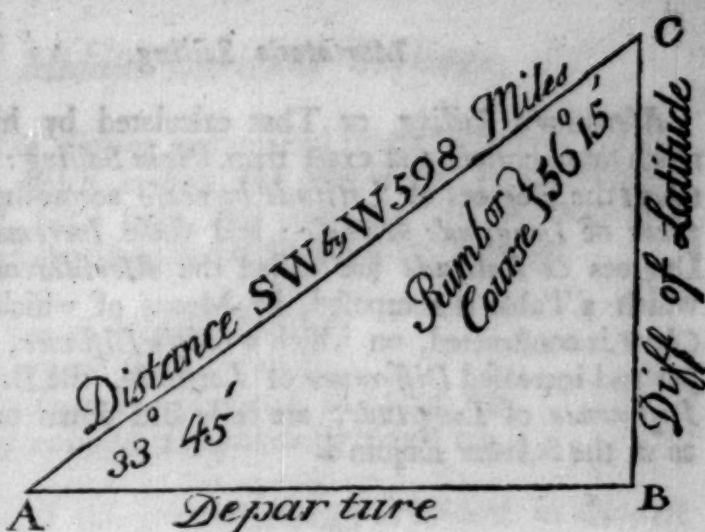
AC 598

Nautic
Miles

whose
Course is
SW b W;

Quere her
Departure

and Difference of Latitude ?



The Analogy for finding the Departure.

As Radius 90° — — = 10,0000000
Is to the Distance run AC 598 — = 2,7767012

So is the Sine of the Course C $56^{\circ} 15'$ — = 9,9198464

To the Departure Westward } 497:2 = 2,6965476
from her former Meridian

The Analogy for the Difference of Latitude.

As the Radius 90° — — = 10,0000000
Is to the Distance run AC 598 — = 2,7767012

So is the Co-sine of the Course $33^{\circ} 45'$ — = 9,7447390

To the Difference of Latitude 332:2 = 2,5214402

But 332,2 Miles are equal to $5^{\circ} 32,2'$, and the Ship's Course
being South-westerly.

Therefore from the Latitude sailed from — $51:30' N.$
Subtract the Difference of Latitude — $5:32,2$

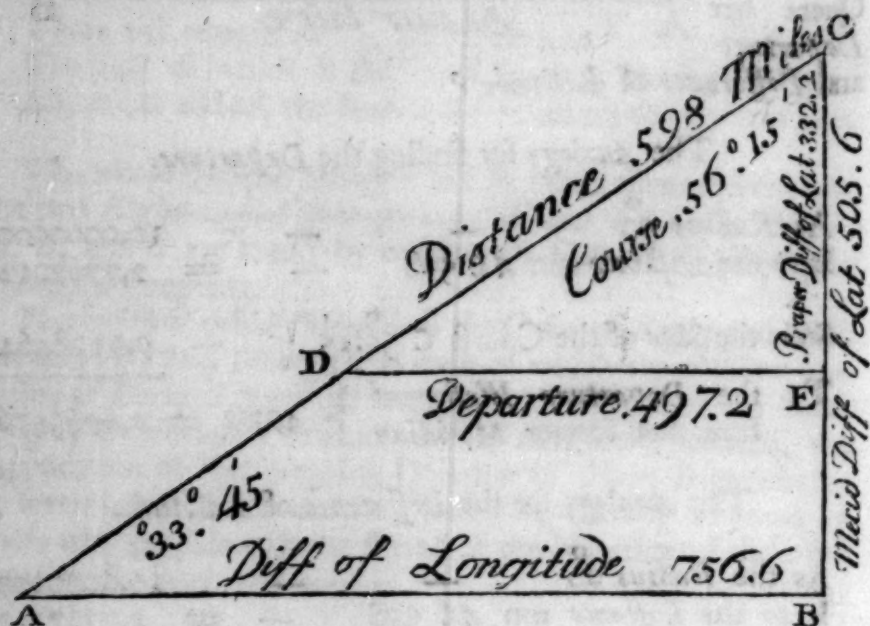
Remains the Latitude come to — = $45:57,8 N.$

The same Case follows in

Mercator's

Mercator's Sailing.

Mercator's Sailing, or That calculated by his *Chart*, is much more correct and exact than *Plain Sailing*: For in this *Chart* the Degrees of *Latitude* increase according as the Degrees of *Longitude* decrease; and these *Increments* of the Degrees of *Latitude* are called the *Meridional Parts*; of which a Table is composed, by Means of which *Mercator's Chart* is constructed, on which a Ship's *Distance*, *Course*, proper and increased *Difference* of *Latitude*, the *Departure* and *Difference* of *Longitude*, are truly laid down or delineated, as in the *Scheme* subjoin'd.



The *Analogy* for the *Difference* of *Longitude*.

As Radius 90°	—	—	—	10,0000000
Is to the Increased Differ. of Lat. 505:6	=			2,7038071
So is the Tangent of the Course 56:15	—			10,1751074
To the Difference of Longitude 756:6	=			2,8789145

The same Case and Data follow in

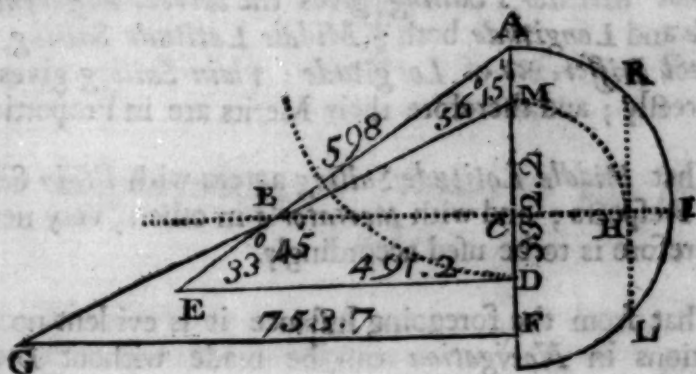
Middle

Middle Latitude Sailing.

This kind of Sailing is computed from the *middle Parallel* of *Latitude*, which is *half the Sum* of the *two Latitudes* of the Places, sail'd from, and come to; and depends altogether on the following *general Theorem* or *Analogy*.

As the *Co-sine* of *Middle Latitude*
Is to the *Tangent* of the *Course*,
So is the *Difference* of *Latitude* in Miles, &c.
To the *Difference* of *Longitude* in Miles, &c.

The Reason of this *general Analogy* is evident in the following Scheme and is deduced therefrom.



Explanation of the Scheme.

- $IK = IL$ Is the *Middle Latitude* 48.44° .
- $CM = CH$ The *Co-sine* of the *Middle Latitude*.
- $AD = MF$ The *Difference* of *Latitude*.
- DE The *Departure* Westward.
- FG The *Difference* of *Longitude*.
- AE The *Distance* sailed.
- EAD The *Rhumb* or *Course* *SW* by *W*.
- BE The *Tangent* of the *Course*.

Now 'tis manifest As $CM : CB :: MF : FG$, which is the same as the *Analogy* above in Words at length.

322 *The Use of Decimals in Fortification.*

The Analogy for the Difference of Longitude.

As the Co-sine of Middle Lat. 48.44°	Arith. C. 0,1807427
Is to the Tangent of the Course 56.15°	— 10,1751074
So is the Difference of Latitude 332.2	= 2,5213996
To the Difference of Longit. 753.7	= 2,8772497

Note 1. The Proportions for finding the *Difference of Latitude* and *Departure* in *Mecator's* and *Middle Latitude Sailing*, are the same as in *Plain Sailing*, and therefore not repeated.

2. That *Mecator's Sailing* gives the *correct Difference of Latitude* and *Longitude* both; *Middle Latitude Sailing*, only the *correct Difference of Longitude*; *Plain Sailing* gives neither *correctly*; and therefore their *Merits* are in *Proportion*.

3. That *Middle Latitude Sailing* agrees with *Plain Sailing* in some *Respects*; and with *Mercator's* in others, very nearly; and therefore is to be used accordingly.

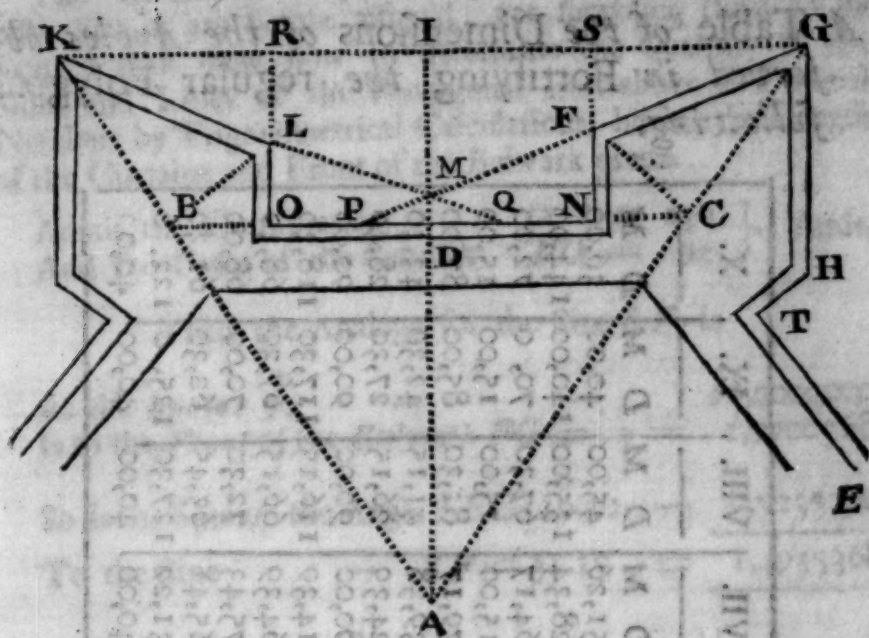
4. That from the foregoing Instance it is evident no exact Calculations in *Navigation* can be made without *Decimal Numbers*; and though I have express'd but one Place of *Decimals*, yet three Places more may be easily found by *Problem 3. of Logarithms*.

The Use of Decimals in Fortification.

A *Fort* is a Piece of Ground in Form of a *Polygon regular* or *irregular*, environ'd with a *Rampier*, or *Wall*, and a *Ditch* to impede the *Assaults* of an *Enemy*.

A Scheme of a *regular Pentagonal Fort*, with its *Explanation*, is here after subjoin'd.

The



The Explanation, or Names of the several Parts.

- | | | | |
|----------------------------------|---|---|-------|
| 1. The Curtaine | — | — | ON |
| 2. The Bulwark or Bastion | — | — | NFGHT |
| 3. The Front of the Bulwark | — | — | FG |
| 4. The Flank | — | — | NF |
| 5. The Gorge of a Bulwark | — | — | NT |
| 6. The Gorge Line | — | — | NC |
| 7. The Head Line | + | — | CG |
| 8. The Shoulder | — | — | F |
| 9. The Flanked Angle | — | — | G |
| 10. The Inward Flanking Angle | — | — | SGF |
| 11. The Outward Flanking Angle | — | — | KMG |
| 12. The Longest Line of Defence | — | — | PG |
| 13. The Shortest Line of Defence | — | — | PF |
| 14. The False Bray | — | — | EC |

T t 2

A Table

324 *The Use of Decimals in Fortification.*

A Table of the Dimensions of the Angles observed in Fortifying the regular Polygons following.

Number of the Sides of the Polygons.	V.		VI.		VII.		VIII.		IX.		X.	
	D	M	D	M	D	M	D	M	D	M	D	M
Angle at the Centre	72,00		60,00		51,26		45,00		40,00		36,00	
Angle of the Polygon	108,00		120,00		128,34		135,00		140,00		144,00	
Half thereof	54,00		60,00		64,17		67,30		70,00		72,00	
To which always add	15,00		15,00		15,00		15,00		15,00		15,00	
The Flanked Angle	69,00		75,00		79,17		82,30		85,00		87,00	
The Half thereof	34,30		37,30		39,38		41,15		42,30		43,30	
Inward Flank Angle	19,30		22,30		24,39		26,15		27,30		28,30	
Which add to a Right Angle	90,00		90,00		90,00		90,00		90,00		90,00	
Angle of the Shoulder	109,00		112,00		114,39		116,15		117,30		118,30	
Angle opposite to the Head Line	59,30		62,30		64,39		66,15		67,30		68,30	
Angle opposite to the Front	86,00		80,00		75,43		72,30		70,00		68,00	
Complement of S G F, viz.	70,00		67,30		65,41		63,45		62,30		61,30	
Outward Flanking Angle	141,00		135,00		131,22		127,30		125,00		123,00	
Angle fronting the Flank	40,00		40,00		40,00		40,00		40,00		40,00	

The Use of Decimals in Fortification. 325

Tho' 'tis not necessary the Angles in Forts should be precisely such as are before assign'd; yet supposing them to be such, I shall shew how to determine the Quantity of the Sides and Lines of the Pentagonal Fort above in Decimal Numbers by Trigonometrical Calculations, having the Length of the Curtaine and Front of the Bulwark given.

Admit the Curtaine be — $ON = 140$ } Yards.
And the Front of the Bulwark $FG = 93$ }

Then the Analogy for the Sine SF , is

As the Radius 90° — — 10,0000000
Is to the Front of the Bulwark $FG = 93$ — 1,9700368

So is the Sine of the Angle SFG 19.30° — 9,5235000

To the Sine — $SF = 31.15$ — 1,4935368

Again; As Radius 90° — — 10,0000000

Is to the Front of the Bulwark $FG = 93$ — 1,9700368

So is the Co-sine of Angle SFG 70.30° — 9,9743466

To the Line — $SG = 87.98$ — 1,9443834

Then $SG = 87.98$, and $SI = 70$. Therefore the whole Sine $KG = 315.96$.

Again; as the Sine of the Angle IAG 36.00° A.C. 0,2307813
Is to $\frac{1}{2} KG = IG = 157.98$ — = 2,1986500

So is Radius 90° — — 10,0000000

To the Semidiameter $AG = 268.8$ — 2,4294313

Again; as the Sine of the Angle IAG 36.00° A.C. 0,2307813
Is to $\frac{1}{2}$ the Side of the Pentagon $IG = 157.98$ — 2,1986500

So is the Sine of the Angle AGI 54.00° — 9,9079576

To the Perpendicular $AI = 217.4$ — 2,3373889

Again;

326 The Use of Decimals in Fortification.

Again, as the Sine of the Angle FCG 86.00 *A.C.* 0.0010592
Is to the Front $FG = 93$ — = $1,9700368$

So is the Sine of the Angle FGC 34.30 $9,7531280$

To the Line $FC = 52.99$ — = $1,7242240$

Also, as the Sine of FCG 86.00 *Arith. C.* 0.0010592
Is to the Front $FG = 93$ — = $1,9700368$

So is the Sine of the Angle GFC 59.30 $9,9353204$

To the Head Line $CG = 80.61$ — = $1,9064164$

Then $AG - CG = AC = 188.19$ the Semidiameter of
the inner Pentagon.

Again, as Radius 90 — = $10,0000000$
Is to the Line $FC = 52.99$ — = $1,7242240$

So is the Sine of the Angle FCN 40.00 $9,8080675$

To the Flank $FN = 34.06$ — = $1,5322915$

Then $FN + SF = ID = 65.21$. And $AI - ID =$
 $AD = 152.19$

Again, as Radius 90 — = $10,0000000$
Is to the Line $FC = 52.99$ — = $1,7242240$

So is the Sine of the Angle NFC 50.00 $9,8842540$

To the Gorge Line $NC = 40.59$ = $1,6084780$

Then $NC + ND = DC = 110.59$ And $2DC = BC = 221.18$

Again, as the Sine of FPN 19.30 *Arith. C.* $0,4765047$
Is to the Flank $FN = 34.06$ — = $1,5322915$

So is the Sine of the Angle PFN 70.30 $9,9743466$

To the Line $PN = 96.19$ — = $1,9831428$

Then $ON - PN = OP = 43.81$ the second Flank.
And $ON + SG = RG = 227.98$. Then in the Trian-
gle ROG As

The Use of Decimals in Altimetry, &c. 327

As the Line $RO (= ID) = 65.21$ *A. C.* 8,1856858
Is to the Line $RG = 227.98$ — = 2,3578967

So is *Radius* 90° — — 10,0000000

To the *Tangent* of the Angle ROG 74.02° 10,5435825

Then, as the *Sine* of ROG 74.02° *Arith. C.* 0,0170850
Is to the *Sine* $RG = 227.28$ — = 2,3578967

So is *Radius* 90° — — 10,0000000

To the *Line of Defence* $OG = 237.13$ — 2,3749817

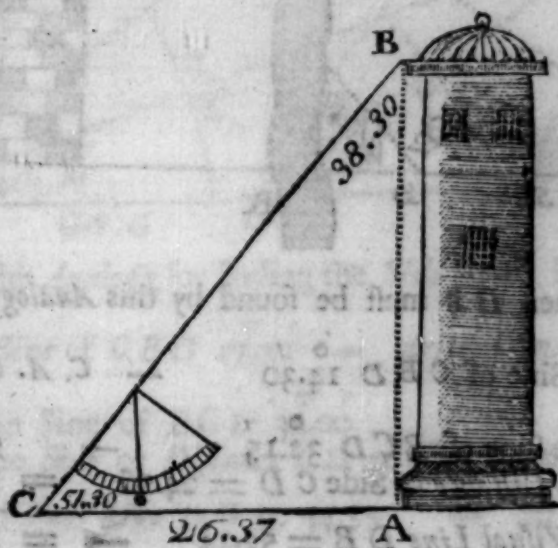
Thus having the *Angles* as in the *Table*, you are here taught the Manner of finding the *Sides* and *Sines* of any *regular Fort*, in any Measure, and *Decimal Parts* thereof.

The Use of Decimals in Measuring Heighths, Depths, and Distances; both accessible and inaccessible.

1. *Altimetry*, or the Mensuration of *Altitudes* and *Depths*, is thus performed.

Let AB represent a *Tower* whose *Height* is required.

Suppose { The *Distance* $AC = 26.37$ Yards.
The Angle ACB (found by a *Quadrant*) 51.30° .



The

The *Analogy* for finding the Height *AB*.

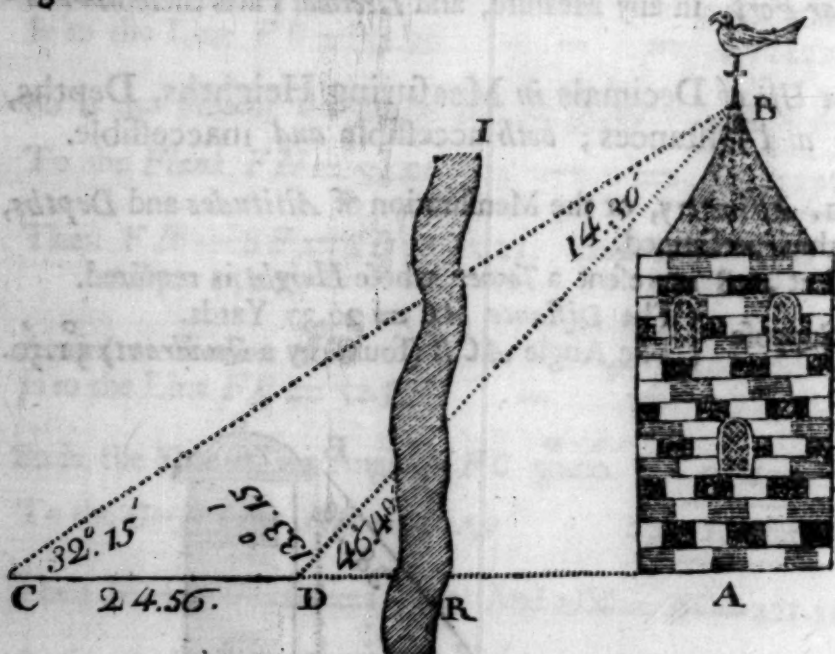
As Radius 90° — — — 10,0000000
Is to the Distance $AC = 26.37$ — — — 1,4211101

So is the Tangent of ACB 51.30° — — — 10,0993948

To the Height of the Tower AB 33.15 — — — 1,5205049

And thus the *Height* or *Altitude* of any other *accessible* Object may be found.

Suppose the *Steeple* AB be *inaccessible* for the River RI ; Then with a *Quadrant* at C take the Angle ACB , and measure a *Distance* to D , where take again the Angle ADB ; and let those *Angles*, and the *Distance* be as in the adjoin'd Figure.



Then DB must be found by this *Analogy*.

As the Sine of CBD 14.30° — — — *C. A.* 0,6014004

Is to the Sine of BCD 32.15° — — — 9,7272276
So is the *Distance* or Side $CD = 24.56$ — — — 1,3902284

To the *Visual Line* $DB = 52.34$ — — — 1,7188564
Having

Having found DB , you may find AB , thus;

As Radius 90° — — — — — 10,000,000
Is to the Line $DB = 52.34$ — — — — — 1,718,856

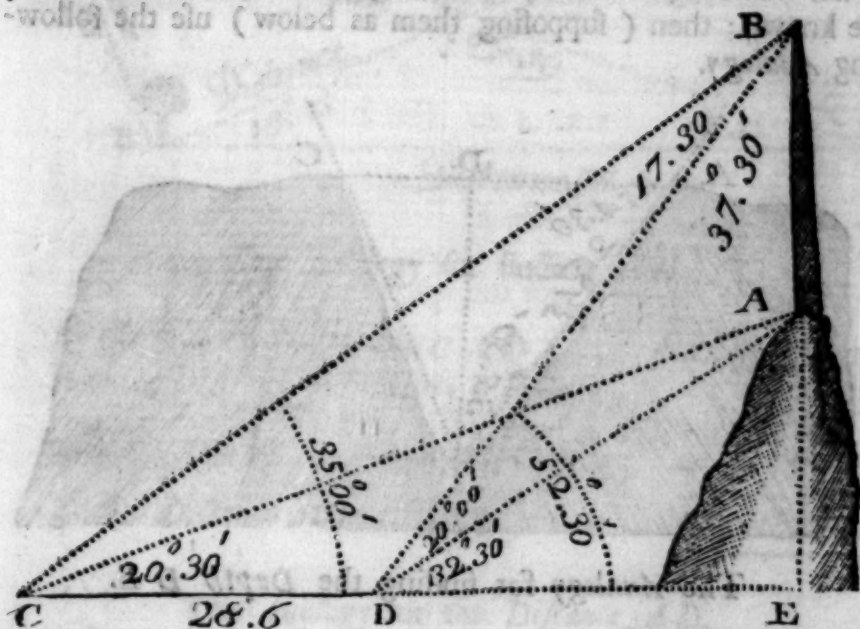
So is the Sine of $ADB = 46.45^\circ$ — — — — — 9,862,352

To the Height required $AB = 38.12$ — — — — — 1,581,209

Thus you find the Height of any inaccessible Objects.

If the Object whose Height you would measure standeth aloft, as on a Hill, &c. as the Tower AB ; then take the

Angles $BCE = 35.00^\circ$, $ACE = 20.30'$; then from C measure the Distance $CD = 28.6$ Yards, and at D , take the Angles $BDE = 52.30'$, and $ADE = 32.30'$, as belcw.



Then the Analogy for finding the Side DB , is

As the Sine of $CBD = 17.30^\circ$ — Arith. Com. 0,521,858

Is to the Sine of $BCD = 35.00^\circ$ — — — — — 9,758,591

So is the Side or Distance $CD = 28.6$ — — — — — 1,456,366

To the Visual Line $DB = 54.55$ — — — — — 1,736,815

U u Then

336 The Use of Decimals in

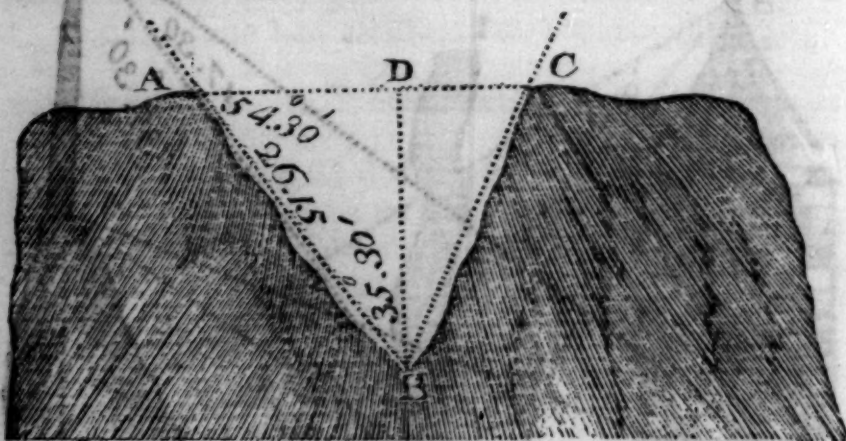
Then say, as the Sine of Complement of BAD } $0,0739708$
 (to 180°) 57.30° $A.C.$ — — —

Is to the Sine of BDA 20.00 — — — $9,5340517$
 So is the Side $DB = 54.55$ — — — $1,7368155$

To the Height of the Object $AB = 22.12 = 1,3448380$

Depths, if *Perpendicular*, are most conveniently measured with a *Line* and *Plummet*; but if the *Depth* be *slanting*, such as *Valleys*, &c. and the *perpendicular Profoundity* be required; do as follows.

Let ABC be a *Valley*, whose *oblique Descents* or *Sides* are AB , and BC ; and its *perpendicular Depth* BD required. Then measure the Side AB , or BC , and take with a *Quadrant* the Angle ABD , or DBC , whereby the others will be known; then (supposing them as below) use the following *Analogy*.



The *Analogy* for finding the *Depth* DB .

As Radius 90° — — — $10,0000000$
 Is to the *slant Descent* $AB = 26.15$ — — — $1,4174717$

So is the Sine of BAD 50.30 — — — $9,9106860$

To the *Perpendicular Depth* $DB = 21.28 = 1,3281577$

The same might have been equally found by the other *Right-angled Triangle* CDB , by the same *Method*.

2. *Longi-*

Longimetry, or measuring Distances. 331

2. *Longimetry*, or the *Menfuration* of the *Distances* of *Ob- jects*, either from *Us*, or from one another, is thus per- formed.

Let *A, B*, be two *Trees*; and let it be required to find the *Distance* of *A* from *C* or *D*; as also of *B* from the same two *Points*; and the *Distance* of *A* from *B*.

Having (by a *Theodolite* or *Semicircle*) at *D*, found the *Angles* *BDC*, and *ADB*; and at *C* the *Angles* *ACD* and *ACB*; and measured the *Distance* of the two *Stations* *CD*, as below: Use the following *Analogies*.



The *Analogy* for finding *AC*.

As the <i>Sine</i> of the Angle <i>CAD</i> $16^{\circ} 15'$	A. C.	0,5531072
Is to the <i>Distance</i> of <i>Stations</i> <i>CD</i> = 25.05	=	1,3988077
So is the <i>Sine</i> of the Angle <i>ADC</i> $76^{\circ} 45'$	—	9,9882821
To the <i>Distance</i> <i>AC</i> = 87.33	—	<u>1,9411972</u>

The *Analogy* for the *Distance* *AD*.

As the <i>Sine</i> of the Angle <i>CAD</i> $16^{\circ} 15'$	A. C.	0,5531072
Is to the <i>Distance</i> of <i>Stations</i> <i>CD</i> = 25.05	=	1,3988077
So is the <i>Sine</i> of <i>ACD</i> $87^{\circ} 00'$	—	9,9994044
To the <i>Distance</i> <i>AD</i> = 89.39	—	<u>1,9513193</u>

The *Analogy* for the Distance *BC*.

$$\begin{array}{rcl}
 \text{As the Sine of the Angle } CBD \ 18.15^\circ & A.C. & 0,5042284 \\
 \text{Is to the Distance of Stations } CD \ 25.05 & = & 1,3988077 \\
 \text{So is the Sine Comp. of the Angle } BDC & & \\
 \quad (to \ 180) \ 88.15^\circ & - & 9,9997974 \\
 \text{To the Distance } BC = 79.95 & - & \underline{1,9028335}
 \end{array}$$

The *Analogy* for the Distance *BD*.

$$\begin{array}{rcl}
 \text{As the Sine of the Angle } CBD \ 18.15^\circ & A.C. & 0,5042284 \\
 \text{Is to the Distance of Stations } CD = 25.05 & = & 1,3988077 \\
 \text{So is the Sine of the Angle } BCD \ 70.30^\circ & & 9,9743466 \\
 \text{To the Distance } BD = 75.4 & - & \underline{1,8775827}
 \end{array}$$

The *Analogy* for the Distance of the *Trees* from each other, viz. *AB*. But first say

$$\begin{array}{rcl}
 \text{As the Sum of the two Sides, } AC + CB = & & \\
 \quad 167.28 \text{ Arith. Comp.} & - & 7,7765050 \\
 \text{Is to their Difference } AC - CB = 7.38 & = & 0,8680564 \\
 \text{So is the Tang. of half the Angles } ABC + BAC & & \\
 \quad 81.45^\circ & - & 9,9954822 \\
 \text{To the Tangent of half their Difference} & & \\
 \quad ABC - BAC \ 2.30^\circ & - & \underline{8,6400436}
 \end{array}$$

Then the Angle *CBA* 84.15° ; and the Angle *BAC* 79.15° .

Wherefore the *Analogy* for the Distance *AB*, is,

$$\begin{array}{rcl}
 \text{As the Sine of the Angle } ABC \ 84.15^\circ & A.C. & 0,0021917 \\
 \text{Is to the Distance of } AC = 87.33 & - & 1,9411972 \\
 \text{So is the Sine of the Angle } ACB \ 16.30^\circ & - & 9,4533418 \\
 \text{To the Dist. of the two Trees } AB = 24.93 & = & \underline{1,3967307}
 \end{array}$$

Thus

The Use of Decimals in Mensuration, &c. 333

Thus I have endeavoured to make it appear how absolutely necessary the Use and Knowledge of the noble Art of *Decimal Arithmetick* is in those Parts of *Mathematical Science*, which require *Trigonometrical Calculations*, by Examples in the most common and useful Arts; I might have gone farther, and shewn its Use in several Parts of *Astronomy*, &c. but I intend only an instructive *Specimen* of its excellent Use in this Kind of Learning, and such I presume this Chapter will be found to be.

C H A P. XIV.

The great Use of DECIMALS in the Mensuration of all Kinds of Superficies and Solids.

IN this *Traet* (wherein the Use of *Decimal Arithmetick* is most obvious, necessary, and excellent) I have only this to advertise the *Reader*, That the Numbers are absolutely taken; and may represent any *Dimensions*, as *Inches*, *Feet*, *Yards*, *Rods*, *Miles*, *Acres*, &c. in the *Area's*, and *Solid Content* of Bodies. And that after the *Area*, or *Content* is found, I shall shew the Manner of Reducing it to any of the *Dimensions* used in *Surveying*, *Guaging*, &c. by means of *Decimals*.

Proposition I. To measure a Square.

Rule. Multiply a Side into it self, the Product is the *Area* or *Square Content*.

Example. Suppose the Side $AB = 12,6$
Multiply by it self $— 12,6$

9) 760

844

2533

12666



The *Square Content* or *Area* is $= 150,4$

Propo.

334 The Use of Decimals in the

Proposition 2. To Measure a Parallelogram.

Rule. Multiply the *Length* by the *Breadth*, the Product is the *Area*, or *Content*.

Example. Mult. the *Length* $AB=16,5$
By the *Breadth* $BD=8,6$

990
1320



The Product is the *Content* = 141,9

Proposition 3. To Measure a Rhombus.

Rule. Multiply one Side into the *perpendicular Height*, the Product is the *Area* or *Content* required.

Exam. Mult. the Side $AB=20,75$
By the *Height* $BD=16,4$

8303
124545
207575



The Product is the *Area* = 340,424

Proposition 4. To Measure a Rhomboides.

Rule. Multiply the *Length* by the *perpendicular Height*, or *Breadth*, the Product is the *Content*.

Ex. Mul. the *Length* $AB=1,28$
By *Breadth* or *Height* $AC=.56$

768
640



The Prod. is the *Area* = 0,7168

Proposition 5. To Measure a Plain Triangle.

Rule. Multiply the *Base* into half the *Perpendicular Height*; or the *Whole perpendicular Height* into half the *Base*; the Product will give the *Area*.

Ex.

Exam. Mult. the Base $AB = 10,8$
By half the Height $CD = 4,2$

217
4355

The Product is the Area = 45,73



Proposition 6. To Measure a Trapezium.

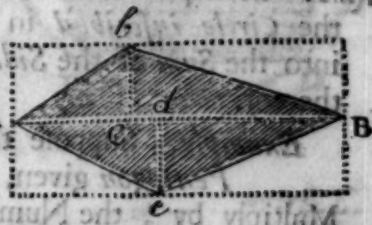
Rule. Multiply the Diagonal into the half Sum of the two Perpendiculars; or the Contrary; and the Product will be the Area or Superficial Content.

Ex. Mul. the Diagonal $AB = 10,5$

By the $\frac{1}{2}$ Sum of the Perpendiculars be de } = 3,3

315
315

The Prod. is the Content = 34,65



Proposition 7. To Measure a Parallelepoleon.

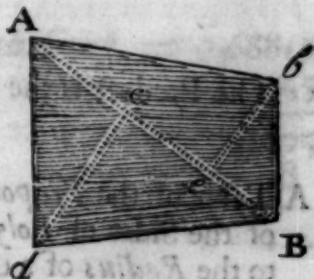
Rule. Multiply the Diagonal by the half Sum of the two Perpendiculars, the Product is the Area.

Exam. Mult. the Diagonal $AB = 14,8$

By $\frac{1}{2}$ Sum of cd and $eb = 6,3$

430
850

The Product is the Content = 90,3

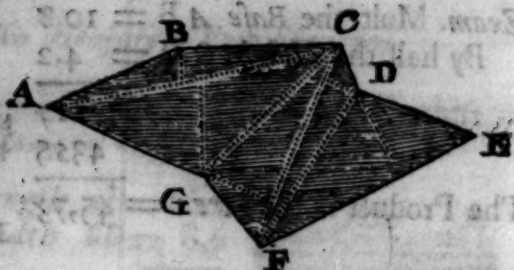


Proposition 8. To Measure an irregular Polygon, or Polygram.

Rule. Divide all such multangular and irregular Figures into Trapeziums and Triangles, then measure them by Prop. 5 and 6.

Example.

Example. Divide the irregular Polygon *ABCDEFG* into the Trapezium *ABCG*, then into the Triangles *GCF*, *CFD*, and *FDE* which are to be measured as taught in Proposition 5, and 6, and the Sum of all added together, will give the Superficial Content or Area of the given Polygram.



Proposition 9. To Measure any regular Polygon, as a Pentagon, Hexagon, Heptagon, Octagon, Nonagon, &c.

Rule. Multiply half the Sum of its Sides into the Radius of the Circle inscribed in the Figure; or half that Radius into the Sum of the Sides, the Product will be the Area thereof.

Example. Suppose the Side of the } $AB = 15,34$
Pentagon given, viz. —
Multiply by $\frac{1}{2}$ the Number of Sides — $2,5$

7670
3068

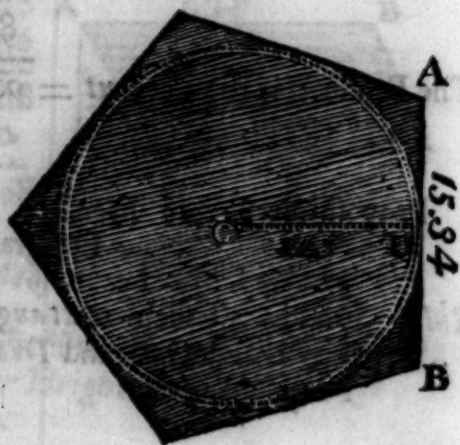
The Product is the $\frac{1}{2}$ Sum of the Sides — $= 38,35$
Multiply that by the Radius *CD* — $= 10,5$

19175
38350

The Product is the Area — $= 402,675$

A Table of the Proportion of the Sides of Polygons to the Radius of a circle inscribed.

Tripon, as 1:0,189675 &c.
Pentagon, as 1:0,68819 &c.
Octagon, as 1:1,207106 &c.
Decagon, as 1:1,535841 &c.
Dodecagon, as 1:1,866320 &c.



Propo-

Proposition 10. To find such constant Multipliers for any of the Regular Polygons, That multiplying the Square of any Side thereby, the Product shall be the Area of the Polygon.

Example. In a Pentagon.

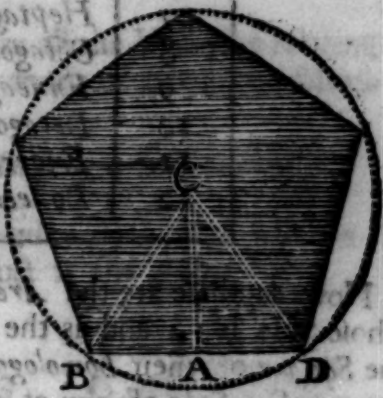
Div. the whole Circle, viz. 360 Deg.

By the Number of Sides; here 5.

The Quotient is the $\angle BCD = 72^\circ$

The $\frac{1}{2}$ thereof is the $\angle ACB = 36^\circ$

Whose Comp. is the $\angle ABC = 54^\circ$



Then make this Proportion;

As the Sine of the Angle $ACB = 36^\circ = 0,2307813$ Arith. C.

Is to half the Side ($= 1$, always) $= ,5 = ,9,6989700$

So is the Sine of the Angle $ABC = 54^\circ = 9,9079576$

To the Perpendicular, or

Radius of the inscribed Circle $AC = ,68819 = ,9,8377089$

Then (by the last Proposition) Arith. Comp. $= 0,68819$

Multiplied into $\frac{1}{2}$ Sum of the Sides $= 2,5$

344095
137638

The Product is the Area $= 1,720475$

And thus may the Area for any other Polygons be found whose Side is 1. And this Area will be the constant Multiplier for that kind of Polygon. A Table of such Multipliers, or Area's, for the several Regular Polygons follow.

X

Sides

<i>Sides.</i>	<i>Names.</i>	<i>Multipliers.</i>
3	<i>Trigon</i>	0,433013
4	<i>Tetragon</i>	1,000000
5	<i>Pentagon</i>	1,720475
6	<i>Hexagon</i>	2,598076
7	<i>Heptagon</i>	3,633959
8	<i>Octagon</i>	4,828427
9	<i>Enneagon</i>	6,181827
10	<i>Decagon</i>	7,694209
11	<i>Endecagon</i>	8,514250
12	<i>Dodecagon</i>	9,330125

Now as these are the *Area's* of each *Polygon* respectively, whose *Side* is 1; and as the *Area's* of *Like Figures*, are as the *Square* of their *homologous*, or *like Sides*; therefore the *Square* of a *Side* of any of those *Polygons* multiplied into its respective *Area* in the *Table*, will produce the true *Area* thereof.

Example. Suppose the *Side* of a *Heptagon* be 10; the *Square* of which is 100; but $100 \times 3,633959 = 363,3959 =$ *Area* of such a *Heptagon*, and the like for any other.

Proposition 11. To Measure a Circle.

Rule. Multiply the *Square* of the *Diameter* (if that be given) by 0,7854; the *Product* is the *Area*. Or, (if the *Periphery* be given) Multiply the *Square* of the *Periphery* by 0,07957; the *Product* is the *Area*, as before.

Exa. Suppose the *Di-* } *AB* = 5,2
ameter of a *Circle* be }

$$\begin{array}{r} 5,2 \\ \times 5,2 \\ \hline 104 \\ 260 \\ \hline \end{array}$$

The *Square* thereof is = 27,04
 Which multiply by —,7854

$$\begin{array}{r} 27,04 \\ \times 7854 \\ \hline 10816 \\ 13520 \\ 21632 \\ 18928 \\ \hline \end{array}$$

The *Area* of the *Circle* is = 21,237216

Put



Put $D = \text{Diameter}$; $P = \text{Periphery}$; and $A = \text{Area}$, of any Circle.

Then it will be $\begin{cases} 3,1416D = P. & \text{And } 0,7854DD = A. \\ 0,3183P = D. & \text{And } 0,07957PP = A. \\ \sqrt{1,2732} = D. & \text{And } \sqrt{12,5664}A = P. \end{cases}$

Thus by those six *Theorems* may all the Varieties relating to the *Diameter*, *Periphery*, and *Area's* of *Circles* be solved.

Proposition 12. To Measure a Semicircle.

Rule. Multiply half the *Semicircular Arch*, into half the *Diameter*; the Product is the *Area*.

Exam. Multiply $\frac{1}{2} ABC = 4,83$
Into $\frac{1}{2} AC = EC = 2,76$

$$\begin{array}{r} 2600 \\ 30333 \\ 86866 \\ \hline \end{array}$$

The Semicircular Area = 11,96



Proposition 13. To Measure the Sector of a Circle.

Rule. Multiply half the *Arch* into the *Radius*, the Product is the *Area* of the *Sector*.

Example. Multiply $\frac{1}{2} AB = 4,53$
Into the Radius $AC = 11,16$

$$\begin{array}{r} 2718 \\ 453 \\ 453 \\ 453 \\ \hline 50,5548 \end{array}$$

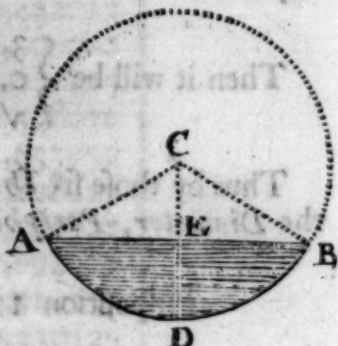


X x 2

Propo.

Proposition 14. To Measure the Segment of a Circle.

Rule. Compleat the Sector $ACBD$, and measure it by the last Proposition; and then find the Area of the Triangle ABC by Proposition 5. Then subduct the Area of the Triangle from the Area of the Sector, the Remainder is the Area of the Segment.



Or thus, (by the Curious Theorems of Mr. Ward.)

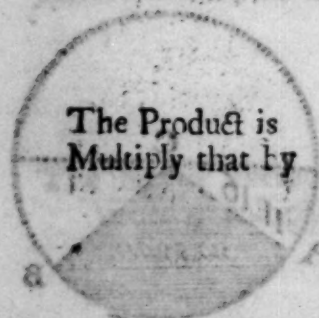
Let $R =$ The Radius, or Semidiameter AC .
 $d =$ The Diff. between the versed Sine and Radius.
 $C =$ Half the Chord or Base of the Seg AE (viz. EC .)

Theorem $\left\{ \frac{2\frac{1}{2} RR - \frac{1}{3} Rd - dd}{1\frac{1}{2} R + d} \right\} \times C = S$, The Area of the Segment.

Proposition 15. To Measure an Ellipsis.

Rule. Multiply the Transverse and Conjugate Diameters into each other; then multiply that Product by the Number 0,7854, the Product is the Area required.

Exam. Mult. the Transverse $CD = 36$
 By the Conjugate $AB = 16$



$$\begin{array}{r}
 36 \\
 \times 16 \\
 \hline
 216 \\
 36 \\
 \hline
 576 \\
 \times 0,7854 \\
 \hline
 2304 \\
 2880 \\
 4608 \\
 4032 \\
 \hline
 \end{array}$$



The Area of the Ellipsis = 452,3904

Propo-

Proposition 16. To Measure the Parabola.

Rule. Multiply the *Greatest Ordinate*, or *Base*, into the perpendicular *Height*, and that *Product* by the single *Repetend*, *8*, the *Product* is the *Area*.

Ex. Mult. the *Ordinate* $AB=53,75$
By the *Abscissa*, or *Height*, $CD=43,3$

$$\begin{array}{r} 16125 \\ 16125 \\ \hline 21500 \end{array}$$

Multiply this *Product* $= 2327,375$
By the *Repetend* $= ,8$

$$\begin{array}{r} 2327,375 \\ \hline 9)13966250 \end{array}$$

The *Area* of the *Parab.* $= 1551,803$



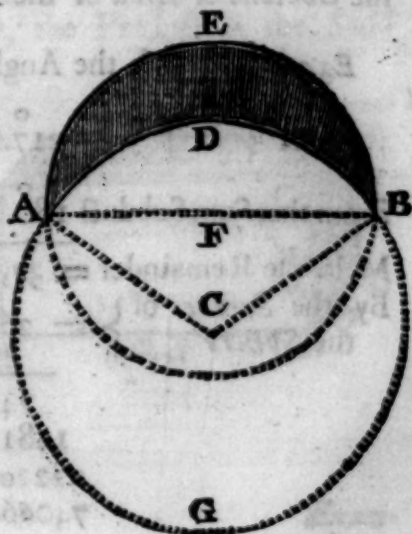
Note, an easier way is to multiply the first *Product* by 2, and then divide by 3 for the *Area*; since $\frac{2}{3} = ,8$

Proposition 17. To Measure the Circular Space, called a Lune; (being like the falcated Moon.)

Rule. In order to find the *Area* of the *Lune* $AEBD$, seek first the *Area* of the *Semicircle* AEB , by *Prop.* 12.

Then find the *Area* of the *Segment* ADB of the *Circle* $ADBG$, by *Prop.* 14.

Lastly; Subtract the *Area* of the *Segment* from the *Area* of the *Semicircle*; there remains the *Area* of the *Lune* required.



Also

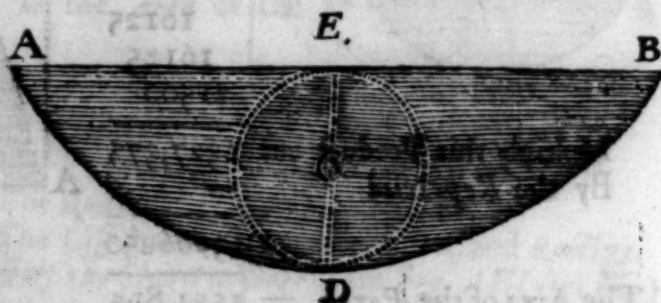
Also the two *Lunes* A, B, are together equal to the *Triangle* C.

Note, This is call'd the *Quadrature*, or *Squaring the Lunes* of *Hippocrates*.



Proposition 18. To Measure the Cycloid.

Rule. Find the *Area* of the *Circle* C, described on the *Axis* DE, and multiply that by 3, the *Product* is the *Area* of the *Cycloid* (called also the *Trochoid*) A E B D.



Proposition 19. To Measure a Spherical Triangle.

Rule. From the *Sum* of the *three Angles*, subduct 180 *Degrees*, multiply the *Superficies* of the whole *Sphere* or *Globe* by the *Remainder*; this *Product* divide by 720, the *Quotient* is the *Content* or *Area* of the *Triangle*.

Exam. Suppose the *Angles* 7

$$A + B + C = 217.02^\circ$$

From that *Sum* Subduct 180°

$$\text{Mult. the Remainder} = 37.02$$

$$\text{By the Surface of the Sphere } \} = 2642$$

$$7406$$

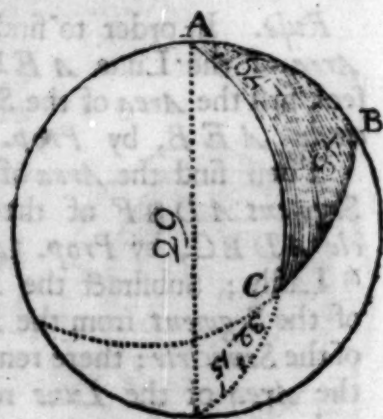
$$148133$$

$$2222000$$

$$7406666$$

Divide by 720) 97842,06 (135,891 = *Area* of the *Triangle*.

Note,



Note, This is a very uncommon, curious, and useful Proposition.

Mensuration of Solids.

Proposition 20. To Measure a Cube.

Rule. Multiply the Side of the Cube into it self, and that Product again by the Side; this last Product will be the Solid Content, or Solidity of the Cube.

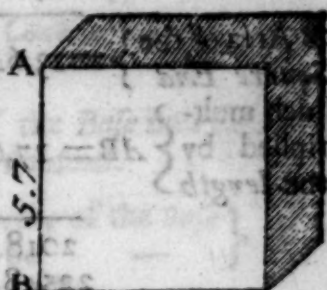
Exam. Mult. the Side $AB = 5,7$
By it self — — $5,7$

399
 285

This Product — $32,49$
Multiply again by — $5,7$

22743
 16245

The Product is — $185,193 = \text{Solidity of the Cube.}$



Proposition 21. To Measure a Parallelopipedon.

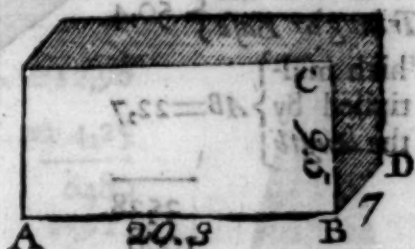
Rule. Find the Area of the End or Base, and Multiply that by the Length of the Piece, the Product is the Solid Content thereof.

Exam. Mult. $CB = 9,5$
By — $ED = 7$

The Area of the } $= 66,5$
End — — — — —

Mult. that by the } $= 20,3$
Length AB — — — — —

The Solidity — $= 1349,95$



Exam-

Example 2. Of a Square Prism, or Parallelopipedon.

Mult. into it self,
or Square the
Side of the
Square End

$$BC = 5,8$$

$$\begin{array}{r} 5,8 \\ \times 5,8 \\ \hline 464 \\ 290 \\ \hline \end{array}$$

The Area of the
Square End } = 23,64

Which mul-
tiplied by
the length } $AB = 17,6$

$$\begin{array}{r} 20184 \\ \times 2364 \\ \hline 23548 \\ 3364 \\ \hline \end{array}$$

The Product is 592,064 = Solid Content.



Example 3. Of a Triangular Prism.

Mult. the Base $BC = 8,4$
By the Height $DE = 6$

The Area of the
Triangular Base } 50,4

Which mul-
tiplied by
the length } $AB = 22,7$

$$\begin{array}{r} 3528 \\ \times 1008 \\ \hline 1008 \\ 1008 \\ \hline \end{array}$$

The Prod. is 1144,00 = Solidity of the Prism.



Exam-

Example 4. Of a Cylinder.

Square the Diameter } = 8,6
of the End *BC* } 8,6

The Area of the } = 44,4
circular Base } 44,4

Which mult. by the } = 23
Length *AB* } 23

The Solidity of } = 1022,2
the Cylinder } 1022,2



Proposition. 22. To Measure the Convex Superficies of a Cylinder.

Rule. Multiply the *Periphery* of the *Base* into the *Length* of the *Cylinder*; the *Product* is the *Content*.

Example. Suppose the *Circumference* of the *Base* } 20,944
(in the last Figure) *BECD* to be — } 20,944

Then multiply that by the Length *AB* = 23

62832
41888

The *Superficial Content* of the *Cylinder* = 481,712

Proposition 23. To Measure a Pyramid.

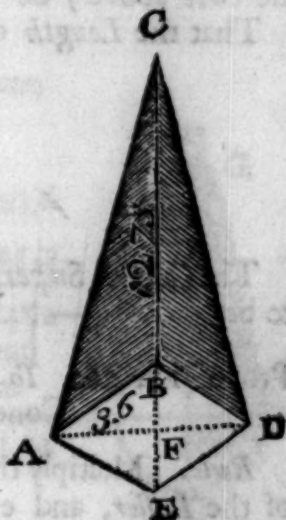
Rule. Multiply the *Area* of the *Base* into one Third of its *Altitude* or *Height*; the *Product* is the *Solid Content*.

Exam. Suppose the *Area* of } = 12,96
the *Square Base* *ABDE* } 12,96

Multiply that into $\frac{1}{3}$ of *FC* = 4,25

6480
2592
5184

The *Solid Content* of the } = 55,08
Pyramid — — } 55,08



Y y

Note

Mensuration of Superficies and Solids. 347

To that *Root*, add the *Sum* of the two *Area's* of the *End*; then multiply this *last Sum* by $\frac{1}{3}$ of the *Frustum's Height*, the *Product* is the *Solid Content*.

Example 1. Of any Pyramid.

Suppose the *Area* of the *Greater Base* } 64
(of a *Square Pyramid* $A D B C =$ }
And the *Area* of the *Lesser* $F E G H =$ 16



The *Product* is — — — 1024
The *Square Root* thereof is — = 32
To which add the *Sum* of the 2 }
Area's — — — = } 80

This *Sum* — — — 112
Multiply by $\frac{1}{3}$ of the *Height* $D E =$ 4,93
— — — 9)336

The *Solid Content* — — = 552,53 of the *Square Frustum*.

And thus the *Frustum* of any other kind of *Pyramid* is to be found.

Example 2. Of a Cone.

Suppose the *Area* of the *Greater Base* } 122,6
(of the *Cone* adjoin'd) $A D B C =$ }
And the *Area* of the *Lesser* $F E G H =$ 11,2



The *Product* is — — — 1373,12

The Square Root thereof is	—	=	37,055 &c.
To which add the Sum of the two Area's	—	=	133,8
This last Sum	—	—	170,855
Multiply by $\frac{1}{3}$ of the Height KE	—	=	6,75
			854275
			1195985
			1025130
The Solid Content	—	=	1153,27125

Proposition 27. To Measure a Sphere, or Body perfectly Round.

Rule. Multiply the Diameter into the Circumference, the Product is the Superficial Content; then multiply that by $\frac{1}{2}$ of the Diameter, the Product will be the Solid Content of the Sphere.

Example. Of the Superficial and Solid Content.

Mult. the Periphery of the Globe or Sphere,	}	62,832
$ACBD$		
By the Axis of the Sphere AB	}	20
The Spherical Super-	}	1256,640
ficies		
Which mult. by $\frac{1}{2}$ of	}	3
AB		
		9) 3769,92



The Solid Content = 4188,8 of the Sphere.

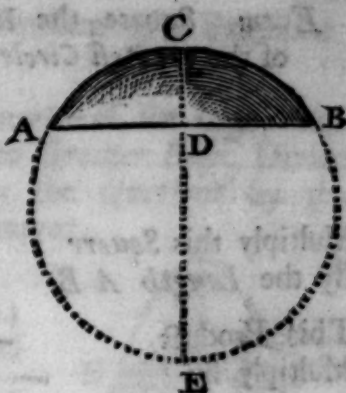
Proposition 28. To Measure any Frustum or Segment of a Sphere or Globe.

1. Let

1. Let $\begin{cases} D = \text{Diameter of the} \\ \text{Sphere } C E \\ H = \text{Height of the Seg-} \\ \text{ment } C D \end{cases}$

Then $\frac{3DHH - 2H^3}{1,91} = \text{The Content.}$

Of the Polar Segment $A C B D$.



2. Let $\begin{cases} D = \text{Diameter } C E, \text{ as} \\ \text{before,} \\ x = \text{the Base } A B = G I. \\ H = \text{the Thickness } D F. \end{cases}$

Then $\frac{2DD + xx}{3,82} \times H = \text{the Middle Segment } A B I G, \text{ called the Zone. By these two Theorems, may the Solidity of any Segment of a Globe or Sphere be found.}$



To find the Superficial Content of any Segment, as

Thus $\begin{cases} \text{As the Axis, or Diameter of the Sphere,} \\ \text{Is to the whole Superficies of the Sphere;} \\ \text{So is the Height of any Segment,} \\ \text{To the Area of its Curve Superficies.} \end{cases}$

Proposition 29. To find the Content of a Spheroid.

Rule. Multiply the Square of the Diameter of the Greatest Circle, by the Length; then multiply that Product by 0,5236; this last Product will be the Solidity of the Spheroid.

Exam.

Exam. Square the Diameter
of the greatest Circle, $CD =$

6,5

6,5

325

390

Multiply this Square

— = 42,25

By the Length AB

— = 10

This Product

— = 422,5

Multiply by

— = 5,236

25350

12675

8450

21125

The Solidity

— = 221,221



of the Spheroid.

Note. The two Theorems, which find the Content of the Segments of a Sphere, find those like Segments of the Spheroid, if in them D be made $= CD$, in this Spheroid.

Proposition 30. To Measure a Parabolic Conoid.

Rule. Multiply the Square of the Diameter of the Base by the Height; and that Product by 0,3927; this last Product is the solid Content.

Exam. Mult. into it self, or }
Square the Diameter $AB =$

9,2

9,2

184

828

This Square

— = 84,64

Multiply by the Height $CD =$

10

And the Product

— = 846,4

Multiply by

— = 3,927

59248

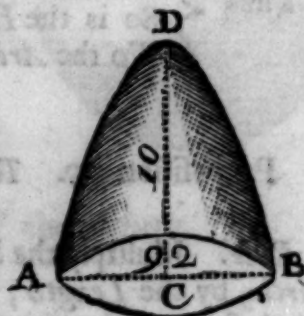
16928

76176

25392

The solid Content = 332,38128 of the Parabolic Conoid.

Propo-



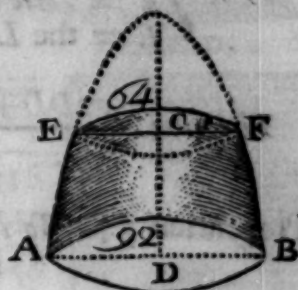
Proposition 31. To Measure the Frustrum of a Parabolic Conoid.

Rule. Add the Square of the Diameter of the Lesser Base to the Square of the Diameter of the Greater Base, Divide that Sum by 2,5464; then multiply the Quotient by the Height; the Product is the Solid Content.

Exam. The Square of $AB = 84,64$

And the Square of $EF = 40,96$

Sum of the Squares $= 125,6$



Then 2,5464) 125,600 (49,3244

101856

5 = CD the Height.

237440

229176

8264

246,622 = the Solidity of the
(Frustrum $ABFE$.)

8264 &c.

Proposition 32. To Measure a Parabolic Spindle, or Pyramid.

Rule. Multiply the Square of the Diameter of the Greatest Circle, by the Length; and that Product by 0,41888; this last Product is the Solid Content of the Spindle.

Exam. 1. Square the
Diameter of the
Greatest Circle AB

6,05

6,05

3025

36300

Mult. this Square 36,6025

By the Length $CD = 8,5$

1830125

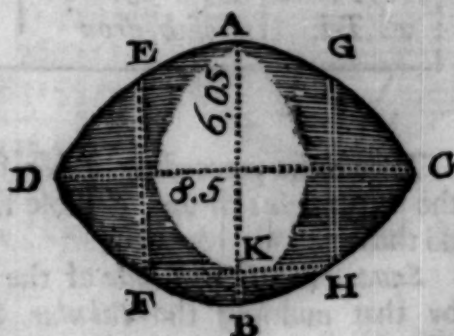
2928200

This Product 311,12125

Multiply by ,41888

Solid Cont. = 130,3224692 of the Parabolic Spindle.

Example.



Example 2. To Measure the Middle Segment of the Spindle, *E G H F*.

Let $\left\{ \begin{array}{l} D = \text{the Diameter of the greatest Circle } A B. \\ C = \text{the Diameter of either Base or End } E F \text{ or } G H. \\ x = 2 B K = \text{the Excess of } A B \text{ above } E F, \text{ or } G H. \\ L = \text{the Length of the Frustum } F H. \end{array} \right.$

Theorem $\left\{ \frac{2DD + CC - 0,4xx}{3,82} \times L = \right\}$ Solidity of the middle Frustum.

Proposition 33. To Measure any of the Five Regular (or Platonic) Bodies.

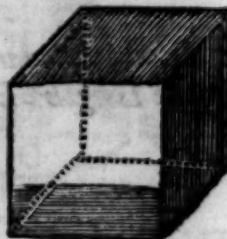
Those Bodies being only an Aggregate of so many Pyramids as they consist of Sides, each Side being the Base of a Pyramid, may, with due Consideration, be measured by Proposition 22. However for the more ready and expeditious Practice, I shall here subjoin a Table of the Solidity and Superficies of each Body whose Side is 1. or Unity.

Sides.	Names.	Superficies.	Solidity.
4 Tri.	Tetrahedron	1,732051	0,1178511
6 Sq.	Hexahedron	6,000000	1,0000000
8 Tri.	Octahedron	3,464102	0,4714045
12 Pent.	Dodecahedron	20,645729	7,663119
20 Tri.	Icosahedron	8,660254	2,181695

To use the preceeding Table for finding the Superficies of any of those five Bodies, do thus;

Square the Given Side of the Body, and by that multiply the Tabular Superficial Number; the Product is the Superficies of the Body, which was sought.

Hexahedron.



Exam.

Example. Suppose the Side of the Dodecahedron be 8, the Square of which is 64.

Then multiply the Tab. Numb. 20,645729
By the Square of the Side

$$\begin{array}{r} 20,645729 \\ \times 64 \\ \hline 82582916 \\ 123874374 \\ \hline 1321,326656 \end{array}$$

Superficial Content = 1321,326656

To find the Solid Content; Multiply the Tabular Number of the Solidity, by the Cube of the Side given, the Product is the Solid Content.

Example. Of the aforesaid Dodecahedron.

Multiply the Tabular Numb. 7,663119
By the Cube of the Side
(=8,) viz. — } 512

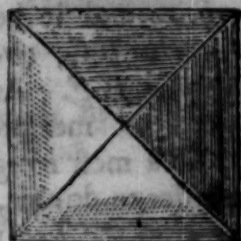
$$\begin{array}{r} 7,663119 \\ \times 512 \\ \hline 15320238 \\ 7663119 \\ 38315525 \\ \hline 3923,516928 \end{array}$$

Solid Content = 3923,516928

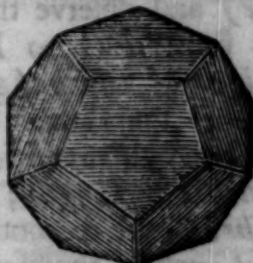
Tetrahedron.



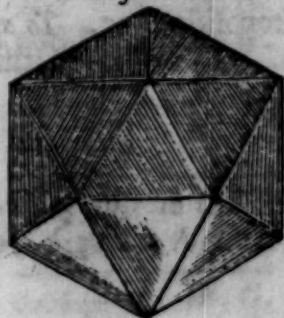
Octahedron.



Dodecahedron.



Icosahedron.



And thus proceed for the Superficies and Solidity of the other Bodies.

Proposition 34. To Measure any solid or hollow Body how irregular soever.

Rule. Take any Vessel in Form of a Parallelopipedon, and fill it with Water to a certain Height, and then immerse
Z z the

the *irregular Body* therein, and observe how much the Water is raised by the Side of the Vessel; for that Water is equal in Quantity, or Solid Content, to the *irregular Body*; and may be found by Proposition 20.

Example. Suppose the Vessel *BCDE*, in Form of a *Parallelopipedon*, whose Length is two Feet, eight Inches; and Breadth one Foot, ten Inches; and it be fill'd with Water to *FG*, twelve Inches Deep = *EF*. And it is required to measure the Log *A*, of a most *irregular Form*. In order to do this, I take and immerge the Log in the Vessel of Water (as in the lower Figure) and observe the Water rise from *FG* to *HI*, the Height of which (viz. *FH*, or *GI*) I measure, and find to be 5,5 Inches.



Then by Proposition 20, I find a Body of Water, 2 Feet, 8 Inches in Length; 1 Foot, ten Inches in Width, and 5,5 Inches Deep, to contain 3520 Solid Inches, or 2,037 Solid Feet; which therefore is the Solid Content of the Log *A*.

Proposition 35. To assign the Dimensions by which the several Artificers measure their Work.

Masons measure their Work by	{ Feet and Inches	{	Superficial	{	As Pavements, Chimneys, Pieces, Corniches, &c.
			Solid		{ Columns, and o- ther solid Parts of Buildings.

Bricklayers mea- sure by the	{	Foot; Arches, Quoins, Cornices, Fascia's, &c.
		Yard; Pavements, Pieces, &c.
		Rod; All manner of Walls, and Chimneys.
		Square; All manner of Tiling, and Slating.

Carpenters mea- sure by the	{	Square of	{	Rocfing, Partitioning, Floor- ing, &c.
		100 Feet		

Joiners

Mensuration of Superficies and Solids. 355

Joiners, Painters, Plasterers measure by the Square Yard for the most part; seldom, by the Foot Square.

Glasiers measure their Work by the Decimal Foot Square; very rarely by Inches and Quarters.

Solid Inches.

Gaugers measure the Area's and Content, of Superficies and Solids, by $\left\{ \begin{array}{l} 231, \text{ For Wine Gallons.} \\ 282, \text{ For Ale Gallons.} \\ 268,8. \text{ For Corn Gallons.} \end{array} \right.$

Surveyers measure Rod or Pole; but mostly by the Chain Land by the $\left\{ \begin{array}{l} \text{of an 100 Links} = 4 \text{ Rod.} \end{array} \right.$

Proposition 36. To assign Multipliers, and Divisors, whereby the Gauger may readily find any Area or Content in Gallons or Bushels, whether the Dimensions be taken in Inches, Feet, or Yards.

This I shall do by disposing the Numbers in their proper Order in the Table subjoin'd.

Dimensions.	Multipliers.	Divisors.
Inches.	0.004329 W. G.	231 W. G.
	0.003546 A. G.	282 A. G.
	0.003722 C. G.	268.8 C. G.
	0.000465 C. B.	2150.42 C. B.
Feet	7.48052 W. G.	0.13368 W. G.
	6.12765 A. G.	0.16352 A. G.
	6.42448 C. G.	0.15565 C. G.
	0.80356 C. B.	1.24446 C. B.
Yards	67.32468 W. G.	0.014853 W. G.
	55.14885 A. G.	0.018168 A. G.
	57.82032 C. G.	0.017294 C. G.
	7.23204 C. B.	0.138273 C. B.
Note,	W. G. } stands for	Wine Gallons.
	A. G. }	Ale Gallons.
	C. G. }	Corn Gallons.
	C. B. }	Corn Bushels.

The Use of the preceeding Table.

If by the foregoing *Propositions* the *Area* or *Content* of any *Superficies* or *Solid* be found; and you would know how many *Wine, Ale, or Corn Gallons, or Bushels* it contains; Multiply, or Divide, the given *Area* or *Solidity*, by the *Tabular* Number corresponding to the *respective Measure, and Dimension*, the *Product* is the *Area* or *Content* sought in *Gallons, or Bushels*.

Example. Suppose the *Parallelopipedon* in *Example I. of Proposition 20.* represent a *Cistern*, and the *Dimensions* there used be *Feet*; then the *Solid Content* of the *Cistern* is there found to be 1349,95 *Solid* or *Cubick Feet*.

$$\text{Then } \left\{ \begin{array}{l} 1349,95 \times 7,48052 = \text{Content in Wine Gallons.} \\ 1349,95 \times 6,12765 = \text{Content in Ale Gallons.} \\ 1349,95 \times 6,42448 = \text{Content in Corn Gallons.} \\ 1349,95 \times 0,80356 = \text{Content in Corn Bushels.} \end{array} \right.$$

Or by Division,

$$\text{Thus } \left\{ \begin{array}{l} ,13368) 1349,95 (= \text{Content in Wine Gallons.} \\ ,16352) 1349,95 (= \text{Content in Ale Gallons.} \\ ,15565) 1349,95 (= \text{Content in Corn Gallons.} \\ 1,24446) 1349,95 (= \text{Content in Corn Bushels.} \end{array} \right.$$

And were *Dimensions* taken by a *Decimal Yard* or *Foot* (which are by far the best *Instruments* for *Mensuration*;) The *Business* of *Gauging* would be easy, and greatly expedited by a *Table* not before extant, that I know of.

Note; If you would find the *Content* of *Circular Area's* at one *Operation* (without reducing them by the common *Multiplier* 0,785398) do thus,

$$\text{Divide the Square of the Diameter by } \left\{ \begin{array}{l} 359,05 \text{ for Ale Gallons.} \\ 294,12 \text{ for Wine Gallons.} \\ 342,24 \text{ for Corn Gallons.} \\ 2738, \text{ for Corn Bushels.} \end{array} \right.$$

But

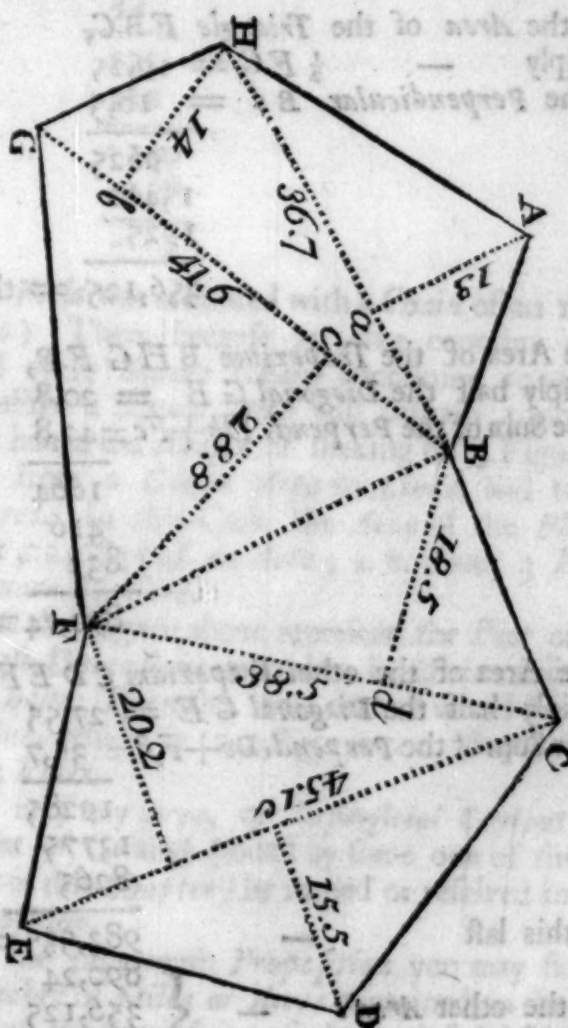
Mensuration of Superficies and Solids. 357

But in this Case the *Dimensions* must be understood of Inches only.

By what I have said in this *Proposition*, I suppose the dexterous young *Artist* will be easily apprised of the Nature, Manner, and Reason of *Gauging*; and how to apply the foregoing *Propositions* to that purpose.

Proposition 37. To apply the foregoing *Propositions* of Superficial Mensuration to Surveying.

Suppose a *Field* in Form of an *Irregular Polygon*; as *ABCDEFGH*, below.



The

The Field being measured, *Plotted*, and the *Plot* resolved into *Trapezia*, and *Triangles*, as per *Proposition* 8.

Proceed to find the *Contents* of the *Triangles* by *Proposition* 5. and of the *Trapezia* by *Proposition* 6.

Thus for the *Area* of the *Triangle* *H A B*,

Multiply half the Base *H B* = 18,35

By the *Perpendicular* *A a* = 13

5505

1835

238,55 = the *Area*.

For the *Area* of the *Triangle* *F B C*,

Multiply — $\frac{1}{2}$ *F C* = 19,25

By the *Perpendicular* *B d* = 18,5

9625

15400

1925

356,125 = the *Area*.

For the *Area* of the *Trapezium* *B H G F B*,

Multiply half the *Diagonal* *G B* = 20,8

By the Sum of the *Perpend.* *H b* + *F c* = 42,8

1664

416

832

890,24 = the *Area*.

For the *Area* of the other *Trapezium* *C D E F C*,

Multiply half the *Diagonal* *C E* = 27,55

By the Sum of the *Perpend.* *D e* + *F f* = 35,7

19285

13775

8265

To this last —

983,535 = the *Area*.

Add the other *Area's* —

890,24

356,125

238,55

The Sum of all is the *Content* = 2468,45 = the *Field*.
Now

Mensuration of Superficies and Solids. 359

Now if the Numbers are supposed to be *Poles* or *Rods*, Divide the Whole *Superficial Content* or *Area* of the *Field*, by 160, the Quotient will be the Number of *Statute Acres* the *Field* contains. See the Work.

$$\begin{array}{r}
 160 \overline{) 2468,45} \text{ (15,4278 Acres;} \\
 \underline{16} \\
 86 \\
 \underline{80} \\
 68 \\
 \underline{64} \\
 44 \\
 \underline{32} \\
 125 \\
 \underline{112} \\
 130 \\
 \underline{128} \\
 2
 \end{array}$$

Acres Rood Rods Yards
Or, 15 : 1 : 28 : 13

But if the *Field* was measured with a *Chain* of an 100 *Links* (= 4 *Rods*.) Then, because an *Acre* contains 10 *Square Chains*, and every *Square Chain* containing 10000 *Square Links*, therefore a *Square Acre* will contain 10000 *Square Links*; and hence the Reason of striking off 5 Figures to the Right-hand from a *Given Area* in *Links*, and taking the Rest for *Acres*. In this Case, the *Area* of the *Plot* above would be but, 246845 of an *Acre*; i. e. about 3 *Poles* and 7 *Yards*, *Square Measure*.

But suppose the *Figure* above represent the *Plot* of a large *Common* whose *Dimensions* are taken in *Chains* and *Decimal Parts* of a *Chain*: Then the *Area* thereof would be 2468,45 *Square Chains*, which is 246,845 *Acres*; Or, 246 *Acres*, 3 *Roods*, 14 *Poles*.

And thus may any *Area*, or *Superficial Content* of any *Field* or *Plot* of Ground (found by some one of the first 19 *Propositions* of this *Chapter*) be turned or resolved into *Acres* by this *Proposition*.

Note, By the nineteenth *Proposition* you may find, with ease, the Number of *Miles* or *Acres* contained in the Whole, or any Part, of the *Superficies* of the *terrestrial Globe*; and of any *Province*, *Kingdom*, *Empire*, or *Nation* of the World;
and

and it ought therefore to be well understood by all who would study *Political Arithmetick*, this being the most certain, curious, and principal Branch of that Art; Tho' it be not in every Tract on that Subject, nor in any Book of Mensuration that I know of.

Thus I have finished a Tract of *Planometry*, and *Stereometry*, or of the *Mensuration* of *Superficies* and *Solids*; containing a *Greater Variety* than I know to be in *many Books*, wrote wholly on the Subject, of two, three, or four *Shillings Price*; having endeavoured here, as in all other Parts of this *System*, to oblige the *Reader* with all that could be *useful* for him to know, in the most plain but *Compendious Manner*, at the easiest Rate.

*Vive, Vale; Si quid novisti rectius istis,
Candidus imperti; si non, his utere mecum.*

F I N I S.



A
TABLE
OF THE
LOGARITHMS
TO ALL
NUMBERS,

Not exceeding 10000, or 4 Places,
whether they be Intire, Broken, or Mixt
Numbers.

Particularly useful in Extracting the
Square Cube, &c. Roots, and solving
Questions in Compound Interest, &c.

A a a

Natural Numbers.	0	1	2	3	4
1	0000000	0413927	0791812	1139433	1461280
2	3010300	3222193	3424227	3517278	3802112
3	4771212	4913617	5051500	5185139	5314789
4	6020600	6127838	6232493	6334684	6434527
5	6989700	7075762	7160033	7242759	7323937
6	7781512	7853298	7923917	7993405	8061800
7	8450980	8512583	8573325	8633229	8692317
8	9030900	9084850	9138138	9190781	9242793
9	9542425	9590414	9637878	9684829	9731278
100	0000000	0004341	0008677	0013009	0017337
101	0043214	0047511	0051805	0056094	0060379
102	0086002	0090257	0094509	0098756	0102999
103	0128372	0132587	0136794	0141003	0145205
104	0170333	0174507	0178677	0182843	0187005
105	0211893	0216027	0220157	0224284	0228400
106	0253059	0257154	0261245	0265333	0269416
107	0293838	0297895	0301948	0305997	0310043
108	0334237	0338257	0342273	0346284	0350293
109	0374265	0378247	0382226	0386202	0390173
110	0413927	0417873	0421816	0425755	0429691
111	0453230	0457140	0461048	0464952	0468852
112	0492180	0496056	0499929	0503797	0507663
113	0530784	0534626	0538464	0542299	0546130
114	0569048	0572856	0576661	0580462	0584260
115	0606978	0610753	0614525	0618293	0622058
116	0644580	0648322	0652061	0655797	0659530
117	0681859	0685569	0689276	0692980	0696681
118	0718820	0722499	0726175	0729847	0733517
119	0755470	0759118	0762762	0766404	0770043
120	0791812	0795430	0799045	0802656	0806265
121	0827854	0831441	0835026	0838608	0842187
122	0863598	0867157	0870712	0874264	0877814
123	0899051	0902580	0906107	0909631	0913151
124	0934217	0937718	0941216	0944711	0948204
125	0969100	0972573	0976043	0979511	0982975
126	1003705	1007151	1010593	1014033	1017471
127	1038037	1041455	1044871	1048284	1051694
128	1072100	1075491	1078880	1082266	1085650
129	1105897	1109262	1112625	1115985	1119343
130	1139433	1142773	1146110	1149444	1152776
131	1172713	1176027	1179338	1182647	1185954
132	1205739	1209028	1212314	1215598	1218880
133	1238516	1241780	1245042	1248301	1251558
134	1271048	1274288	1277525	1280760	1283993

Natural Numbers.	5	6	7	8	9
1	1760912	2041200	2304489	2552725	2787536
2	3979400	4149733	4313637	4471580	4623980
3	5440680	5563015	5681017	5797836	5910646
4	6532125	6627578	6720978	6812412	6901961
5	7403627	7481880	7558748	7634280	7708520
6	8129133	8195439	8260748	8325089	8388491
7	8750613	8808136	8864907	8920946	8976271
8	9294189	9344984	9395192	9444827	9493900
9	9777236	9822712	9867717	9912261	9956352
100	0021661	0025980	0030295	0034605	0038912
101	0064660	0068937	0073209	0077478	0081742
102	0107239	0111473	0115704	0119931	0124154
103	0149403	0153597	0157787	0161973	0166155
104	0191163	0195317	0199467	0203613	0207755
105	0232524	0236639	0240750	0244857	0248960
106	0273496	0277572	0281644	0285712	0289777
107	0314085	0318123	0322157	0326188	0330214
108	0354297	0358298	0362295	0366289	0370279
109	0394141	0398105	0402066	0406023	0409977
110	0433623	0437551	0441476	0445398	0449315
111	0472749	0476642	0480532	0484418	0488301
112	0511525	0515384	0519239	0523091	0526939
113	0549958	0553783	0557605	0561423	0565237
114	0588055	0591846	0595634	0599419	0603200
115	0625820	0629578	0633334	0637085	0640834
116	0663259	0666985	0670708	0674428	0678145
117	0700379	0704073	0707765	0711453	0715138
118	0737183	0740847	0744507	0748164	0751818
119	0773679	0777312	0780941	0784568	0788192
120	0809870	0813473	0817073	0820669	0824263
121	0845763	0849336	0852906	0856473	0860037
122	0881361	0884905	0888446	0891984	0895519
123	0916669	0920185	0923696	0927206	0930712
124	0951693	0955180	0958664	0962146	0965624
125	0986437	0989896	0993353	0996806	1000257
126	1020905	1024337	1027766	1031192	1034616
127	1055102	1058506	1061909	1065308	1068705
128	1089031	1092410	1095785	1099159	1102529
129	1122698	1126050	1129400	1132746	1136091
130	1156105	1159432	1162756	1166077	1169396
131	1189257	1192559	1195858	1199154	1202448
132	1222159	1225435	1228709	1231981	1235250
133	1254813	1258064	1261314	1264561	1267806
134	1287223	1290450	1293676	1296890	1300110

Natural Numbers.	0	1	2	3	4
135	1303338	1306553	1309767	1312978	1316187
136	1335389	1338581	1341771	1344958	2348144
137	1367206	1370374	1373541	1376705	1379867
138	1398791	1401937	1405080	1408222	1411361
139	1430148	1433271	1436392	1439511	1442628
140	1461280	1464381	1467480	1470577	1473671
141	1492191	1495270	1498347	1501422	1504494
142	1522883	1525941	1528996	1532049	1535100
143	1553360	1556396	1559430	1562462	1565491
144	1583625	1586640	1589653	1592663	1595672
145	1613680	1616674	1619666	1622656	1625644
146	1643528	1646502	1649474	1652443	1655411
147	1673173	1676127	1679078	1682027	1684975
148	1702617	1705550	1708482	1711411	1714339
149	1731863	1734776	1737688	1740598	1743506
150	1760913	1763807	1766699	1769590	1772478
151	1789769	1792645	1795518	1798389	1801259
152	1818436	1821292	1824146	1826999	1829850
153	1846941	1849752	1852588	1855421	1858253
154	1875207	1878026	1880844	1883659	1886473
155	1903317	1906118	1908917	1911714	1914510
156	1931246	1934029	1936810	1939590	1942367
157	1958996	1961762	1964525	1967287	1970047
158	1986571	1989319	1992065	1994809	1997552
159	2013971	2016702	2019431	2022158	2024883
160	2041200	2043913	2046625	2049335	2052044
161	2068259	2070955	2073650	2076344	2079035
162	2095150	2097830	2100508	2103185	2105860
163	2121876	2124540	2127201	2129862	2132521
164	2148438	2151086	2153732	2156376	2159018
165	2174839	2177471	2180100	2182727	2185355
166	2201081	2203696	2206310	2208922	2211533
167	2227165	2229764	2232363	2234959	2237554
168	2253093	2255677	2258260	2260841	2263421
169	2278867	2281436	2284003	2286570	2289134
170	2304489	2307043	2309596	2312146	2314696
171	2329961	2332500	2335038	2337574	2340108
172	2355284	2357809	2360331	2362853	2365373
173	2380461	2382971	2385479	2387986	2390491
174	2405492	2407988	2410481	2412974	2415465
175	2430380	2432861	2435341	2437819	2440296
176	2455127	2457593	2460059	2462523	2464986
177	2479733	2482186	2484637	2487085	2489536
178	2504200	2506639	2509077	2511513	2513948

Logarithms (to 1789.)

365

Natural Number.	5	6	7	8	9
135	1319393	1322597	1325798	1328998	1332194
136	1351326	1354507	1357685	1360861	1364034
137	1383027	1386184	1389339	1392492	1395643
138	1414498	1417632	1420765	1423895	1427022
139	1445742	1448854	1451964	1455072	1458177
140	1476763	1479853	1482941	1486026	1489110
141	1507564	1510632	1513698	1516762	1519824
142	1538149	1541195	1544240	1547282	1550322
143	1568519	1571544	1574568	1577589	1580608
144	1598678	1601683	1604685	1607686	1610684
145	1628630	1631614	1634595	1637575	1640553
146	1658376	1661310	1664301	1667260	1670218
147	1687920	1690863	1693803	1696744	1699682
148	1717264	1720188	1723110	1726029	1728947
149	1746412	1749316	1752218	1755118	1758016
150	1775365	1778250	1781132	1784013	1786892
151	1804126	1806992	1809856	1812718	1815578
152	1832698	1835545	1838390	1841233	1844075
153	1861084	1863912	1866739	1869563	1872386
154	1889285	1892095	1894903	1897709	1900514
155	1917304	1920096	1922886	1925674	1928461
156	1945143	1947917	1950690	1953460	1956229
157	1972806	1975562	1978317	1981070	1983821
158	2000293	2003032	2005769	2008505	2011239
159	2027607	2030329	2033049	2035768	2038485
160	2054750	2057455	2060159	2062869	2065560
161	2081725	2084413	2087100	2089785	2092468
162	2108534	2111203	2113876	2116544	2119211
163	2135178	2137833	2140487	2143139	2145789
164	2161659	2164298	2166935	2169572	2172206
165	2187980	2190603	2193225	2195845	2198464
166	2214142	2216750	2219356	2221960	2224563
167	2240148	2242740	2245331	2247920	2250507
168	2265999	2268576	2271151	2273724	2276296
169	2291697	2294258	2296818	2299377	2301934
170	2317244	2319790	2322335	2324879	2327421
171	2342641	2345173	2347703	2350232	2352759
172	2367891	2370408	2372923	2375437	2377950
173	2392995	2395497	2397998	2400498	2402996
174	2417954	2420442	2422929	2425414	2427898
175	2442771	2445245	2447718	2450189	2452658
176	2467447	2469907	2472365	2474823	2477278
177	2491984	2494430	2496874	2499317	2501759
178	2516382	2518814	2521245	2523675	2526103

Natural Numbers.	0	1	2	3	4
179	2528530	2530956	2533380	2535803	2538224
180	2552725	2555137	2557548	2559957	2562365
181	2576786	2579184	2581582	2583978	2586373
182	2600714	2603099	2605484	2607867	2610248
183	2624511	2626883	2629255	2631625	2633993
184	2648178	2650538	2652896	2655253	2657609
185	2671717	2674064	2676410	2678752	2681097
186	2695129	2697464	2699797	2702128	2704459
187	2718416	2720738	2723058	2725375	2727696
188	2741578	2743888	2746196	2748503	2750809
189	2764618	2766915	2769211	2771506	2773800
190	2787536	2789821	2792105	2794388	2796669
191	2810334	2812607	2814879	2817150	2819419
192	2833012	2835274	2837534	2839793	2842051
193	2855573	2857823	2860071	2862318	2864565
194	2878017	2880255	2882492	2884728	2886963
195	2900346	2902573	2904798	2907022	2909246
196	2922561	2924776	2926990	2929203	2931415
197	2944662	2946866	2949069	2951271	2953471
198	2966652	2968845	2971036	2973227	2975417
199	2988531	2990713	2992893	2995073	2997151
200	3010300	3012471	3014641	3016809	3018977
201	3031961	3034121	3036280	3038438	3040595
202	3053514	3055663	3057811	3059959	3062105
203	3074960	3077099	3079237	3081374	3083509
204	3096302	3098430	3100557	3102684	3104809
205	3117539	3119657	3121774	3123889	3126004
206	3138672	3140780	3142887	3144992	3147097
207	3159703	3161801	3163897	3165993	3168087
208	3180633	3182721	3184807	3186893	3188977
209	3201463	3203540	3205617	3207692	3209767
210	3222193	3224260	3226327	3228393	3230457
211	3242825	3244882	3246939	3248995	3251050
212	3263359	3265407	3267454	3269500	3271545
213	3283796	3285834	3287872	3289909	3291944
214	3304138	3306167	3308195	3310222	3312248
215	3324385	3326404	3328423	3330440	3332457
216	3344557	3346548	3348557	3350565	3352572
217	3364597	3366598	3368598	3370597	3372595
218	3384565	3386557	3388547	3390537	3392526
219	3404441	3406424	3408405	3410386	3412366
220	3424227	3426200	3428173	3430145	3432116
221	3443923	3445887	3447851	3449814	3451776
222	3463530	3465486	3467441	3469395	3471348

Logarithms (to 2229.)

367

Natural Numbers.	5	6	7	8	9
179	2540645	2543063	2545481	2547897	2550312
180	2564772	2567177	2569581	2571984	2574386
181	2588766	2591158	2593549	2595939	2598327
182	2612629	2615008	2617385	2619762	2622137
183	2636361	2638727	2641092	2643455	2645817
184	2659964	2662317	2664669	2667020	2669369
185	2683439	2685780	2688119	2690457	2692794
186	2706788	2709116	2711443	2713769	2716093
187	2730013	2732328	2734643	2736956	2739268
188	2753113	2755417	2757719	2760020	2762320
189	2776092	2778383	2780673	2782962	2785250
190	2798950	2801229	2803507	2805784	2808059
191	2821688	2823955	2826221	2828486	2830750
192	2844307	2846563	2848817	2851070	2853322
193	2866810	2869054	2871296	2873538	2875778
194	2889196	2891428	2893659	2895889	2898118
195	2911468	2913688	2915908	2918127	2920344
196	2933626	2935835	2938044	2940251	2942457
197	2955671	2957869	2960067	2962263	2964458
198	2977605	2979792	2981979	2984164	2986348
199	2999420	3001605	3003781	3005955	3008128
200	3021144	3023309	3025474	3027637	3029799
201	3042751	3044905	3047059	3049211	3051363
202	3064240	3066394	3068537	3070679	3072820
203	3085644	3087778	3089910	3092042	3094172
204	3106933	3109056	3111173	3113299	3115420
205	3128118	3130231	3132343	3134454	3136563
206	3149200	3151303	3153405	3155505	3157605
207	3170181	3172273	3174365	3176455	3178545
208	3191061	3193143	3195224	3197305	3199384
209	3211840	3213913	3215984	3218055	3220124
210	3232521	3234584	3236645	3238705	3240766
211	3253104	3255157	3257209	3259260	3261310
212	3273589	3275633	3277675	3279716	3281757
213	3293979	3296012	3298045	3300077	3302108
214	3314273	3316297	3318320	3320343	3322364
215	3334473	3336488	3338501	3340514	3342526
216	3354579	3356585	3358589	3360593	3362596
217	3374593	3376589	3378584	3380579	3382572
218	3394514	3396501	3398488	3400473	3402458
219	3414345	3416323	3418301	3420277	3422252
220	3434086	3436055	3438023	3439991	3441957
221	3453737	3455698	3457657	3459615	3461573
222	3473300	3475252	3477202	3479152	3481101

Natural. Numbers	0	1	2	3	4
223	3483049	3484996	3486942	3488887	3490832
224	3502480	3504419	3506356	3508293	3510228
225	3521825	3523755	3525684	3527612	3529539
226	3541084	3543006	3544926	3546845	3548764
227	3560259	3562171	3564083	3565994	3567905
228	3579348	3581253	3583156	3585059	3586961
229	3598355	3600251	3602146	3604040	3605934
230	3617278	3619166	3621053	3622939	3624825
231	3636120	3638000	3639878	3641756	3643633
232	3654880	3656751	3658622	3660492	3662361
233	3673559	3675423	3677285	3679147	3681008
234	3692159	3694014	3695869	3697723	3699576
235	3710679	3712526	3714373	3716219	3718065
236	3729120	3730960	3732799	3734637	3736475
237	3747483	3749316	3751147	3752977	3754807
238	3765769	3767594	3769418	3771240	3773062
239	3783979	3785796	3787612	3789427	3791241
240	3802112	3803922	3805730	3807538	3809345
241	3820170	3821972	3823773	3825573	3827373
242	3838154	3839948	3841741	3843534	3845326
243	3856063	3857850	3859636	3861421	3863206
244	3873898	3875678	3877457	3879235	3881012
245	3891661	3893433	3895205	3896975	3898746
246	3909351	3911116	3912880	3914644	3916407
247	3926969	3928727	3930485	3932241	3933997
248	3944517	3946268	3948018	3949767	3951516
249	3961993	3963734	3965480	3967223	3968964
250	3979400	3981137	3982873	3984603	3986343
251	3996737	3998467	4000196	4001925	4003653
252	4014005	4015728	4017451	4019172	4020893
253	4031205	4032921	4034637	4036352	4038066
254	4048337	4050047	4051755	4053464	4055171
255	4065402	4067105	4068807	4070508	4072209
256	4082400	4084096	4085791	4087486	4089180
257	4099331	4101021	4102710	4104398	4106085
258	4116197	4117880	4119562	4121244	4122925
259	4132998	4134674	4136350	4138025	4139700
260	4149733	4151404	4153073	4154742	4156410
261	4166405	4168069	4169732	4171394	4173056
262	4183013	4184670	4186327	4187983	4189638
263	4199557	4201208	4202859	4204509	4206158
264	4216039	4217684	4219328	4220972	4222614
265	4232459	4234097	4235735	4237372	4239009
266	4248816	4250449	4252080	4253712	4255342

Natural Numbers.	5	6	7	8	9
223	3492775	3494718	3496660	3498601	3500541
224	3512163	3514098	3516031	3517963	3519895
225	3531465	3533391	3535316	3537239	3539162
226	3550682	3552599	3554515	3556430	3558345
227	3569813	3571723	3573630	3575537	3577443
228	3588862	3590762	3592662	3594560	3596458
229	3607827	3609719	3611610	3613500	3615390
230	3626709	3628593	3630476	3632358	3634239
231	3645510	3647380	3649260	3651134	3653007
232	3664230	3666097	3667964	3669830	3671695
233	3682869	3684728	3686587	3688445	3690302
234	3701428	3703280	3705131	3706981	3708830
235	3719909	3721753	3723596	3725438	3727279
236	3738311	3740147	3741983	3743817	3745651
237	3756636	3758464	3760292	3762118	3763944
238	3774884	3776704	3778524	3780343	3782161
239	3793055	3794868	3796680	3798492	3800302
240	3811151	3812956	3814761	3816565	3818368
241	3829171	3830969	3832766	3834563	3836359
242	3847117	3848908	3850698	3852487	3854275
243	3864990	3866773	3868555	3870337	3872118
244	3882789	3884565	3886340	3888114	3889888
245	3900515	3902284	3904052	3905819	3907585
246	3918162	3919931	3921691	3923452	3925211
247	3935742	3937506	3939260	3941013	3942765
248	3953254	3955011	3956758	3958504	3960249
249	3970705	3972446	3974185	3975924	3977662
250	3988077	3989811	3991543	3993275	3995007
251	4005380	4007106	4008832	4010557	4012282
252	4022614	4024333	4026052	4027771	4029488
253	4039780	4041492	4043205	4044916	4046627
254	4056878	4058584	4060289	4061994	4063698
255	4073909	4075608	4077307	4079005	4080703
256	4090874	4092567	4094259	4095950	4097641
257	4107772	4109459	4111144	4112829	4114513
258	4124605	4126285	4127964	4129643	4131320
259	4141374	4143047	4144719	4146391	4148063
260	4158077	4159744	4161410	4163076	4164741
261	4174717	4176377	4178037	4179696	4181355
262	4191293	4192947	4194601	4196254	4197906
263	4207806	4209454	4211101	4212748	4214394
264	4224257	4225898	4227539	4229180	4230820
265	4240645	4242281	4243915	4245550	4247183
266	4256972	4258601	4260230	4261858	4263486

Natural. Numbers	0	1	2	3	4
267	4265113	4266739	4268365	4269990	4271614
268	4281348	4282968	4284588	4285207	4287825
269	4297523	4299137	4300751	4302364	4303976
270	4313638	4315246	4316853	4318460	4320067
271	4329693	4331295	4332897	4334498	4336098
272	4345689	4347285	4348881	4350476	4352071
273	4361626	4363217	4364807	4366396	4367985
274	4377506	4379090	4380674	4382258	4383841
275	4393327	4394906	4396484	4398062	4399639
276	4409091	4410664	4412237	4413809	4415380
277	4424798	4426365	4427932	4429499	4431065
278	4442448	4442010	4443571	4445132	4446692
279	4456042	4457598	4459154	4460709	4462264
280	4471580	4473131	4474681	4476231	4477780
281	4487063	4488608	4490153	4491697	4493241
282	4502491	4504031	4505570	4507109	4508647
283	4517864	4519399	4520932	4522465	4523998
284	4533183	4534712	4536241	4537769	4539296
285	4548449	4549972	4551495	4553018	4554540
286	4563660	4565179	4566696	4568213	4569731
287	4578819	4580332	4581844	4583356	4584868
288	4593925	4595433	4596940	4598446	4599953
289	4608978	4610481	4611983	4613484	4614985
290	4623980	4625477	4626974	4628470	4629966
291	4638930	4640422	4641914	4643405	4644895
292	4653828	4655316	4656802	4658288	4659774
293	4668676	4670158	4671640	4673120	4674601
294	4683473	4684950	4686427	4687903	4689378
295	4698220	4699692	4701163	4702634	4704105
296	4712917	4714384	4715850	4717317	4718782
297	4727564	4729027	4730488	4731949	4733410
298	4742163	4743620	4745076	4746533	4747988
299	4756712	4758164	4759616	4761067	4762518
300	4771212	4772660	4774107	4775553	4776999
301	4785665	4787108	4788550	4789991	4791432
302	4800069	4801507	4802945	4804381	4805818
303	4814426	4815859	4817292	4818724	4820156
304	4828736	4830164	4831592	4833019	4834446
305	4842998	4844422	4845845	4847268	4848690
306	4857214	4858633	4860052	4861470	4862888
307	4871384	4872798	4874212	4875626	4877039
308	4885507	4886917	4888326	4889735	4891144
309	4899585	4900990	4902395	4903799	4905203
310	4913617	4915018	4916418	4917818	4919217

Natural Numbers.	5	6	7	8	9
267	4273238	4274861	4276484	4278106	4279727
268	4289443	4291060	4292677	4294293	4295908
269	4305588	4307199	4308809	4310419	4312029
270	4321673	4323278	4324883	4326487	4328090
271	4337698	4339298	4340896	4342491	4344092
272	4353665	4355258	4356851	4358444	4360035
273	4369573	4371161	4372748	4374334	4375920
274	4385423	4387005	4388587	4390167	4391747
275	4401216	4402792	4404368	4405943	4407517
276	4416951	4418522	4420092	4421661	4423229
277	4432630	4434195	4435759	4437322	4438885
278	4448252	4449811	4451370	4452928	4454485
279	4463818	4465372	4466925	4468477	4470029
280	4479329	4480877	4482424	4483971	4485517
281	4494784	4496326	4497868	4499410	4500951
282	4510184	4511721	4513258	4514794	4516329
283	4525531	4527062	4528593	4530124	4531654
284	4540823	4542349	4543875	4545400	4546924
285	4556061	4557582	4559102	4560622	4562142
286	4571246	4572762	4574277	4575791	4577305
287	4586378	4587889	4589399	4590908	4592417
288	4601458	4602963	4604468	4605972	4607475
289	4616486	4617986	4619485	4620984	4622482
290	4631461	4632956	4634450	4635944	4637437
291	4646386	4647875	4649364	4650853	4652341
292	4661259	4662743	4664227	4665711	4667194
293	4676081	4677560	4679039	4680518	4681996
294	4690853	4692327	4693801	4695275	4696748
295	4705575	4707044	4708513	4709982	4711450
296	4720247	4721711	4723175	4724639	4726102
297	4734870	4736329	4737788	4739247	4740705
298	4749443	4750898	4752352	4753806	4755259
299	4763968	4765418	4766867	4768316	4769765
300	4778445	4779890	4781334	4782778	4784222
301	4792873	4794313	4795754	4797192	4798631
302	4807254	4808689	4810124	4811559	4812993
303	4821587	4823018	4824448	4825878	4827307
304	4835873	4837299	4838725	4840150	4841574
305	4850112	4851533	4852954	4854375	4855795
306	4864305	4865721	4867138	4868554	4869969
307	4878451	4879863	4881275	4882686	4884097
308	4892552	4893959	4895366	4896773	4898179
309	4906607	4908009	4909412	4910814	4912216
310	4920616	4922014	4923413	4924810	4926207

Natural Numbers.	0	1	2	3	4
311	4927604	4929000	3930396	4931791	4933186
312	4941546	4942938	4944329	4945720	4947110
313	4955443	4956831	4958218	4959604	4960990
314	4969296	4970679	4972062	4973444	4974825
315	4983106	4984484	4985862	4987240	4988617
316	4996871	4998245	4999619	5000992	5002365
317	5010593	5011962	5013332	5014701	5016069
318	5024271	5025637	5027002	5028366	5029731
319	5037907	5039268	5040629	5041989	5043349
320	5051500	5052857	5054213	5055569	5056925
321	5065050	5066403	5067755	5069107	5070459
322	5078559	5079907	5081255	5082603	5083950
323	5092025	5093370	5094711	5096057	5097400
324	5105450	5106790	5108130	5109469	5110808
325	5118834	5120170	5121505	5122841	5124175
326	5132176	5133508	5134840	5136171	5137501
327	5145478	5146805	5148133	5149460	5150787
328	5158738	5160062	5161386	5162709	5164031
329	5171959	5173279	5174598	5175917	5177236
330	5185139	5186455	5187771	5189086	5190400
331	5198280	5199592	5200903	5202214	5203525
332	5211381	5212689	5213996	5215303	5216610
333	5224442	5225746	5227050	5228353	5229656
334	5237465	5238765	5240064	5241364	5242663
335	5250448	5251744	5253040	5254335	5255631
336	5263393	5264685	5265977	5267269	5268560
337	5276299	5277588	5278876	5280163	5281451
338	5289167	5290452	5291736	5293020	5294303
339	5301997	5303278	5304558	5305839	5307118
340	5314789	5316066	5317343	5318619	5319895
341	5327544	5328817	5330090	5331363	5332635
342	5340261	5341531	5342800	5344069	5345338
343	5352941	5354207	5355473	5356738	5358003
344	5365584	5366847	5368109	5369370	5370631
345	5378191	5379450	5380708	5381966	5383223
346	5390761	5392016	5393271	5394525	5395779
347	5403295	5404546	5405797	5407048	5408298
348	5415792	5417040	5418288	5419535	5420781
349	5428254	5429498	5430742	5431986	5433229
350	5440680	5441921	5443161	5444401	5445641
351	5453071	5454308	5455545	5456781	5458017
352	5465427	5466660	5467894	5469126	5470359
353	5477747	5478977	5480207	5481436	5482665
354	5490033	5491259	5492486	5493712	5494937

Natural Numbers.	5	6	7	8	9
311	4934580	4935974	4937368	4938761	4940154
312	4948500	4949890	4951279	4952667	4954056
313	4962375	4963761	4965145	4966529	4967913
314	4976206	4977587	4978967	4980347	4981727
315	4989994	4991370	4992746	4994121	4995496
316	5003737	5005109	5006481	5007852	5009222
317	5017437	5018805	5020172	5021539	5022905
318	5031094	5032458	5033821	5035183	5036545
319	5044709	5046068	5047426	5048785	5050142
320	5058280	5059635	5060990	5062344	5063697
321	5071810	5073160	5074511	5075860	5077210
322	5085297	5086644	5087990	5089335	5090680
323	5098743	5100085	5101427	5102768	5104109
324	5112147	5113485	5114823	5116160	5117497
325	5125510	5126844	5128178	5129511	5130844
326	5138832	5140162	5141491	5142820	5144149
327	5152113	5153439	5154764	5156089	5157414
328	5165354	5166676	5167997	5169318	5170639
329	5178554	5179872	5181189	5182506	5183823
330	5191715	5193028	5194342	5195655	5196968
331	5204835	5206145	5207455	5208764	5210073
332	5217916	5219222	5220528	5221833	5223138
333	5230958	5232260	5233562	5234863	5236164
334	5243961	5245259	5246557	5247854	5249151
335	5256925	5258219	5259513	5260807	5262100
336	5269851	5271141	5272431	5273721	5275010
337	5282738	5284024	5285311	5286596	5287882
338	5295587	5296869	5298152	5299434	5300716
339	5308398	5309677	5310955	5312234	5313512
340	5321171	5322446	5323721	5324996	5326270
341	5333907	5335179	5336450	5337721	5338991
342	5346606	5347874	5349141	5350408	5351675
343	5359267	5360532	5361795	5363059	5364322
344	5371892	5373153	5374413	5375672	5376932
345	5384481	5385737	5386994	5388250	5389506
346	5397032	5398286	5399538	5400791	5402043
347	5409548	5410798	5412047	5413296	5414544
348	5422028	5423274	5424519	5425765	5427010
349	5434472	5435714	5436956	5438198	5439439
350	5446880	5448119	5449358	5450596	5451834
351	5459253	5460489	5461724	5462958	5464193
352	5471591	5472823	5474055	5475286	5476517
353	5483894	5485123	5486351	5487578	5488806
354	5496162	5497387	5498612	5499836	5501060

Natural Numbers.	0	1	2	3	4
355	5502283	5503507	5504730	5505952	5507174
356	5514500	5515720	5516939	5518158	5519377
357	5526682	5527898	5529114	5530330	5531545
358	5538830	5540043	5541256	5542468	5543680
359	5550944	5552154	5553362	5554572	5555781
360	5563025	5564231	5565437	5566643	5567848
361	5575072	5576275	5577477	5578680	5579881
362	5587086	5588285	5589484	5590683	5591882
363	5599066	5600262	5601458	5602654	5603849
364	5611014	5612207	5613399	5614592	5615784
365	5622929	5624118	5625308	5626497	5627685
366	5634811	5635997	5637183	5638369	5639555
367	5646661	5647844	5649027	5650209	5651392
368	5658478	5659658	5660838	5662017	5663196
369	5670264	5671440	5672617	5673793	5674969
370	5682017	5683191	5684364	5685537	5686710
371	5693739	5694910	5696080	5697249	5698419
372	5705429	5706597	5707764	5708930	5710097
373	5717088	5718252	5719416	5720580	5721743
374	5728716	5729877	5731038	5732198	5733358
375	5740313	5741471	5742628	5743786	5744943
376	5751878	5753033	5754188	5755342	5756496
377	5763413	5764565	5765717	5766868	5768019
378	5774917	5776057	5777215	5778363	5779511
379	5786392	5787538	5788683	5789828	5790973
380	5797836	5798979	5800121	5801263	5802405
381	5809250	5810389	5811529	5812668	5813807
382	5820634	5821770	5822907	5824043	5825179
383	5831988	5833122	5834255	5835388	5836521
384	5843312	5844443	5845574	5846704	5847834
385	5854617	5855735	5856863	5857990	5859117
386	5865873	5866998	5868123	5869247	5870371
387	5877110	5878232	5879353	5880475	5881596
388	5888317	5889436	5890555	5891674	5892792
389	5899496	5900612	5901728	5902844	5903959
390	5910646	5911759	5912873	5913985	5915098
391	5921768	5922878	5923988	5925098	5926208
392	5932861	5933968	5935076	5936183	5937290
393	5943925	5945030	5946135	5947239	5948344
394	5954962	5956064	5957166	5958268	5959369
395	5965971	5967070	5968169	5969268	5970367
396	5976952	5978048	5979145	5980241	5981336
397	5987905	5988999	5990092	5991186	5992279
398	5998831	5999922	6001013	6002102	6003193

Natural Numbers.	5	6	7	8	9
355	5508396	5509618	5510839	5512059	5513280
356	5520595	5521813	5523031	5524248	5525465
357	5532760	5533975	5535189	5536403	5537617
358	5544892	5546103	5547314	5548524	5549735
359	5556989	5558197	5559404	5560612	5561818
360	5569053	5570257	5571461	5572665	5573869
361	5581083	5582284	5583485	5584686	5585886
362	5593080	5594278	5595476	5596673	5597870
363	5605044	5606239	5607433	5608627	5609820
364	5616975	5618167	5619358	5620548	5621739
365	5628873	5630062	5631250	5632437	5633624
366	5640740	5641925	5643109	5644293	5645477
367	5652573	5653755	5654936	5656117	5657298
368	5664373	5665553	5666731	5667909	5669087
369	5676144	5677320	5678494	5679669	5680843
370	5687882	5689054	5690226	5691397	5692568
371	5699588	5700757	5701926	5703094	5704262
372	5711263	5712428	5713594	5714759	5715924
373	5722906	5724069	5725231	5726393	5727555
374	5734518	5735678	5736837	5737996	5739154
375	5746099	5747256	5748412	5749568	5750723
376	5757650	5758803	5759956	5761109	5762261
377	5769169	5770320	5771470	5772620	5773769
378	5780659	5781806	5782953	5784100	5785246
379	5792118	5793262	5794406	5795550	5796693
380	5803547	5804688	5805829	5806969	5808110
381	5814945	5816084	5817222	5818359	5819497
382	5826314	5827450	5828585	5829719	5830854
383	5837654	5838786	5839918	5841050	5842181
384	5848963	5850093	5851222	5852351	5853479
385	5860244	5861370	5862496	5863622	5864748
386	5871495	5872618	5873742	5874865	5875987
387	5882717	5883838	5884958	5886078	5887198
388	5893910	5895028	5896145	5897262	5898379
389	5905075	5906189	5907304	5908418	5909532
390	5916210	5917322	5918434	5919546	5920657
391	5927318	5928427	5929536	5930644	5931753
392	5938397	5939503	5940609	5941715	5942820
393	5949447	5950551	5951654	5952757	5953860
394	5960470	5961571	5962671	5963771	5964871
395	5971465	5972563	5973660	5974758	5975855
396	5982432	5983527	5984622	5985717	5986811
397	5993371	5994464	5995556	5996648	5997739
398	6004282	6005373	6006462	6007551	6008640

Natural Numbers.	0	1	2	3	4
399	6009729	6010817	6011905	6012993	6014081
400	6020600	6021685	6022771	6023856	6024941
401	6031444	6032527	6033609	6034692	6035774
402	6042261	6043341	6044421	6045500	6046580
403	6053050	6054128	6055205	6056282	6057359
404	6063814	6064888	6065963	6067037	6068111
405	6074550	6075622	6076694	6077766	6078837
406	6085260	6086330	6087399	6088468	6089537
407	6095944	6097011	6098078	6099144	6100210
408	6106602	6107666	6108730	6109794	6110857
409	6117233	6118295	6119355	6120417	6121478
410	6127839	6128898	6129957	6131015	6132073
411	6138418	6139475	6140531	6141587	6142643
412	6148972	6150026	6151080	6152133	6153187
413	6159501	6160552	6161603	6162654	6163705
414	6170003	6171052	6172101	6173149	6174197
415	6180481	6181527	6182573	6183619	6184665
416	6190933	6191977	6193021	6194064	6195107
417	6201360	6202402	6203443	6204484	6205524
418	6211763	6212802	6213840	6214879	6215917
419	6222140	6223177	6224213	6225249	6226284
420	6232493	6233527	6234560	6235594	6236627
421	6242821	6243852	6244884	6245915	6246945
422	6253154	6254183	6255212	6256241	6257269
423	6263404	6264430	6265457	6266483	6267509
424	6273659	6274683	6275707	6276730	6277754
425	6283889	6284911	6285933	6286954	6287975
426	6294096	6295115	6296134	6297153	6298172
427	6304279	6305296	6306312	6307329	6308345
428	6314438	6315452	6316467	6317481	6318495
429	6324573	6325585	6326597	6327609	6328620
430	6334685	6335694	6336704	6337713	6338723
431	6344773	6345780	6346788	6347795	6348801
432	6354837	6355843	6356848	6357852	6358857
433	6364879	6365882	6366884	6367887	6368889
434	6374897	6375898	6376898	6377898	6378898
435	6384893	6385891	6386889	6387887	6388884
436	6394865	6395861	6396857	6397852	6398847
437	6404814	6405808	6406802	6407795	6408788
438	6414741	6415733	6416724	6417715	6418705
439	6424645	6425634	6426623	6427612	6428601
440	6434527	6435514	6436500	6437487	6438473
441	6444386	6445371	6446355	6447339	6448323
442	6454223	6455205	6456187	6457169	6458153

Natural Numbers.	5	6	7	8	9
399	6015168	6016255	6017341	6018428	6019514
400	6026025	6027109	6028193	6029277	6030361
401	6036855	6037937	6039018	6040099	6041180
402	6047659	6048738	6049816	6050895	6051973
403	6058435	6059512	6060587	6061663	6062738
404	6069185	6070259	6071332	6072405	6073478
405	6079909	6080979	6082050	6083120	6084190
406	6090605	6091674	6092742	6093809	6094877
407	6101275	6102342	6103407	6104472	6105537
408	6111921	6112948	6114046	6115109	6116171
409	6122539	6123599	6124660	6125720	6126779
410	6133132	6134189	6135247	6136304	6137361
411	6143698	6144754	6145809	6146863	6147918
412	6154240	6155292	6156345	6157397	6158449
413	6164755	6165805	6166855	6167905	6168954
414	6175245	6176293	6177340	6178387	6179434
415	6185710	6186755	6187800	6188845	6189889
416	6196150	6197193	6198235	6199277	6200319
417	6206565	6207605	6208645	6209684	6210724
418	6216955	6217992	6219030	6220067	6221104
419	6227320	6228355	6229300	6230424	6231459
420	6237660	6238693	6239725	6240757	6241789
421	6247976	6249006	6250036	6251066	6252095
422	6258267	6259295	6260322	6261350	6262377
423	6268534	6269559	6270585	6271610	6272634
424	6278777	6279800	6280823	6281845	6282867
425	6289995	6290016	6291036	6292057	6293076
426	6299190	6300208	6301226	6302244	6303262
427	6309361	6310377	6311392	6312408	6313423
428	6319508	6320522	6321535	6322548	6323560
429	6329632	6330643	6331653	6332664	6333674
430	6339732	6340740	6341749	6342757	6343765
431	6349808	6350814	6351820	6352826	6353832
432	6359861	6360865	6361869	6362872	6363876
433	6369891	6370893	6371894	6372895	6373896
434	6379898	6380897	6381896	6382895	6383894
435	6389882	6390879	6391876	6392872	6393869
436	6399842	6400837	6401832	6402826	6403820
437	6409781	6410773	6411765	6412758	6413749
438	6419696	6420686	6421676	6422666	6423656
439	6429589	6430577	6431565	6432552	6433540
440	6439459	6440445	6441430	6442416	6443401
441	6449307	6450291	6451274	6452257	6453240
442	6459133	6460114	6461095	6462076	6463057

Natural Numbers.	0	1	2	3	4
443	6464027	6465017	6465997	6466977	6467957
444	6473830	6474808	6475785	6476763	6477740
445	6483600	6484576	6485552	6486527	6487502
446	6493349	6494322	6495296	6496269	6497242
447	6503075	6504047	6505018	6505989	6506960
448	6512780	6513749	6514719	6515687	6516656
449	6522463	6523430	6524397	6525364	6526331
450	6532125	6533090	6534055	6535019	6535984
451	6541765	6542728	6543691	6544653	6545616
452	6551384	6552345	6553306	6554266	6555226
453	6560982	6561941	6562899	6563857	6564815
454	6570559	6571515	6572471	6573427	6574383
455	6580114	6581068	6582023	6582976	6583930
456	6589648	6590601	6591553	6592505	6593456
457	6599162	6600112	6601062	6602012	6602962
458	6608655	6609603	6610551	6611499	6612446
459	6618127	6619073	6620019	6620954	6621910
460	6627578	6628522	6629466	6630410	6631353
461	6637009	6637951	6638893	6639835	6640776
462	6646420	6647360	6648299	6649239	6650178
463	6655810	6656748	6657685	6658623	6659560
464	6665180	6666116	6667051	6667987	6668922
465	6674530	6675463	6676397	6677331	6678264
466	6683859	6684791	6685723	6686654	6687585
467	6693169	6694099	6695028	6695958	6696887
468	6702459	6703386	6704314	6705242	6706169
469	6711728	6712654	6713580	6714506	6715431
470	6720979	6721903	6722826	6723750	6724673
471	6730209	6731131	6732053	6732974	6733896
472	6739420	6740340	6741260	6742179	6743099
473	6748611	6749529	6750447	6751365	6752283
474	6757783	6758700	6759615	6760531	6761447
475	6766936	6767850	6768764	6769678	6770592
476	6776069	6776982	6777894	6778806	6779718
477	6785184	6786094	6787004	6787914	6788824
478	6794279	6795187	6796095	6797004	6797912
479	6803355	6804262	6805168	6806074	6806980
480	6812412	6813317	6814222	6815126	6816030
481	6821451	6822354	6823256	6824159	6825061
482	6830470	6831371	6832272	6833173	6834073
483	6839471	6840370	6841269	6842168	6843066
484	6848454	6849351	6850248	6851145	6852041
485	6857417	6858313	6859208	6860103	6860998
486	6866363	6867256	6868149	6869043	6869936

Natural. Numbers	5	6	7	8	9
443	6468936	6469915	6470894	6471873	6472851
444	6478718	6479695	6480671	6481648	6482624
445	6488477	6489452	6490426	6491401	6492375
446	6498215	6499187	6500160	6501132	6502104
447	6507930	6508901	6509871	6510841	6511811
448	6517624	6518593	6519561	6520528	6521496
449	6527297	6528263	6529229	6530195	6531160
450	6536948	6537912	6538876	6539839	6540802
451	6546578	6547539	6548501	6549462	6550423
452	6556186	6557145	6558105	6559064	6560023
453	6565773	6566730	6567688	6568645	6569602
454	6575339	6576294	6577250	6578205	6579159
455	6584884	6585837	6586790	6587743	6588696
456	6594408	6595359	6596310	6597261	6598212
457	6603911	6604860	6605809	6606758	6607706
458	6613393	6614340	6615287	6616234	6617181
459	6622855	6623800	6624745	6625690	6626634
460	6632296	6633239	6634182	6635125	6636067
461	6641717	6642658	6643599	6644539	6645480
462	6651117	6652056	6652995	6653933	6654872
463	6660497	6661434	6662371	6663307	6664244
464	6669857	6670792	6671727	6672661	6673595
465	6679197	6680130	6681062	6681995	6682927
466	6688516	6689447	6690378	6691308	6692239
467	6697816	6698745	6699674	6700602	6701530
468	6707096	6708023	6708950	6709875	6710802
469	6716356	6717281	6718206	6719130	6720054
470	6725596	6726519	6727442	6728365	6729287
471	6734817	6735738	6736659	6737574	6738490
472	6744018	6744937	6745856	6746775	6747693
473	6753200	6754117	6755034	6755951	6756867
474	6762362	6763277	6764192	6765107	6766022
475	6771505	6772418	6773332	6774244	6775157
476	6780629	6781540	6782452	6783362	6784273
477	6789734	6790643	6791552	6792461	6793370
478	6798819	6799727	6800634	6801541	6802448
479	6807886	6808792	6809697	6810602	6811507
480	6816934	6817838	6818741	6819645	6820548
481	6825963	6826865	6827766	6828668	6829569
482	6834973	6835873	6836773	6837673	6838572
483	6843965	6844863	6845761	6846659	6847556
484	6852938	6853834	6854730	6855626	6856522
485	6861892	6862787	6863681	6864575	6865469
486	6870828	6871721	6872613	6873506	6874398

Natural Numbers.	0	1	2	3	4
487	6875290	6876181	6877073	6877964	6878855
488	6884198	6885088	6885978	6886867	6887757
489	6893099	6893977	6894864	6895752	6896640
490	6901961	6902847	6903733	6904616	6905505
491	6910815	6911699	6912584	6913468	6914352
492	6919651	6920534	6921416	6922298	6923180
493	6928469	6929350	6930231	6931111	6931991
494	6937269	6938148	6939027	6939906	6940785
495	6946052	6946929	6947806	6948683	6949560
496	6954817	6955692	6956568	6957443	6958318
497	6963564	6964438	6965311	6966185	6967058
498	6972293	6973165	6974037	6974909	6975780
499	6981005	6981876	6982746	6983616	6984485
500	6989700	6990569	6991437	6992305	6993173
501	6998377	6999244	7000111	7000977	7001843
502	7007037	7007902	7008767	7009632	7010496
503	7015680	7016543	7017406	7018269	7019132
504	7024305	7025167	7026028	7026890	7027751
505	7032914	7033774	7034633	7035493	7036352
506	7041505	7042363	7043221	7044079	7044937
507	7050080	7050936	7051792	7052649	7053505
508	7058637	7059492	7060347	7061201	7062055
509	7067178	7068031	7068884	7069737	7070589
510	7075702	7076553	7077405	7078256	7079107
511	7084206	7085059	7085908	7086758	7087607
512	7092700	7093548	7094396	7095244	7096091
513	7101174	7102020	7102866	7103713	7104559
514	7109631	7110476	7111321	7112165	7113010
515	7118072	7118915	7119759	7120601	7121444
516	7126497	7127339	7128180	7129021	7129862
517	7134905	7135745	7136585	7137425	7138264
518	7143298	7144136	7144974	7145812	7146650
519	7151674	7152510	7153347	7154183	7155019
520	7160033	7160869	7161703	7162538	7163373
521	7168377	7169211	7170044	7170877	7171710
522	7176705	7177537	7178369	7179200	7180032
523	7185017	7185847	7186677	7187507	7188337
524	7193313	7194142	7194970	7195799	7196627
525	7201593	7202420	7203247	7204074	7204901
526	7209857	7210683	7211508	7212334	7213159
527	7218106	7218930	7219754	7220578	7221401
528	7226339	7227162	7227984	7228806	7229628
529	7234557	7235378	7236198	7237019	7237839
530	7242759	7243578	7244397	7245216	7246035

Logarithms (to 5309.)

381

Natural Numbers.	5	6	7	8	9
487	6879746	6880637	6881528	6882418	6883308
488	6888646	6889535	6890423	6891312	6892200
489	6897527	6898414	6899301	6900188	6901074
490	6906390	6907275	6908161	6909046	6909930
491	6915235	6916119	6917002	6917885	6918768
492	6924062	6924944	6925826	6926707	6927588
493	6932872	6933752	6934631	6935511	6936390
494	6941663	6942541	6943419	6944297	6945174
495	6950437	6951313	6952189	6953065	6953941
496	6959193	6960067	6960942	6961816	6962690
497	6967931	6968804	6969676	6970549	6971421
498	6976652	6977523	6978394	6979264	6980135
499	6985355	6986224	6987093	6987963	6988831
500	6994041	6994908	6995776	6996643	6997510
501	7002709	7003575	7004441	7005307	7006172
502	7011361	7012225	7013089	7013953	7014816
503	7019995	7020857	7021719	7022582	7023444
504	7028612	7029472	7030333	7031193	7032054
505	7037212	7038071	7038929	7039788	7040647
506	7045793	7046652	7047509	7048366	7049223
507	7054360	7055216	7056072	7056927	7057782
508	7062910	7063764	7064617	7065471	7066324
509	7071442	7072294	7073146	7073998	7074850
510	7079957	7080808	7081659	7082509	7083359
511	7088456	7089305	7090154	7091003	7091851
512	7096939	7097786	7098633	7099480	7100327
513	7105404	7106250	7107096	7107941	7108786
514	7113854	7114698	7115542	7116385	7117229
515	7122287	7123129	7123971	7124813	7125655
516	7130703	7131544	7132385	7133225	7134065
517	7139104	7139943	7140782	7141620	7142459
518	7147488	7148325	7149162	7150000	7150837
519	7155856	7156691	7157527	7158363	7159198
520	7164207	7165042	7165876	7166710	7167544
521	7172543	7173376	7174208	7175041	7175873
522	7180863	7181694	7182525	7183356	7184186
523	7189167	7189996	7190826	7191655	7192484
524	7197455	7198283	7199111	7199938	7200766
525	7205727	7206554	7207380	7208206	7209032
526	7213984	7214809	7215633	7216458	7217282
527	7222225	7223048	7223871	7224694	7225517
528	7230450	7231272	7232093	7232914	7233736
529	7238660	7239480	7240300	7241120	7241939
530	7246854	7247672	7248491	7249309	7250127

Natural Numbers.	0	1	2	3	4
531	7250945	7251763	7252581	7253398	7254215
532	7259116	7259933	7260749	7261565	7262380
533	7267272	7268087	7268901	7269716	7270531
534	7275413	7276226	7277039	7277852	7278664
535	7283538	7284349	7285161	7285972	7286784
536	7291648	7292458	7293268	7294078	7294888
537	7299743	7300551	7301360	7302168	7302977
138	7307823	7308630	7309437	7310244	7311051
539	7315888	7316693	7317499	7318304	7319109
540	7323938	7324749	7325546	7326350	7327153
541	3331973	7332775	7333578	7334380	7335182
542	7339993	7340794	7341595	7342396	7343197
543	7347998	7348798	7349598	7350397	7351196
544	7355989	7356787	7357585	7358383	7359181
545	7363965	7364762	7365558	7366355	7367151
546	7371926	7372722	7373517	7374312	7375107
547	7379873	7380667	7381461	7382254	7383048
548	7387806	7388598	7389390	7390182	7390974
549	7395723	7396514	7397305	7398096	7398886
550	7403627	7404416	7405206	7405995	7406784
551	7411516	7412304	7413092	7413880	7414668
552	7419391	7420177	7420964	7421750	7422537
553	7427251	7428037	7428822	7429607	7430392
554	7435098	7435881	7436665	7437449	7438232
555	7442930	7443712	7444495	7445277	7446059
556	7450748	7451529	7452310	7453091	7453871
557	7458552	7459332	7460111	7460890	7461670
558	7466342	7467120	7467898	7468676	7469454
559	7474118	7474895	7475672	7476448	7477225
560	7481880	7482656	7483431	7484206	7484981
561	7489629	7490403	7491177	7491950	7492724
562	7497363	7498136	7498908	7499681	7500453
563	7505087	7505855	7506626	7507398	7508168
564	7512791	7513561	7514331	7515100	7515870
565	7520484	7521253	7522022	7522790	7523558
566	7528164	7528932	7529699	7530466	7531232
567	7535831	7536596	7537362	7538128	7538893
568	7543483	7544248	7545012	7545777	7546541
569	7551123	7551886	7552649	7553412	7554178
570	7558749	7559510	7560279	7561034	7561795
571	7566361	7567122	7567882	7568642	7569402
572	7573960	7574719	7575479	7576237	7576996
573	7581546	7582304	7583062	7583819	7584577
574	7589119	7589875	7590632	7591388	7592144

Natural Numbers.	5	6	7	8	9
531	7255033	7255850	7256667	7257483	7258300
532	7263196	7264012	7264827	7265642	7266457
533	7271344	7272158	7272972	7273786	7274599
534	7279477	7280290	7281101	7281914	7282726
535	7287595	7288406	7289216	7290027	7290838
536	7295697	7296506	7297316	7298125	7298934
537	7303785	7304593	7305400	7306208	7307015
538	7311857	7312663	7313470	7314276	7315082
539	7319914	7320719	7321524	7322329	7323133
540	7327957	7328760	7329564	7330367	7331170
541	7335985	7336787	7337588	7338390	7339191
542	7343997	7344798	7345598	7346398	7347198
543	7351995	7352794	7353593	7354392	7355191
544	7359979	7360776	7361574	7362371	7363168
545	7367948	7368744	7369540	7370335	7371131
546	7375902	7376696	7377491	7378285	7379076
547	7383841	7384634	7385427	7386220	7387013
548	7391766	7392558	7393350	7394141	7394932
549	7399677	7400467	7401257	7402047	7402837
550	7407571	7408362	7409151	7409939	7410728
551	7415455	7416243	7417030	7417817	7418604
552	7423323	7424109	7424895	7425680	7426466
553	7431176	7431961	7432745	7433530	7434314
554	7439015	7439799	7440582	7441365	7442147
555	7446841	7447622	7448404	7449187	7449967
556	7454652	7455432	7456212	7456992	7457772
557	7462449	7463228	7464006	7464785	7465564
558	7470232	7471009	7471787	7472564	7473341
559	7478001	7478777	7479553	7480329	7481105
560	7485756	7486531	7487306	7488080	7488854
561	7493498	7494271	7495044	7495817	7496590
562	7501225	7501997	7502769	7503541	7504312
563	7508939	7509710	7510480	7511251	7512021
564	7516639	7517409	7518178	7518947	7519716
565	7524326	7525094	7525862	7526629	7527397
566	7531999	7532766	7533532	7534298	7535065
567	7539659	7540424	7541189	7541954	7542719
568	7547305	7548069	7548832	7549596	7550359
569	7554937	7555700	7556462	7557224	7557987
570	7562556	7563318	7564079	7564840	7565600
571	7570162	7570922	7571682	7572441	7573201
572	7577755	7578513	7579272	7580030	7580788
573	7585334	7586091	7586848	7587605	7588362
574	7592900	7593656	7594412	7595168	7595923

Natural Numbers.	0	1	2	3	4
575	7596678	7597434	7598189	7598944	7599699
576	7604225	7604979	7605733	7609486	7607240
577	7611758	7612511	7613263	7614016	7614768
578	7619278	7620030	7620781	7621532	7622283
579	7626786	7627536	7628286	7629035	7629785
580	7634280	7635029	7635777	7636526	7637274
581	7641761	7642509	7643256	7644003	7644750
582	7649230	7649976	7650722	7651468	7652214
583	7656616	7657430	7658175	7658920	7659664
584	7664128	7664872	7665616	7666359	7667102
585	7671559	7672301	7673043	7673785	7674527
586	7678976	7679717	7680458	7681199	7681940
587	7686381	7687121	7687860	7688600	7689339
588	7693773	7694512	7695250	7695988	7696727
589	7701153	7701890	7702627	7703364	7704101
590	7708520	7709256	7709992	7710728	7711463
591	7715875	7716610	7717344	7718079	7718813
592	7723217	7723951	7724684	7725417	7726150
593	7730547	7731279	7732011	7732743	7733475
594	7737864	7738596	7739326	7740057	7740788
595	7745170	7745899	7746629	7747359	7748088
596	7752463	7753191	7753920	7754648	7755376
597	7759743	7760471	7761198	7761925	7762652
598	7767012	7767738	7768464	7769190	7769916
599	7774268	7774993	7775718	7776443	7777167
600	7781513	7782236	7782960	7783683	7784407
601	7788745	7789467	7790190	7790912	7791634
602	7795965	7796686	7797408	7798129	7798850
603	7803173	7803893	7804613	7805333	7806053
604	7810369	7811088	7811807	7812526	7813245
605	7817554	7818272	7818989	7819707	7820424
606	7824726	7825443	7826159	7826876	7827592
607	7831887	7832602	7833318	7834033	7834748
608	7839036	7839750	7840464	7841178	7841892
609	7846173	7846886	7847599	7848312	7849024
610	7853298	7854010	7854722	7855434	7856145
611	7860412	7861123	7861833	7862544	7863254
612	7867514	7868224	7868933	7869643	7870352
613	7874605	7875313	7876021	7876730	7877438
614	7881684	7882391	7883098	7883805	7884512
615	7888751	7889457	7890163	7890869	7891575
616	7895807	7896512	7897217	7897922	7898626
617	7902852	7903555	7904259	7904963	7905666
618	7909885	7910587	7911290	7911992	7912695

Logarithms (to 6189.)

385

Natural Numbers.	5	6	7	8	9
575	7600453	7601208	7601962	7602717	7603471
576	7607993	7608746	7609500	7610253	7611005
577	7615520	7616272	7617024	7617775	7618527
578	7623034	7623784	7624535	7625285	7626035
579	7630534	7631284	7632033	7632782	7633531
580	7638022	7638770	7639518	7640266	7641014
581	7645497	7646244	7646991	7647737	7648484
582	7652959	7653705	7654450	7655195	7655941
583	7660409	7661153	7661897	7662641	7663385
584	7667845	7668588	7669331	7670074	7670816
585	7675269	7676011	7676752	7677494	7678235
586	7682680	7683421	7684161	7684901	7685641
587	7690079	7690818	7691557	7692296	7693035
588	7697465	7698203	7698940	7699678	7700416
589	7704838	7705575	7706311	7707048	7707784
590	7712199	7712934	7713670	7714405	7715140
591	7719547	7720282	7721016	7721750	7722483
592	7726884	7727616	7728349	7729082	7729814
593	7734207	7734939	7735670	7736402	7737133
594	7741519	7742249	7742979	7743710	7744440
595	7748818	7749547	7750276	7751005	7751734
596	7756104	7756832	7757560	7758288	7759016
597	7763379	7764106	7764833	7765559	7766286
598	7770642	7771367	7772093	7772818	7773543
599	7777892	7778616	7779340	7780065	7780789
600	7785130	7785853	7786576	7787299	7788022
601	7792356	7793078	7793800	7794522	7795243
602	7799571	7800291	7801012	7801732	7802453
603	7806773	7807492	7808212	7808931	7809650
604	7813963	7814681	7815400	7816118	7816836
605	7821141	7821859	7822576	7823293	7824010
606	7828308	7829024	7829740	7830456	7831171
607	7835463	7836178	7836892	7837607	7838321
608	7842606	7843319	7844033	7844746	7845460
609	7849737	7850450	7851162	7851874	7852586
610	7856857	7857568	7858279	7858990	7859701
611	7863965	7864675	7865385	7866095	7866805
612	7871061	7871770	7872479	7873188	7873896
613	7878146	7878853	7879561	7880269	7880976
614	7885219	7885926	7886632	7887339	7888045
615	7892281	7892986	7893691	7894397	7895102
616	7899331	7900035	7900739	7901444	7902148
617	7906370	7907073	7907776	7908479	7909182
618	7913397	7914099	7914801	7915503	7916205

Natural Numbers.	0	1	2	3	4
619	7916906	7917608	7918309	7919011	7919712
620	7923917	7924617	7925318	7926018	7926715
621	7930916	7931615	7932314	7933014	7933712
622	7937904	7938602	7939300	7939998	7940696
623	7944880	7945578	7946274	7946971	7947668
624	7951846	7952542	7953238	7953933	7954629
625	7958800	7959495	7960190	7960884	7961578
626	7965743	7966437	7967131	7967824	7968517
627	7972675	7973368	7974060	7974753	7975445
628	7979596	7980288	7980979	7981671	7982362
629	7986506	7987197	7987887	7988577	7989267
630	7993405	7994097	7994784	7995473	7996162
631	8000294	8000982	8001670	8002358	8003046
632	8007171	8007858	8008545	8009232	8009919
633	8014037	8014723	8015409	8016095	8016781
634	8020893	8021578	8022262	8022947	8023632
635	8027737	8028421	8029105	8029789	8030472
636	8034571	8035254	8035937	8036619	8037302
637	8041394	8042076	8042758	8043439	8044121
638	8048207	8048887	8049568	8050248	8050929
639	8055009	8055688	8056368	8057047	8057726
640	8061800	8062478	8063157	8063835	8064513
641	8068580	8069258	8069935	8070612	8071290
642	8075350	8076027	8076703	8077379	8078055
643	8082110	8082785	8083460	8084136	8084811
644	8088859	8089533	8090207	8090881	8091555
645	8095597	8096270	8096944	8097617	8098290
646	8102325	8102997	8103670	8104342	8105013
647	8109043	8109714	8110385	8111056	8111727
648	8115750	8116420	8117090	8117760	8118430
649	8122447	8123116	8123785	8124454	8125123
650	8129134	8129802	8130470	8131138	8131805
651	8135810	8136477	8137144	8137811	8138478
652	8142476	8143142	8143808	8144474	8145140
653	8149132	8149797	8150462	8151127	8151791
654	8155777	8156441	8157105	8157769	8158433
655	8162413	8163076	8163739	8164402	8165064
656	8169038	8169700	8170362	8171024	8171686
657	8175654	8176315	8176976	8177636	8178297
658	8182259	8182919	8183579	8184239	8184898
659	8188854	8189513	8190172	8190831	8191489
660	8195439	8196097	8196755	8197413	8198071
661	8202015	8202672	8203328	8203987	8204642
662	8208580	8209236	8209892	8210548	8211203

Natural Numbers.	5	6	7	8	9
619	7920413	7921114	7921815	7922516	7923216
620	7927418	7928118	7928817	7929517	7930217
621	7934411	7935110	7935809	7936507	7937206
622	7941394	7942091	7942789	7943486	7944185
623	7948365	7949061	7949757	7950454	7951150
624	7955324	7956020	7956715	7957410	7958105
625	7962273	7962967	7963662	7964356	7965050
626	7969211	7969904	7970597	7971290	7971983
627	7976137	7976829	7977521	7978213	7978905
628	7983053	7983744	7984435	7985125	7985816
629	7989957	7990647	7991337	7992027	7992716
630	7996851	7997540	7998228	7998917	7999605
631	8003734	8004421	8005109	8005796	8006484
632	8010605	8011292	8011978	8012665	8013351
633	8017466	8018152	8018837	8019522	8020208
634	8024316	8025001	8025685	8026369	8027053
635	8031156	8031839	8032522	8033205	8033888
636	8037984	8038666	8039348	8040031	8040712
637	8044802	8045483	8046164	8046845	8047526
638	8051609	8052289	8052969	8053649	8054329
639	8058405	8059085	8059763	8060442	8061121
640	8065191	8065869	8066547	8067225	8067903
641	8071967	8072643	8073320	8073997	8074674
642	8078731	8079407	8080083	8080759	8081434
643	8085485	8086160	8086835	8087510	8088184
644	8092229	8092903	8093577	8094250	8094924
645	8098962	8099635	8100308	8100980	8101653
646	8105685	8106357	8107029	8107700	8108371
647	8112398	8113068	8113739	8114409	8115080
648	8119100	8119769	8120439	8121108	8121778
649	8125792	8126460	8127129	8127797	8128465
650	8132473	8133141	8133808	8134475	8135143
651	8139144	8139811	8140477	8141144	8141810
652	8145805	8146471	8147136	8147801	8148467
653	8152456	8153120	8153785	8154449	8155113
654	8159096	8159760	8160423	8161087	8161750
655	8165727	8166389	8167052	8167714	8168376
656	8172347	8173009	8173670	8174331	8174993
657	8178958	8179618	8180278	8180939	8181599
658	8185558	8186217	8186877	8187536	8188195
659	8192146	8192806	8193455	8194123	8194781
660	8198728	8199386	8200043	8200700	8201358
661	8205298	8205955	8206611	8207268	8207924
662	8211859	8212514	8213170	8213825	8214480

Natural Numbers.	0	1	2	3	4
663	8215135	8215790	8216445	8217100	8217755
664	8221681	8222335	8222989	8223643	8224296
665	8228216	8228869	8229522	8230175	8230828
666	8234742	8235394	8236046	8236698	8237350
667	8241258	8241909	8242560	8243211	8243862
668	8247765	8248415	8249065	8249715	8250364
669	8254261	8254910	8255559	8256208	8256857
670	8260748	8261396	8262044	8262692	8263340
671	8267225	8267872	8268519	8269166	8269813
672	8273693	8274339	8274985	8275631	8276277
673	8280151	8280796	8281441	8282086	8282731
674	8286599	8287243	8287887	8288532	8289176
675	8293038	8293681	8294324	8294967	8295611
676	8299467	8300109	8300752	8301394	8302036
677	8305887	8306528	8307169	8307811	8308452
678	8312297	8312937	8313578	8314218	8314858
679	8318698	8319337	8319977	8320616	8321255
680	8325089	8325728	8326366	8327005	8327643
681	8331471	8332109	8332746	8333384	8334021
682	8337844	8338480	8339117	8339754	8340390
683	8344207	8344843	8345479	8346114	8346750
684	8350561	8351196	8351831	8352465	8353100
685	8356906	8357540	8358174	8358807	8359441
686	8363241	8363874	8364507	8365140	8365773
687	8369567	8370199	8370832	8371463	8372095
688	8375884	8376516	8377147	8377778	8378409
689	8382192	8382822	8383453	8384083	8384713
690	8388491	8389120	8389750	8390379	8391008
691	8394780	8395409	8396037	8396666	8397294
692	8401061	8401688	8402316	8402943	8403571
693	8407332	8407959	8408586	8409212	8409838
694	8413595	8414220	8414846	8415472	8416097
695	8419848	8420473	8421098	8421722	8422347
696	8426092	8426716	8427340	8427964	8428588
697	8432328	8432951	8433574	8434197	8434819
698	8438554	8439176	8439798	8440420	8441042
699	8444772	8445393	8446014	8446635	8447256
700	8450980	8451601	8452221	8452841	8453461
701	8457180	8457800	8458419	8459038	8459658
702	8463371	8463990	8464608	8465227	8465845
703	8469553	8470171	8470789	8471406	8472024
704	8475727	8476343	8476960	8477577	8478193
705	8481891	8482507	8483123	8483739	8484355
706	8488047	8488662	8489277	8489892	8490507

Natural Numbers.	5	6	7	8	9
663	8218409	8219064	8219718	8220372	8221027
664	8224950	8225603	8226257	8226910	8227563
665	8231481	8232133	8232786	8233438	8234090
666	8238032	8238683	8239335	8239986	8240637
667	8244513	8245163	8245814	8246464	8247114
668	8251014	8251664	8252313	8252963	8253612
669	8257506	8258154	8258803	8259451	8260100
670	8263988	8264635	8265283	8265931	8266578
671	8270460	8271107	8271753	8272400	8273046
672	8276923	8277569	8278214	8278860	8279505
673	8283376	8284021	8284665	8285310	8285955
674	8289820	8290463	8291107	8291751	8292394
675	8296254	8296896	8297539	8298182	8298824
676	8302678	8303320	8303962	8304603	8305245
677	8309093	8309734	8310375	8311016	8311656
678	8315499	8316139	8316778	8317418	8318058
679	8321895	8322534	8323173	8323812	8324450
680	8328281	8328919	8329558	8330195	8330833
681	8334659	8335296	8335933	8336570	8337207
682	8341027	8341663	8342299	8342937	8343571
683	8347385	8348021	8348656	8349291	8349926
684	8353735	8354369	8355003	8355638	8356272
685	8360075	8360708	8361341	8361975	8362608
686	8366405	8367038	8367670	8368303	8368935
687	8372727	8373359	8373990	8374622	8375253
688	8379039	8379670	8380301	8380931	8381562
689	8385343	8385973	8386602	8387232	8387861
690	8391637	8392266	8392895	8393523	8394152
691	8397922	8398550	8399178	8399806	8400433
692	8404198	8404825	8405452	8406079	8406706
693	8410465	8411091	8411717	8412343	8412969
694	8416722	8417348	8417973	8418598	8419223
695	8422971	8423596	8424220	8424844	8425468
696	8429211	8429835	8430458	8431081	8431705
697	8435442	8436065	8436687	8437310	8437932
698	8441664	8442286	8442907	8443529	8444150
699	8447877	8448498	8449119	8449739	8450360
700	8454081	8454701	8455321	8455941	8456561
701	8460277	8460896	8461515	8462134	8462752
702	8466463	8467081	8467700	8468318	8468935
703	8472641	8473258	8473876	8474493	8475110
704	8478810	8479426	8480043	8480659	8481275
705	8484970	8485586	8486201	8486817	8487432
706	8491122	8491736	8492351	8492965	8493580

Natural Numbers.	0	1	2	3	4
707	8494194	8494808	8495423	8496037	8496651
708	8500333	8500946	8501559	8502172	8502786
709	8506462	8507075	8507687	8508300	8508912
710	8512583	8513195	8513807	8514418	8515030
711	8518696	8519307	8519917	8520528	8521139
712	8524800	8525410	8526020	8526629	8527239
713	8530895	8531504	8532113	8532722	8533331
714	8536982	8537590	8538198	8538806	8539414
715	8543060	8543668	8544275	8544882	8545489
716	8549130	8549737	8550343	8550949	8551556
717	8555192	8555797	8556403	8557008	8557614
718	8561241	8561849	8562454	8563059	8563663
719	8567289	8567893	8568497	8569101	8569704
720	8573325	8573928	8574531	8575134	8575737
721	8579353	8579955	8580557	8581159	8581761
722	8585372	8585973	8586575	8587176	8587777
723	8591383	8591984	8592584	8593185	8593785
724	8597386	8597985	8598585	8599185	8599784
725	8603380	8603979	8604578	8605177	8605776
726	8609366	8609964	8610562	8611160	8611758
727	8615344	8615941	8616539	8617136	8617733
728	8621314	8621910	8622507	8623103	8623699
729	8627275	8627871	8628467	8629062	8629658
730	8633229	8633823	8634418	8635013	8635608
731	8639174	8639768	8640362	8640956	8641550
732	8645111	8645704	8646297	8646890	8647483
733	8651040	8651632	8652225	8652817	8653409
734	8656961	8657552	8658144	8658735	8659327
735	8662873	8663464	8664055	8664646	8665236
736	8668778	8669368	8669958	8670548	8671138
737	8674675	8675264	8675853	8676442	8677031
738	8680564	8681152	8681740	8682329	8682917
739	8686444	8687032	8687620	8688207	8688794
740	8692317	8692904	8693491	8694077	8694664
741	8698182	8698768	8699354	8699940	8700526
742	8704039	8704624	8705209	8705795	8706380
743	8709888	8710473	8711057	8711641	8712226
744	8715729	8716313	8716897	8717480	8718064
745	8721563	8722146	8722728	8723311	8723894
746	8727388	8727970	8728552	8729134	8729716
747	8733206	8733788	8734369	8734950	8735531
748	8739016	8739597	8740177	8740757	8741338
749	8744818	8745398	8745978	8746557	8747137
750	8750613	8751192	8751771	8752349	8752928

Natural Numbers.	5	6	7	8	9
707	8497264	8497878	8498492	8499106	8499719
708	8503399	8504011	8504624	8505237	8505850
709	8509524	8510136	8510748	8511360	8511972
710	8515641	8516252	8516863	8517474	8518085
711	8521749	8522359	8522970	8523580	8524190
712	8527849	8528458	8529068	8529677	8530286
713	8533940	8534548	8535157	8535765	8536374
714	8540022	8540630	8541238	8541845	8542453
715	8546096	8546703	8547310	8547917	8548524
716	8552162	8552768	8553374	8553980	8554586
717	8558219	8558824	8559429	8560035	8560640
718	8564268	8564872	8565476	8566081	8566685
719	8570308	8570912	8571515	8572118	8572722
720	8576340	8576943	8577545	8578148	8578750
721	8582363	8582965	8583567	8584169	8584770
722	8588379	8588980	8589581	8590181	8590782
723	8594385	8594986	8595586	8596186	8596786
724	8600384	8600983	8601583	8602182	8602781
725	8606374	8606973	8607571	8608170	8608768
726	8612356	8612954	8613552	8614149	8614747
727	8618330	8618927	8619524	8620120	8620717
728	8624296	8624892	8625488	8626084	8626679
729	8630253	8630848	8631443	8632039	8632634
730	8636202	8636797	8637391	8637985	8638580
731	8642143	8642737	8643331	8643924	8644517
732	8648076	8648669	8649262	8649855	8650447
733	8654001	8654593	8655185	8655777	8656369
734	8659918	8660509	8661100	8661691	8662282
735	8665827	8666417	8667008	8667598	8668188
736	8671728	8672317	8672907	8673496	8674086
737	8677620	8678209	8678798	8679387	8679975
738	8683505	8684093	8684681	8685269	8685857
739	8689382	8689969	8690556	8691143	8691730
740	8695251	8695837	8696423	8697010	8697596
741	8701112	8701697	8702283	8702868	8703454
742	8706965	8707549	8708134	8708719	8709304
743	8712810	8713394	8713978	8714562	8715146
744	8718647	8719230	8719814	8720397	8720980
745	8724476	8725059	8725641	8726224	8726806
746	8730298	8730880	8731461	8732043	8732625
747	8736112	8736693	8737274	8737855	8738435
748	8741918	8742498	8743078	8743658	8744238
749	8747716	8748296	8748875	8749454	8750034
750	8753507	8754086	8754664	8755243	8755821

Natural Numbers.	0	1	2	3	4
751	8756399	8756978	8757556	8758134	8758712
752	8762178	8762756	8763333	8763911	8764488
753	8767950	8768526	8769103	8769680	8770256
754	8773713	8774289	8774865	8775441	8776017
755	8779469	8780045	8780620	8781195	8781770
756	8785218	8785792	8786367	8786941	8787515
757	8790959	8791532	8792106	8792680	8793253
758	8796692	8797265	8797838	8798411	8798983
759	8802418	8802990	8803562	8804134	8804706
760	8808136	8808707	8809279	8809850	8810421
761	8813847	8814417	8814988	8815558	8816129
762	8819550	8820120	8820689	8821259	8821829
763	8825245	8825815	8826384	8826953	8827522
764	8830934	8831502	8832070	8832639	8833207
765	8836614	8837182	8837750	8838317	8838885
766	8842288	8842855	8843421	8843988	8844555
767	8847954	8848520	8849086	8849652	8850218
768	8853612	8854178	8854743	8855308	8855874
769	8859263	8859828	8860393	8860957	8861522
770	8864907	8865471	8866035	8866599	8867163
771	8870544	8871107	8871670	8872233	8872796
772	8876173	8876736	8877298	8877860	8878423
773	8881795	8882357	8882918	8883480	8884042
774	8887410	8887971	8888532	8889093	8889653
775	8893017	8893577	8894138	8894698	8895258
776	8898617	8899177	8899736	8900296	8900855
777	8904210	8904765	8905328	8905887	8906445
778	8909796	8910354	8910912	8911470	8912028
779	8915375	8915932	8916489	8917047	8917604
780	8920946	8921503	8922059	8922616	8923173
781	8926510	8927066	8927622	8928178	8928734
782	8932068	8932623	8933178	8933733	8934288
783	8937618	8938172	8938727	8939281	8939836
784	8943161	8943715	8944268	8944822	8945376
785	8948697	8949250	8949803	8950356	8950909
786	8954225	8954778	8955330	8955883	8956435
787	8959747	8960299	8960851	8961403	8961954
788	8965262	8965813	8966364	8966915	8967466
789	8970770	8971320	8971871	8972421	8972971
790	8976271	8976821	8977370	8977920	8978469
791	8981765	8982314	8982863	8983412	8983960
792	8987252	8987800	8988348	8988897	8989445
793	8992732	8993279	8993827	8994375	8994922
794	8998205	8998752	8999299	8999846	9000392

Natural Numbers.	5	6	7	8	9
751	8759290	8759868	8760445	8761023	8761601
752	8755065	8765642	8766219	8766796	8767373
753	8770833	8771409	8771985	8772561	8773137
754	8776592	8777168	8777743	8778319	8778894
755	8782345	8782919	8783494	8784069	8784643
756	8788089	8788663	8789237	8789811	8790385
757	8793826	8794400	8794973	8795546	8796119
758	8799556	8800128	8800701	8801273	8801846
759	8805278	8805850	8806421	8806993	8807564
760	8810992	8811563	8812134	8812705	8813276
761	8816699	8817269	8817840	8818410	8818980
762	8822398	8822968	8823537	8824107	8824676
763	8828090	8828659	8829228	8829797	8830365
764	8833775	8834343	8834911	8835479	8836047
765	8839452	8840019	8840586	8841154	8841721
766	8845122	8845688	8846255	8846821	8847387
767	8850784	8851350	8851915	8852481	8853047
768	8856439	8857004	8857569	8858134	8858699
769	8866086	8866651	8867215	8867779	8868343
770	8867726	8868290	8868854	8869417	8869980
771	8873359	8873922	8874485	8875048	8875610
772	8878985	8879547	8880109	8880671	8881233
773	8884603	8885165	8885726	8886287	8886848
774	8890214	8890775	8891336	8891896	8892457
775	8895818	8896378	8896935	8897498	8898058
776	8901415	8901974	8902533	8903092	8903651
777	8907004	8907562	8908121	8908679	8909238
778	8912586	8913144	8913702	8914259	8914817
779	8918161	8918718	8919275	8919832	8920389
780	8923729	8924285	8924842	8925398	8925954
781	8929290	8929846	8930401	8930957	8931512
782	8934843	8935398	8935953	8936508	8937063
783	8940390	8940944	8941498	8942053	8942607
784	8945929	8946483	8947037	8947590	8948143
785	8951462	8952015	8952567	8953120	8953673
786	8956987	8957539	8958092	8958644	8959195
787	8962506	8963057	8963608	8964160	8964711
788	8968017	8968568	8969118	8969669	8970219
789	8973521	8974071	8974621	8975171	8975721
790	8979019	8979568	8980117	8980667	8981215
791	8984509	8985058	8985606	8986155	8986703
792	8989993	8990541	8991089	8991636	8992184
793	8995469	8996017	8996564	8997111	8997658
794	9000939	9001486	9002032	9002579	9003125

Natural Numbers.	0	1	2	3	4
795	9003671	9004218	9004764	9005310	9005856
796	9009131	9009676	9010222	9010767	9011313
797	9014583	9015128	9015673	9016218	9016762
798	9020029	9020573	9021117	9021661	9022205
799	9025468	9026011	9026555	9027098	9027641
800	9030900	9031443	9031985	9032528	9033071
801	9036325	9036867	9037409	9037951	9038493
802	9041744	9042285	9042827	9043368	9043909
803	9047155	9047696	9048237	9048778	9049318
804	9052560	9053101	9053641	9054181	9054721
805	9057960	9058498	9059038	9059577	9060116
806	9066351	9066889	9067428	9067967	9068505
807	9068735	9069273	9069812	9070350	9070887
808	9074114	9074651	9075188	9075726	9076263
809	9079485	9080022	9080559	9081095	9081632
810	9084850	9085386	9085922	9086458	9086994
811	9090209	9090744	9091279	9091815	9092350
812	9095560	9096095	9096630	9097165	9097699
813	9100905	9101440	9101974	9102508	9103042
814	9106244	9106778	9107311	9107844	9108378
815	9111576	9112109	9112642	9113174	9113707
816	9116902	9117434	9117966	9118498	9119030
817	9122220	9122752	9123284	9123815	9124346
818	9127533	9128064	9128595	9129126	9129656
819	9132839	9133369	9133899	9134430	9134960
820	9138139	9138668	9139198	9139727	9140257
821	9143432	9143961	9144489	9145018	9145547
822	9148718	9149246	9149775	9150303	9150831
823	9153998	9154526	9155054	9155581	9156109
824	9159272	9159799	9160326	9160853	9161380
825	9164539	9165066	9165592	9166118	9166645
826	9169800	9170326	9170852	9171378	9171903
827	9175055	9175580	9176105	9176630	9177155
828	9180303	9180828	9181352	9181877	9182401
829	9185545	9186069	9186593	9187117	9187640
830	9190781	9191304	9191827	9192350	9192873
831	9196010	9196533	9197055	9197578	9198100
832	9201233	9201755	9202277	9202799	9203321
833	9206450	9206971	9207493	9208014	9208535
834	9211661	9212181	9212702	9213222	9213743
835	9216865	9217385	9217905	9218425	9218945
836	9222063	9222582	9223102	9223621	9224140
837	9227255	9227773	9228292	9228811	9229330
838	9232440	9232958	9233477	9233995	9234513

Logarithms (to 8389.)

395

Natural Numbers.	5	6	7	8	9
795	9006402	9006948	9007494	9008039	9008585
796	9011858	9012403	9012984	9013448	9014038
797	9017307	9017851	9018396	9018940	9019485
798	9022749	9023293	9023837	9024381	9024924
799	9028185	9028728	9029271	9029814	9030357
800	9033613	9034156	9034698	9035241	9035783
801	9039035	9039577	9040119	9040661	9041202
802	9044450	9044992	9045533	9046073	9046615
803	9049859	9050399	9050940	9051480	9052020
804	9055261	9055800	9056340	9056880	9057419
805	9060655	9061195	9061734	9062274	9062812
806	9066044	9066582	9067121	9067659	9068197
807	9071425	9071963	9072501	9073038	9073576
808	9076800	9077337	9077874	9078411	9078948
809	9082169	9082705	9083241	9083778	9084314
810	9087530	9088066	9088602	9089137	9089673
811	9092885	9093420	9093955	9094490	9095025
812	9098234	9098768	9099303	9099837	9100371
813	9103576	9104109	9104643	9105177	9105710
814	9108911	9109444	9109977	9110510	9111043
815	9114240	9114772	9115305	9115837	9116369
816	9119562	9120094	9120626	9121157	9121689
817	9124878	9125409	9125940	9126471	9127002
818	9130187	9130717	9131248	9131778	9132309
819	9135490	9136019	9136549	9137079	9137609
820	9140786	9141315	9141844	9142373	9142903
821	9146076	9146604	9147133	9147661	9148190
822	9151359	9151887	9152415	9152943	9153471
823	9156636	9157163	9157691	9158218	9158745
824	9161907	9162433	9162960	9163487	9164013
825	9167171	9167697	9168223	9168749	9169275
826	9172429	9172954	9173479	9174005	9174530
827	9177680	9178205	9178730	9179254	9179779
828	9182925	9183449	9183973	9184497	9185021
829	9188164	9188687	9189211	9189734	9190258
830	9193396	9193919	9194442	9194965	9195488
831	9198623	9199145	9199667	9200189	9200711
832	9203842	9204364	9204886	9205407	9205929
833	9209056	9209577	9210098	9210619	9211140
834	9214263	9214784	9215304	9215824	9216345
835	9219465	9219984	9220504	9221024	9221543
836	9224659	9225179	9225698	9226217	9226736
837	9229848	9230367	9230885	9231404	9231922
838	9235031	9235549	9236066	9236584	9237102

Natural Numbers.	0	1	2	3	4
839	9237620	9238137	9238655	9239172	9239690
840	9242793	9243310	9243827	9244344	9244860
841	9247960	9248476	9248993	9249509	9250025
842	9253121	9253637	9254152	9254668	9255184
843	9258276	9258791	9259306	9259821	9260336
844	9263424	9263939	9264453	9264968	9265482
845	9268567	9269081	9269595	9270109	9270622
846	9273704	9274217	9274730	9275243	9275757
847	9278854	9279347	9279859	9280372	9280885
848	9283959	9284471	9284983	9285495	9286007
849	9289077	9289588	9290100	9290611	9291123
850	9294189	9294700	9295211	9295722	9296233
851	9299296	9299806	9300316	9300826	9301336
852	9304396	9304906	9305415	9305925	9306434
853	9309490	9309999	9310508	9311017	9311526
854	9314579	9315087	9315596	9316104	9316612
855	9319661	9320169	9320677	9321185	9321692
856	9324738	9325245	9325752	9326259	9326767
857	9329808	9330315	9330822	9331328	9331835
858	9334873	9335379	9335885	9336391	9336897
859	9339932	9340437	9340943	9341448	9341953
860	9344984	9345489	9345994	9346499	9347004
861	9350032	9350536	9351040	9351544	9352049
862	9355073	9355576	9356080	9356584	9357087
863	9360108	9360611	9361114	9361617	9362120
864	9365137	9365640	9366143	9366645	9367148
865	9370161	9370663	9371165	9371667	9372169
866	9375179	9375680	9376182	9376683	9377184
867	9380191	9380692	9381193	9381693	9382194
868	9385197	9385697	9386198	9386698	9397198
869	9390198	9390697	9391197	9391697	9392196
870	9395193	9395692	9396191	9396690	9397189
871	9400182	9400680	9401179	9401677	9402176
872	9405165	9405663	9406161	9406659	9407157
873	9410142	9410640	9411137	9411635	9412132
874	9415114	9415611	9416108	9416605	9417101
875	9420081	9420577	9421073	9421569	9422065
876	9425041	9425537	9426032	9426528	9427024
877	9429996	9430491	9430986	9431481	9431976
878	9434945	9435440	9435934	9436429	9436923
879	9439889	9440383	9440877	9441371	9441865
880	9444827	9445320	9445814	9446307	9446800
881	9449759	9450252	9450745	9451238	9451730
882	9454686	9455178	9455671	9456163	9456655

Natural Numbers.	5	6	7	8	9
839	9240208	9240724	9241246	9241759	9242276
840	9245377	9245894	9246410	9246927	9247444
841	9250541	9251057	9251573	9252089	9252605
842	9255699	9256215	9256730	9257245	9257761
843	9260851	9261366	9261880	9262395	9262910
844	9265995	9266511	9267025	9267539	9268053
845	9271136	9271650	9272163	9272677	9273190
846	9276270	9276783	9277296	9277808	9278321
847	9281397	9281909	9282422	9282934	9283446
848	9286518	9287030	9287542	9288054	9288565
849	9291634	9292145	9292656	9293167	9293678
850	9296743	9297254	9297764	9298275	9298785
851	9301847	9302357	9302866	9303376	9303886
852	9306944	9307453	9307963	9308472	9308981
853	9312035	9312544	9313053	9313561	9314070
854	9317121	9317629	9318137	9318645	9319153
855	9322200	9322708	9323215	9323723	9324230
856	9327274	9327781	9328288	9328795	9329301
857	9332341	9332848	9333354	9333860	9334367
858	9337403	9337909	9338415	9338920	9339426
859	9342459	9342964	9343469	9343974	9344479
860	9347509	9348013	9348518	9349022	9349527
861	9352553	9353057	9353561	9354065	9354569
862	9357591	9358095	9358598	9359101	9359605
863	9362623	9363126	9363629	9364132	9364635
864	9367650	9368152	9368655	9369157	9369659
865	9372671	9373172	9373674	9374176	9374677
866	9377686	9378187	9378688	9379189	9379690
867	9382695	9383195	9383696	9384196	9384697
868	9387698	9388198	9388698	9389198	9389698
869	9392696	9393195	9393695	9394194	9394693
870	9397688	9398187	9398685	9399184	9399683
871	9402674	9403172	9403670	9404169	9404667
872	9407654	9408152	9408650	9409147	9409645
873	9412629	9413126	9413623	9414120	9414617
874	9417598	9418095	9418591	9419088	9419584
875	9422561	9423058	9423554	9424049	9424545
876	9427519	9428015	9428510	9429005	9429501
877	9432471	9432966	9433461	9433956	9434450
878	9437418	9437912	9438406	9438900	9439395
879	9442358	9442852	9443346	9443840	9444333
880	9447294	9447787	9448280	9448773	9449266
881	9452223	9452716	9453208	9453701	9454193
882	9457147	9457639	9458131	9458623	9459115

Natural Numbers.	0	1	2	3	4
883	9459607	9460099	9460591	9461082	9461574
884	9464323	9465014	9465505	9465996	9466487
885	9469433	9469923	9470414	9470905	9471395
886	9474337	9474827	9475317	9475807	9476297
887	9479236	9479726	9480215	9480705	9481194
888	9484130	9484619	9485108	9485597	9486085
889	9489018	9489506	9489994	9490483	9490971
890	9493900	9494388	9494876	9495364	9495852
891	9498777	9499264	9499872	9500239	9500726
892	9503649	9504135	9504622	9505109	9505596
893	9508515	9509001	9509487	9509973	9510459
894	9513375	9513861	9514347	9514832	9515318
895	9518230	9518716	9519201	9519686	9520171
896	9523080	9523565	9524049	9524534	9525018
897	9527924	9528409	9528893	9529377	9529861
898	9532763	9533247	9533730	9534214	9534697
899	9537597	9538080	9538563	9539046	9539529
900	9542425	9542908	9543390	9543872	9544355
901	9547248	9547730	9548212	9548694	9549176
902	9552065	9552547	9553028	9553510	9553991
903	9556877	9557358	9557839	9558320	9558801
904	9561684	9562165	9562645	9563125	9563605
905	9566486	9566966	9567445	9567925	9568405
906	9571282	9571761	9572241	9572720	9573199
907	9576073	9576552	9577030	9577509	9577988
908	9580858	9581337	9581815	9582293	9582771
909	9585639	9586117	9586594	9587072	9587549
910	9590414	9590891	9591368	9591845	9592322
911	9595184	9595660	9596137	9596614	9597090
912	9599948	9600425	9600901	9601377	9601853
913	9604708	9605183	9605659	9606135	9606610
914	9609462	9609937	9610412	9610887	9611362
915	9614211	9614686	9615160	9615635	9616109
916	9618955	9619429	9619903	9620377	9620851
917	9623693	9624167	9624640	9625114	9625587
918	9628427	9628900	9629373	9629846	9630319
919	9633155	9633628	9634100	9634573	9635045
920	9637878	9638350	9638822	9639294	9639766
921	9642596	9643068	9643539	9644011	9644482
922	9647309	9647780	9648251	9648722	9649193
923	9652017	9652488	9652958	9653428	9653899
924	9656720	9657190	9657660	9658130	9658599
925	9661417	9661887	9662356	9662826	9663295
926	9666110	9666579	9667048	9667517	9667985

Natural Numbers.	5	6	7	8	9
883	9462066	9462557	9463048	9463540	9464031
884	9466978	9467469	9467960	9468451	9468942
885	9471886	9472376	9472866	9473357	9473847
886	9476787	9477277	9477767	9478257	9478747
887	9481684	9482173	9482662	9483151	9483641
888	9486574	9487063	9487552	9488040	9488529
889	9491460	9491948	9492436	9492924	9493412
890	9495330	9496827	9497314	9497802	9498290
891	9501213	9501701	9502188	9502675	9503162
892	9506082	9506569	9507055	9507542	9508028
893	9510946	9511432	9511918	9512404	9512889
894	9515803	9516289	9516774	9517260	9517745
895	9520656	9521141	9521626	9522111	9522595
896	9525503	9525987	9526472	9526956	9527440
897	9530345	9530828	9531312	9531796	9532280
898	9535181	9535664	9536147	9536631	9537114
899	9540012	9540494	9540977	9541460	9541943
900	9544837	9545319	9545802	9546284	9546766
901	9549657	9550139	9550621	9551102	9551584
902	9554472	9554953	9555434	9555915	9556397
903	9559282	9559762	9560243	9560723	9561204
904	9564086	9564566	9565046	9565526	9566006
905	9568885	9569364	9569844	9570323	9570803
906	9573678	9574157	9574636	9575115	9575594
907	9578466	9578945	9579423	9579902	9580380
908	9583249	9583727	9584205	9584683	9585161
909	9588027	9588505	9588982	9589459	9589937
910	9592799	9593276	9593754	9594230	9594707
911	9597567	9598043	9598520	9598996	9599472
912	9602329	9602805	9603280	9603756	9604232
913	9607086	9607561	9608036	9608511	9608987
914	9611837	9612312	9612787	9613261	9613736
915	9616583	9617058	9617532	9618006	9618481
916	9621325	9621799	9622272	9622746	9623220
917	9626061	9626534	9627007	9627481	9627954
918	9630792	9631264	9631737	9632210	9632683
919	9635517	9635990	9636462	9636934	9637406
920	9640238	9640710	9641181	9641653	9642125
921	9644953	9645425	9645896	9646367	9646838
922	9649664	9650134	9650605	9651076	9651546
923	9654369	9654839	9655309	9655780	9656250
924	9659069	9659539	9660009	9660478	9660948
925	9663764	9664233	9664703	9665172	9665641
926	9668454	9668923	9669392	9669860	9670329

Natural Numbers.	0	1	2	3	4
927	9670797	9671266	9671734	9672203	9672671
928	9675480	9675948	9676416	9676883	9677351
929	9680157	9680625	9681092	9681559	9682027
930	9684829	9685296	9685763	9686230	9686697
931	9689497	9689963	9690430	9690896	9691362
932	9694159	9694625	9695091	9695557	9696023
933	9698816	9699282	9699747	9700213	9700678
934	9703459	9703934	9704399	9704863	9705328
935	9708116	9708581	9709045	9709509	9709974
936	9712758	9713222	9713686	9714150	9714614
937	9717396	9717859	9718323	9718786	9719249
938	9722028	9722491	9722954	9723417	9723880
939	9726656	9727118	9727581	9728043	9728506
940	9731278	9731741	9732202	9732664	9733126
941	9735896	9736358	9736819	9737281	9737742
942	9740509	9740970	9741431	9741892	9742353
943	9745117	9745577	9746038	9746498	9746959
944	9749720	9750180	9750640	9751100	9751560
945	9754318	9754778	9755237	9755695	9756156
946	9758911	9759370	9759829	9760288	9760747
947	9763500	9763958	9764417	9764875	9765334
948	9768083	9768541	9768999	9769457	9769915
949	9772662	9773120	9773577	9774035	9774492
950	9777236	9777693	9778150	9778607	9779064
951	9781805	9782262	9782718	9783175	9783631
952	9786369	9786826	9787282	9787738	9788194
953	9790929	9791385	9791840	9792296	9792751
954	9795484	9795939	9796394	9796849	9797304
955	9800034	9800488	9800943	9801398	9801852
956	9804579	9805033	9805487	9805942	9806396
957	9809119	9809573	9810027	9810481	9810934
958	9813655	9814108	9814562	9815015	9815468
959	9818186	9818639	9819092	9819544	9819997
960	9822712	9823165	9823617	9824069	9824522
961	9827234	9827686	9828138	9828589	9829041
962	9831751	9832202	9832654	9833105	9833556
963	9836263	9836714	9837165	9837616	9838066
964	9840770	9841221	9841671	9842122	9842572
965	9845273	9845723	9846173	9846623	9847073
966	9849771	9850221	9850670	9851120	9851569
967	9854265	9854714	9855163	9855612	9856061
968	9858754	9859202	9859651	9860099	9860548
969	9863238	9863686	9864134	9864582	9865030
970	9867717	9868165	9868613	9869060	9869508

Logarithms (to 9709.)

401

Natural Numbers.	5	6	7	8	9
927	9673139	9673607	9674076	9674544	9675012
928	9677819	9678287	9678754	9679222	9679690
929	9682494	9682961	9683428	9683895	9684362
930	9687164	9687630	9688097	9688564	9689030
931	9691829	9692295	9692761	9693227	9693693
932	9696488	9696954	9697420	9697885	9698351
933	9701143	9701608	9702074	9702539	9703004
934	9705793	9706258	9706722	9707187	9707652
935	9710438	9710902	9711366	9711830	9712294
936	9715078	9715542	9716005	9716469	9716932
937	9719713	9720176	9720639	9721102	9721565
938	9724343	9724805	9725268	9725731	9726193
939	9728968	9729430	9729892	9730354	9730816
940	9733588	9734050	9734511	9734973	9735435
941	9738203	9738664	9739126	9739587	9740048
942	9742814	9743274	9743735	9744196	9744656
943	9747419	9747879	9748340	9748800	9749260
944	9752020	9752479	9752939	9753399	9753858
945	9756615	9757075	9757534	9757993	9758452
946	9761206	9761665	9762124	9762582	9763041
947	9765792	9766251	9766709	9767167	9767625
948	9770373	9770831	9771289	9771747	9772204
949	9774950	9775407	9775864	9776322	9776779
950	9779521	9779978	9780435	9780892	9781348
951	9784088	9784544	9785001	9785457	9785913
952	9788650	9789106	9789562	9790017	9790473
953	9793207	9793662	9794118	9794573	9795028
954	9797759	9798214	9798669	9799124	9799579
955	9802307	9802761	9803216	9803670	9804125
956	9806850	9807304	9807758	9808212	9808666
957	9811388	9811841	9812295	9812748	9813202
958	9815921	9816374	9816827	9817280	9817733
959	9820450	9820902	9821355	9821807	9822260
960	9824974	9825426	9825878	9826330	9826782
961	9829493	9830945	9830396	9830848	9831299
962	9834007	9834459	9834910	9835361	9835812
963	9838517	9838958	9839419	9839869	9840320
964	9843022	9843473	9843923	9844373	9844823
965	9847523	9847973	9848422	9848872	9849322
966	9852019	9852468	9852917	9853366	9853816
967	9856510	9856959	9857407	9857856	9858305
968	9860956	9861445	9861893	9862341	9862790
969	9865478	9865926	9866374	9866822	9867270
970	9869955	9870403	9870850	9871298	9871745

Natural Numbers.	0	1	2	3	4
971	9872192	9872640	9873087	9873534	9873981
972	9876663	9877109	9877556	9878003	9878449
973	9881128	9881575	9882021	9882467	9882913
974	9885590	9886035	9886481	9886927	9887373
975	9890046	9890492	9890937	9891382	9891828
976	9894498	9894943	9895388	9895833	9896278
977	9898946	9899390	9899835	9900279	9900723
978	9903389	9903833	9904277	9904721	9905164
979	9907827	9908270	9908714	9909158	9909601
980	9912261	9912704	9913147	9913590	9914033
981	9916697	9917133	9917575	9918018	9918461
982	9921115	9921557	9921999	9922441	9922884
983	9925535	9925977	9926419	9926860	9927302
984	9929951	9930392	9930834	9931275	9931716
985	9934362	9934803	9935244	9935685	9936126
986	9938769	9939210	9939650	9940090	9940531
987	9943172	9943612	9944051	9944491	9944931
988	9947569	9948009	9948448	9948888	9949327
989	9951963	9952402	9952841	9953280	9953719
990	9956352	9956791	9957229	9957668	9958106
991	9960737	9961175	9961613	9962051	9962489
992	9965117	9965554	9965992	9966430	9966868
993	9969492	9969930	9970367	9970804	9971242
994	9973864	9974301	9974738	9975174	9975611
995	9978231	9978667	9979104	9979540	9979976
996	9982593	9983029	9983465	9983901	9984337
997	9986952	9987387	9987823	9988258	9988694
998	9991305	9991740	9992176	9992611	9993046
999	9995655	9996090	9996524	9996959	9997393

Natural Numbers.	5	6	7	8	9
971	9874428	9874875	9875322	9875769	9876216
972	9878896	9879343	9879789	9880236	9880682
973	9883360	9883806	9884252	9884698	9885144
974	9887818	9888264	9888710	9889155	9889601
975	9892273	9892718	9893163	9893608	9894050
976	9896722	9897167	9897612	9898056	9898501
977	9901168	9901612	9902056	9902500	9902944
978	9905608	9906052	9906496	9906940	9907383
979	9910044	9910488	9910931	9911374	9911818
980	9914476	9914919	9915362	9915805	9916247
981	9918903	9919345	9919788	9920230	9920673
982	9923326	9923768	9924210	9924651	9925093
983	9927744	9928185	9928627	9929068	9929510
984	9932157	9932598	9933039	9933480	9933921
985	9936566	9937007	9937448	9937888	9938329
986	9940971	9941411	9941851	9942291	9942731
987	9945371	9945811	9946251	9946690	9947130
988	9949767	9950206	9950645	9951085	9951524
989	9954158	9954597	9955036	9955474	9955913
990	9958545	9958983	9959422	9959860	9960298
991	9962927	9963365	9963803	9964241	9964679
992	9967305	9967743	9968180	9968618	9969055
993	9971679	9972116	9972553	9972990	9973427
994	9976048	9976485	9976921	9977358	9977794
995	9980413	9980849	9981285	9981721	9982157
996	9984773	9985209	9985645	9986080	9986516
997	9989129	9989564	9990000	9990435	9990870
998	9993481	9993916	9994350	9994785	9995220
999	9997828	9998262	9998697	9999131	9999566

10000 it's Log. = 4.0000000

The End of the Table of the Logarithms.

THE CONTENTS.

THE Introduction — Page 1

CHAP. I.

Addition of Decimals — 6

CHAP. II.

Subtraction — — 12

CHAP. III.

Multiplication — — 16

CHAP. IV.

Division of Decimals — 28

Division contracted — — 34

CHAP. V.

Reduction — — — 46

Reduction of Vulgar Fractions to Decimals

47

Reduction by Logarithms — 48

Reduction of Mixed Numbers to Decimals 49

Reduction by various Tables of Money,

Weight, Measure, Time and Motion 52 to 54

Common Tables of Money, Weights, Measure,

and Time — — 55 to 58

Deci-

The CONTENTS.

Decimal Tables of Money, Weight, &c.	Page 59 to 69
The Description and Use of a Sett of New Tables for that Purpose	71 to 80
The Tables themselves —	81 to 125

C H A P. VI.

The Use of Decimals in all the Rules of Proportion, viz. The Golden Rule Direct	126 to 127
<i>Ditto</i> Inverse —	129
The Double Rule of Three —	131

C H A P. VII.

A New Method of Decimal Practice	136
A large General Table —	137
Its extraordinary Use —	138 to 141

C H A P. VIII.

The Use of Decimals in Fellowship	142
The Use of Decimals in Tare and Trett	145
The Use of Decimals in Barter —	149
The Use of Decimals in Gain and Loss	151
The Use of Decimals in Exchange —	152
Table of the Course and Par of Exchange	156 to 158
Table of the Course and Par in Alligation	160
Table of the Course and Par in the Rule of False Position —	166
Tables of the Course and Par in the Rule of Extraction of Roots —	169

C H A P.

The CONTENTS.

C H A P. IX.

The Use of Decimals in Interest	Page 173
The Use of Decimals in Simple	— 174
The Use of Decimals in Compound	176
Table of Time	— — — 201
The Nature, Construction, and Use of the Decimal Tables of Simple Interest	— 203
Decimal <i>Tables</i> of Simple Interest	— 210
The Nature, Construction, and Use of the Decimal Tables of Compound Interest	221
The Decimal Tables of Compound Interest	235 to 258

C H A P. X.

The Use of Decimals in the Arithmetick of Vulgar Fractions	— — 259
The Use of Decimals in the Arithmetick of Duodecimals	— — 267
The Use of Sexagesimals	— — 273

C H A P. XI.

The Use and Management of Decimals (after a new Manner) by Logarithms	— 276
Addition and Subtraction thereby	— 278
To find the Arithmetical Complement	279
To find the Logarithm of any Repeating De- cimal	— — — 280
Multiplication by Logarithms	— 284
Division	— — — 286
The Golden Rule	— — 287
Extraction of Roots	— — 289

C H A P.

The CONTENTS.

C H A P. XII.

The Use of Decimals in Algebra exemplified
in the Solution of thirty four Select, Curious,
and Useful Questions. — Page 290 to 311

C H A P. XIII.

The Use of Decimals in Plain Trigonometry	312
The Use of Decimals in Navigation	318
The Use of Decimals in Fortification	322
The Use of Decimals in Altimetry —	327
The Use of Decimals in Longimetry	331

C H A P. XIV.

The great Use of Decimals in all Kinds of Menfuration, viz. of Superficies —	333
— of Solids — —	343
— of Artificers Work —	354
— of Gauging — —	355
— in Surveying — —	357
A Table of the Logarithms to all Numbers, not exceeding One Thousand, or Four Pla- ces, whether they be Intire, Broken, or Mixt Numbers —	361 to the End.

F I N I S.

BOOKS printed for JOHN NOON, at the White-Hart near Mercer's-Chapel, Cheapſide.

I. **A** Paraphraſe and Notes on the Epiftles of St. Paul to the Colloſſians, Philippians, and Hebrews: after the Manner of Mr. LOCKE. To which are annex'd, Several critical Diſſertations on particular Texts of Scripture. By the late Reverend Mr. JAMES PEIRCE of Exon. The ſecond Edition. price 16s.

II. A Paraphraſe and Notes on the Revelation of St. John; with large Hiſtorical Obſervations, and a Preface giving an Account of the Deſign and Uſe of this Book; with a Scheme and Order of the Prophecies. By MOSES LOWMAN. price 9s.

III. A Paraphraſe with Notes on the Revelation of St. John, in the Manner of Dr. CLARKE; with a Preface concerning the Uſefulneſs and Intelligibility of this Book. By THOMAS PYLE, M. A. Prebendary of the Cathedral Church of Salisbury. price 4s.

IV. A Paraphraſe on the ſeven Catholick Epiftles: after the Manner of Dr. CLARKE on the four Evangelifts. Uſeful for Families. price 3s. 6d.

V. A Paraphraſe on Chriſt's Sermon on the Mount, contain'd in the 5th, 6th, and 7th Chapters of St. Matthew; with proper Soliloquies, at every Period. In a Letter from a Father to his Son. price 1s.

VI. SERMONS on various Subjects, in two Volumes, 8vo. Vol. I. treating, 1. Of the univerſal Senſe of Good and Evil. 2. The Characters of the righteous and good Man compared; or Benevolence the nobleſt Branch of ſocial Virtue. 3. The perfection of the Chriſtian Scheme of Benevolence; in answer to the Objection from its not having particularly recommended private Friendſhip, and the Love of our Country. 5. Of the Image of God in Man; or, The Excellency of human Nature. 5. God not an arbitrary Being. 6. Of the Abuses of Free-thinking. 7. Of Myſteries. 8. Agur's Prayer; or, The middle Condition of Life, generally, the moſt eligible. 9. The Miſchiefs of ſlaviſh Complaiſance, and Cowardice. 10. Rules for the profitable reading the Holy Scriptures. 11. Of Hereſy. 12. Of Schiſm. 13. Of the Pleaſures of a religious Life. 14. Religion founded on Reason, and the Right of private Judgment. 15. The Evidence of a future State, on the Principles of Reason and Revelation, diſtinctly conſider'd. 16. The Nature, Folly, and Danger of ſcoffing at Religion.

VII. Vol. II.] 1. Of moral and natural Evil. 2. Of the true Principle of Virtue. 3. Of the Nature, Divine Original, and Authority of Conſcience. 4. The Plea of an erroneous Conſcience diſtinctly conſidered, and argued. 5. The Influences of the Spirit entirely perſuaſive and moral; and its Fruits of the ſame kind with the Dictates of natural Conſcience. 6. Of Anger. 7. The time when Chriſtianity was made known proved to be the fitteſt, from the State and Circumſtances of the World. 8. The Wiſdom of God in the various Ranks and Subordinations of Human Life.

A 9. The

Books Printed for J. NOON.

9. The Glory of GOD best promoted by the moral Rectitude and Happiness of his Creatures. 10. The Folly of imitating popular and fashionable Vices. 11. The Sublimity and Extent of Christian Morals. 12. Of Sincerity, as opposed to Prejudice. 13. Of the Idea and Improvement of Human Life. 14. The Kingdom of GOD, under the Dispensation of the Gospel, not an external and ritual, but a moral Constitution. 15. Universal Charity the Essence and Life of Religion. 16. The peculiar Guilt and Infamy of prevailing Wickedness in an enlightned and polite Age. 2 vol. price 10s.

VIII. The Usefulness, Truth, and Excellency of the Christian Revelation defended against the Objections contain'd in a late Book, intituled, Christianity as old as the Creation, &c. The third Edition, corrected.

IX. Three Letters to DR. STEBBING, on the Subject of Heresy. price 2 s. 6 d. These three last by the Rev. Mr. JAMES FOSTER.

X. A Free and Impartial Study of the Holy Scriptures recommended: In three Vols. 8vo. Containing, Notes on many peculiar Texts of Scripture, and Discourses on various Subjects; some of which are, viz. Of the Septuagint Version of the Bible, and the Difference between the Citations as they lie in the New Testament, and the original Passages in the Old. Christians not bound by any Authority of the Law of Moses in the Ten Commandments. Of the Soul, its Immortality, Immateriality, &c. with the Impossibility of proving a Future State by the Light of Nature; and of the Place where Good Men shall dwell after the Resurrection. Of our Saviour's Miracles. The Nature of Sacrifices, particularly of the Sacrifice of Christ. The Origin of Evil. The original Meaning of the Ten Commandments. The Lord's Day. Moral and Positive Duties. The Agapæ, or Love Feasts. Circumcision and Baptism. Schism and Heresy. The Restoration of the Jews. The End of the World. price 15 s.

XI. The Laws and Liberties of the Gospel: or, The Duties and Privileges common to all Christians explain'd and recommended, in several practical Discourses. Design'd to promote Christian Truth and Charity, Peace and Unity. By GILBERT MICHELL, M. A. Rector of Bredfall near Derby. price 5 s.

XII. Eight Sermons preach'd on several Occasions. By the late Mr. JAMES BURROUGHS. price 3 s. 6 d.

XIII. Fifteen Sermons on several Occasions. To which is added, a Scripture-Catechism, or the Principles of the Christian Religion laid down in the Words of the Bible. By the late Reverend Mr. JAMES PEIRCE. price 5 s.

XIV. Funeral Discourses: In two Parts. Containing, 1. Consolations on the Death of our Friends. 2. Preparations for our own Death. By WILLIAM HARRIS, D. D. price 5 s.

XV. Bibliotheca Technologica: Or, a philological Library of Literary Arts and Sciences, viz. 1. Theology, or the first Principles of Natural Religion. 2. Ethics, or Morality, the Doctrine of moral Virtues. 3. Christianity, or the Substance of the Christian Religion. Judaism, or the Religion and State of the Jews.

Books Printed for J. Noon.

5. Mahometanism, or the Life, Religion, and Polity of Mahomet. 6. Gentilism, or the Deities and Religion of the Heathen. 7. Mythology, or an Explanation of Fabulous Histories. 8. Grammar and Language, particularly of the English Tongue. 9. Rhetoric and Oratory, or the Art of speaking eloquently. 10. Logic, or the Art of Reasoning and Persuasion. 11. Ontology, or the Science of Being abstractedly consider'd. 12. Poetry, or the Art of making Verses or Poems. 13. Criticism, or the Art of judging well of Men and Things. 14. Geography, or a Description of the World. 15. Chronology, or the Doctrine of Time. 16. History, with the Original of Nations and Kingdoms. 17. Physiology, or Science of Natural Philosophy. 18. Botany, or the Doctrine of Plants and Vegetables. 19. Anatomy, or a Description of the Parts of an Human Body. 20. Pharmacy, or the Art of making Medicines. 21. Medicine, or the Theory of Physick and Diseases. 22. Polity and Oeconomics, or the Doctrine of Society and Government. 23. Jurisprudence, or the Knowledge of Law or Right. 24. Heraldry, or the Art of Blazoning Coat-Armour. 25. Miscellanies, an Account of the Mathematical Arts and Sciences. price 6 s. 6 d.

XVI. The Philosophical Grammar of Experimental Natural Philosophy, in the familiar Way of Dialogue; adapted to the Capacities of Youth, and illustrated with Variety of Copper-plates, Maps, &c. The Second Edition, with large Additions.

XVII. The young Student's Memorial-Book, or Pocket-Library of the Mathematicks; containing for Illustration, above 120 Cuts. Price 3 s.

XVIII. A new and universal System, or Body of Decimal Arithmetick; containing the Doctrine, Application and Use, in all the Parts of Arithmetick, Mathematicks, &c. price 6 s.

XIX. The young Trigonometer's Complete Guide, being plain and spherical Trigonometry made plain and easy. In 2 vols. 8vo. price 10 s. These five last by BENJAMIN MARTIN, Teacher of the Mathematicks, &c.

XX. Novatiani Opera, cum copiosissimis Observationibus & Notis, in quibus totum argumentum auctoris de Regula Fidei ex veterum Patrum Monumentis latè discutitur. Præmittitur Dissertatio de Filii Dei Homœousio, five Coessentialitate uni Deo Patri. Studio JOHANNIS JACKSON, Eccles. Angl. Presb. price 7 s.

XXI. A second Edition, of a Defence of Human Liberty, in answer to the principal Arguments which have been alledged against it, and particularly to CATO's Letters on that Subject. In which Defence, the Opinion of the Ancients, concerning Fate, is also distinctly and largely consider'd. To which is added a Vindication of Human Liberty: In answer to a Dissertation on Liberty and Necessity; Written by A. C. Esq; By JOHN JACKSON Rector of Rossington in the County of York, and Master of Wigston's Hospital in Leicester. price 4 s. 6 d.

Books Printed for J. NOON.

XXII. *CYROPÆDIA*: or, the Institution of *CYRUS*, concerning Religion and Government. Translated from the Greek Original of *XENOPHON*, by the late Honourable *MAURICE ASHLEY*, Esq; (Brother to the late Earl of *SHAFTSBURY*.) Address'd to the Lady *ELIZABETH HARRIS*, in a Dissertation upon the true Liberty of Thinking, in Matters both Ecclesiastical and Civil. By the Translator. In two Vol. 8vo. price 9s.

XXIII. The History of the *SEVARAMBIANS*, a People of the South-Continent: In five Parts. Containing an Account of the Government, Laws, Religion, Manners, and Language of that Nation. Translated from the Memoirs of Captain *SIDEN*, who liv'd near fifteen Years amongst them. price 5s.

XXIV. *ALKIBLA*. A Disquisition upon Worshipping towards the East: In two Parts. Part I. Contains the general Antiquity, the Rise and Reasonableness of this religious Ceremony in the Gentile World: Its early Adoption into the Church of Christ; with a free and impartial Examination of the Reasons assign'd for it by the ancient Fathers. Part II. Contains an historical Account of this Ceremony in the Christian Church, from the primitive to the present Times: With a serious and impartial Examination of the Reasons assigned for the Practice by our *Modern Divines*. To the second Part are prefixed, Some Thoughts by way of Preface, concerning the proper Use of Ridicule in Controversies stil'd Religious. By *WILLIAM ASPLIN*, M. A. Rector of Bowthorp. Price 3 s. 6 d.

XXV. The Nature and Design of Christianity consider'd: In two Discourses. Dis. I. Sheweth, That the Peace and Happiness of this World is the immediate Design of Christianity; with an Address in the close to the Deists, or those who deny the Christian Revelation. Dis. II. Is concerning the Simplicity and Reasonableness of the Christian Institution. Design'd to obviate those Prejudices against it, which are both the most common, and likewise observ'd to have the greatest Influence. By *THOMAS BOTT*, M. A. Rector of Spixwell near Norwich. price 3 s. 6 d.

XXVI. Human Osteogeny explain'd, in two Lectures read in the Anatomical Theatre of the Surgeons of London. In which not only the Beginning and gradual Increase of the Bones of Human Foetus's are described, but also the Nature of Ossification is considered; and the general Notion, that all Bones are formed from Cartilages, is demonstrated to be a Mistake. By *ROBERT NESBITT*, M. D. Fellow of the Royal College of Physicians, and of the Royal Society, and Reader of Anatomy at Surgeons-Hall. price 6s.

XXVII. *Anatomy Epitomized and Illustrated*: Containing, 1. A concise and plain Description of all the Parts of the Human Body; together with their various Uses in the Animal Oeconomy. 2. A large and choice Collection of Sculptures, exhibiting a just Idea of the Figure, Size, Situation, Connection, and Uses of the several Parts of the Body, in seventeen Folio Copper-plates. By *M. N. B. A.* price 6s.

XXVIII. A Paraphrase and critical Commentary on the Prophecy of Joel. By *SAMUEL CHANDLER*. price 4 s.

Books Printed for J. NOON.

XXIX. An Essay towards explaining the History and Revelation of Scripture in their several Periods. To which is added, a Dissertation on the Fall of Man. By JEREMIAH HUNT. D. D. pr. 4 s.

XXX. The plain Account of the Nature and End of the Sacrament of the Lord's-Supper, vindicated from the Misrepresentations and false Reasonings of *Richard Warren*, D. D. and particularly from his abusive Charge of Socinianism; in Remarks on the three Parts of his Answer to that Book, and also on his Appendix. price 2 s. 6 d.

XXXI. The Morality of Religion and the Doctrine of the Sacrament put in a true Light. price 1 s.

XXXII. The Doctrine of Hell-Torments distinctly and impartially discuss'd. price 1 s. The three last by one Author.

XXXIII. An Enquiry into the Law of Nature and Revelation. To which is annexed, A Discourse concerning the Mediator between God and Man. price 1 s. 6 d.

XXXIV. The Argument *à priori* concerning the Existence and Perfections of God, and its Importance to Virtue and True Religion, stated and consider'd. Together with the Difficulties and Objections which have any where occurred; particularly in a Dissertation, by a learned Hand, at the End of Mr. *Law's* Treatise of Space, &c. price 1 s.

XXXV. A Discourse concerning Virtue and Religion: Occasioned by some late Writings. pr. 6 d. The two last writ by one Author.

XXXVI. Divine Benevolence: or an Attempt to prove that the principal End of the Divine Providence and Government is the Happiness of his Creatures. Being an Answer to a Pamphlet, entitled, Divine Rectitude; or an Enquiry concerning the Moral Perfections of the Deity. With the Refutation of the Notions therein advanced concerning Beauty and Order, the Reason of Punishment, and the Necessity of a State of Tryal antecedent to perfect Happiness. price 1 s.

XXXVII. An Introduction to the Doctrine of Fluxions, and Defence of the Mathematicians against the Objections of the Author of the Analyst, so far as they are design'd to affect their general Methods of Reasoning. pr. 1 s. These two last by the same Author.

XXXVIII. An Essay in Vindication of the Use and Advantage of Prayer. Wherein Objections raised from the Attributes and Decrees of God, and the settled Course and Order of Things, are consider'd and answer'd. By the late Rev. Mr. *John Archer*. pr. 6 d.

XXXIX. A Question proposed by the Author of an Address to the Inhabitants of London and Westminster, viz. Whether God is an arbitrary Being and commands only for Commanding's sake, such Things as only oblige by Virtue of the Command, and can oblige no longer, consider'd. price 6 d.

XL. A Discourse of Natural and Reveal'd Religion, and the Relation they bear to each other. By the late Right Hon. the Lord Viscount *Barrington*. price 6 d.

XLI. An Argument to prove the Unity and Perfections of God *à priori*. price 6 d.

XLII.

Books Printed for J. Noon.

XLII. The Principles of Popery schismatical. A Sermon. By *Moses Lowman*. price 6 d.

XLIII. The Period of Human Life not unalterably fix'd, but variable by Men's virtuous or vicious Behaviour, by *Henry Fyfe*, M. A. Vicar of Middleton in Norfolk. price 4 d.

XLIV. A Sermon on the Death of the Queen, with a short Account or Character of her Majesty. By *John Dickenson*, M. A. price 6 d.

XLV. The Acceptableness of Sincerity to God, and the Unprofitableness of a mere speculative Faith to any Man's Salvation. A Sermon. By *Jacob Ball*. price 6 d.

XLVI. The Importance of right Apprehensions of God, with respect to Religion and Virtue, consider'd. By *Jacob Ball*. price 6 d.

XLVII. A Sermon occasion'd by the Death of Mrs. *Mary Wilks*. By the Rev. Mr. *James Foster*. price 6 d.

XLVIII. The Suddenness of Christ's Coming consider'd and improv'd. A Sermon occasion'd by the Death of the Rev. Mr. *Henry Grove*. By *James Strong*. price 6 d.

XLIX. Of the Use of publick Instruction. A Sermon. By *Rich. White*, Vicar of Abbottham, Devon. price 4 d.

L. Reflections on the Duty of Masters, Mistresses and Servants, in which several Irregularities are reprov'd, and certain plain and useful Rules propos'd for promoting the Peace and Tranquility of Families. By *Matthew Randall*. price 6 d.

LI. The Hope of a good Man in the View of Death, and the proper Temper of his Mind with Reference to it; consider'd in a Sermon on the Death of Mrs. *Mary Mottershed*. By *Jer. Tidcomb*. price 6 d.

LII. The wonderful Works of God in the Deep, consider'd and recommended to all the sea-faring Men. A Sermon on the Death of a Sea Captain. By *Jer. Tidcomb*. price 6 d.

LIII. Eight Discourses upon the Mystery of Godliness, from 1 Tim. 3. 16. price 2 s. 6 d.

LIV. The uninterrupted Succession of Bishops, prov'd not necessary to the Conveyance of the Ministerial Office, and the Validity of Ordinances in the Church: Wherein is consider'd the Nature of the Sanhedrim, the Synagogue, and the Rights of Societies, before the Writings of the Sacred Books, and since they were written. price 1 s.

LV. An Essay for allaying the Animosities among *British* Protestants, in a Discourse founded upon the 14th and part of the 15th Chapter of the Epistle to the *Romans*. price 4 d.

LVI. An Historical Account of the several Attempts for a further Reformation of the Establish'd Church of *England*. price 6 d. The four last by the late Rev. Mr. *John Platts*.

LVII. The principal and peculiar Notion advanc'd in a late Book, intitled, *The Religion of Nature delineated; consider'd and refuted*. In a Letter to a Gentleman. price 4 d.

Books Printed for J. NOON.

LVIII. Remarks upon Dr. *Butler's* sixth Chapter of *The Analogy of Religion, &c. Concerning Necessity*; and also upon *The Dissertation of the Nature of Virtue*. By *Philanthropus*. The two last by one Author.

LIX. A Defence of Christianity against the Power of Enthusiasm, in Answer to Mr. *Bradbury*. By *Thomas Morgan*. price 4 d.

LX. *Jesus Christ* the Mediator between God and Man, an Advocate for us with the Father, and a Propitiation for the Sin of the World. price 2 s.

LXI. An Answer to Mr. *Watts* concerning the Trinity. Being a sober Appeal to a *Turk* or an *Indian*. price 2 s.

LXII. A calm Enquiry, whether we have any Warrant from Scripture for addressing ourselves in a way of Prayer or Praise directly to the Holy Spirit. Humbly offer'd to the Consideration of all Christians, particularly of Protestant Dissenters. To which is prefix'd, a Preface to the Rev. Mr. *Barker* of Hackney. By *Martin Tomkins*. price 6 d.

LXIII. The Nature of the Lord's Supper, and the Obligations to it briefly consider'd; with a serious Exhortation to a due Attendance on it: Four Discourses, preach'd by *William Harris*, D. D. price 1 s.

LXIV. A Defence of a Discourse on the Impossibility of proving a future State by the Light of Nature. With an Answer to the Reverend Mr. *Groves's* Thoughts on the same Subject. price 1 s. 6 d.

LXV. A Letter to the Rev. Mr. *Enty*, being a Defence of several Notes and Discourses contain'd in a Book, intitled, A free and impartial Study of the Scriptures recommended. price 6 d.

LXVI. An Answer to Mr. *Mudge's* Sermon, intitled, Liberty. price 6 d.

LXVII. The consistent Christian: Being a Confutation of the Errors advanc'd in Mr. *Chubb's* late Book, entitled, The true Gospel of *Jesus Christ* asserted; relating to the Necessity of Faith, the Nature of the Gospel, the Inspiration of the Apostles, &c. With Remarks on his Dissertation on Providence. price 6 d.

LXVIII. The Immorality of the moral Philosopher and its Vindication; being Answers to a Book, intitled, The moral Philosopher. price 1 s. 6 d.

LXIX. An Essay on the Use of Miracles. Design'd against the Assertion, That they are no proper Proof of a Divine Mission. To which is prefix'd, an Answer to some other Objections against Reveal'd Religion, contain'd in a late Book, intitled, *Christianity as old as the Creation*. price 1 s.

LXX. The Christian Creed, concerning the Son of God as profess'd by those Christians, who are (falsly) call'd *Arians*. price 3 d.

LXXI. The Truth and Importance of the Scripture-Doctrine of the Trinity and Incarnation demonstrated, in a Defence of the late learned Mr. *Peirce's* thirteen Queries, and a Reply to Dr. *W———d*, and a Gentleman's Answers to them; together with a full Confutation of Dr. *Waterland's* late Book of the Importance, &c. price 1 s.

Books Printed for J. Noon.

LXXII. The Reconciler; or an Essay to shew that Christians are much more agreed in their Notions concerning the Holy Trinity than has been commonly represented. With a Reply to Mr. Ball's (of Honiton) Answer to some common Objections. price 6d.

LXXIII. Index Librorum MSS. Græcorum & Versionum Antiquarum Novi Fœderis viri Eruditissimi J. Millius, & L. Kusterus, cum tertia Editione Stephanica contulerunt. price 1s. The ten last writ by Mr. Joseph Hallett, Jun.

LXXIV. Remarks on Dr. Waterland's second Defence. With an Appendix, shewing the true Sense of Creation, Eternity, and Consubstantiality. In a Letter to the Doctor. By Philaethes Cantabrigiensis. price 1s.

LXXV. Farther Remarks on Dr. Waterland's farther Vindication of Christ's Divinity. By Philaethes Cantabrigiensis. price 1s.

LXXVI. A true Narrative of the Controversy concerning the Doctrine of the Trinity; being a Reply to Dr. Berriman's Historical Account. Wherein the Partiality and Mis-representations of that Author are fully shewn. By the Author of the Reply to Dr. Waterland's Defences. price 1s. 6d.

LXXVII. Christian Liberty asserted, and the Scripture-Doctrine of the Trinity vindicated; against a Book written by Dr. Waterland, and entitled, *The Importance of the Doctrine of the Holy Trinity Asserted*, &c. By a Clergyman in the Country. price 2s.

LXXVIII. A Confutation of the Fifth of Mr. Moore's Propositions of Natural and Revealed Religion, which relates to the Doctrine of the Trinity. In a Letter to a Clergyman. To which is added, a Letter to Dr. Waterland. price 1s.

LXXIX. Memoirs on the Life and Writings of Dr. Waterland: being a summary View of the Trinitarian Controversy for twenty Years, between the Dr. and a Clergyman in the Country; wherein, in defence of a Book, entitled, *Christian Liberty Asserted*, &c. in answer to some *Animadversions* upon it, and to a *Defence* of Dr. Waterland's, is shewn the Pravity of the Dr.'s Book, called, *The Importance*, &c. and the Tendency of it to introduce Heresy, Schism, and Persecution into the Church. By a Clergyman. price 2s.

LXXX. An Answer to a Book, entitled, *Things Divine and Supernatural*, conceiv'd by Analogy with Things Natural and Human. price 1s.

LXXXI. A Dissertation on Matter and Spirit, with some Remarks on a Book, entitled, *An Enquiry into the Nature of the Human Soul*. price 1s.

LXXXII. The second Part of the Plea for Human Reason, in answer to a Letter written to the Author of the Plea for Human Reason. price 1s.

LXXXIII. A Narrative of the Case of the Rev. Mr. Jackson, being refused the Sacrament of the Lord's Supper at Bath, by Dr. Coney, Minister of Bath: with some Observations upon it, worthy the Consideration of all Friends to true Religion, and Liberty of Conscience. price 3d. The ten last Pieces were written by a Country Clergyman.

